The Cost Allocation Problem in a Joint Replenishment Transportation Model

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1 Introduction

In today's business world it becomes more prevalent to lease the transportation and storage activities as well as other related services of a supply chain to an external third party logistics (3PL) service provider. The use of 3PL started in the 1980's but has grown significantly in recent years. Today many businesses prefer to lease several of their activities to a single 3PL company by a long-term commitment, see e.g., Leahy et al. (1995). However, as it is well recognized today, mainly large firms as Minnesota Mining & Manufacturing Co. (3M), Eastman Kodak, Dow Chemical, Time Warner and Sears Roebuck are leasing large parts of their logistical activities to 3PL providers. Small companies tend to be more skeptical regarding the advantage of using 3PL, see e.g., Simchi-Levi et al. (2000). One of the main reasons for this reservation of small companies is related to the cost schemes, which often contain some economies of scale benefits that cannot be exploited by small firms.

In the last two decades, the importance of combining forces within a supply chain has been strongly recognized by the OR/MS community as well as by practitioners. As a result, the body of research on joint replenishment problems has flourished. The main emphasis of this research has been on minimizing total system-wide costs. However, once the total cost is minimized, a further question has to be asked, which is how to allocate this cost among the various parties in the supply chain. The cost allocation problem is important for cost accounting purposes as well as for enabling the management to decide upon the profitability of the various entities in the supply chain. The cost allocation scheme should be fair in the sense that no facility would feel as subsidizing the others. Sharkey (1995) provides an excellent review of cost allocation problems in the context of transportation models.

In this research we focus on the cost allocation problem of an infinite-horizon single warehouse joint replenishment model, where n retailers in the set $N = \{1,...,n\}$ lease the transportation of their supplies as well as their storage activities to a 3PL provider. The cost structure considered here is as follows: Each time a delivery is requested by any subset of the retailers, a fixed transportation cost K_0, called major transportation setup cost, is incurred. Moreover, each retailer i is associated with a minor fixed transportation cost K_i that is retailer dependent, and is possibly a function of the distance or the travel time between the warehouse and the retailer. Thus, if a set of retailers S, $\Phi \subset S \subseteq N$, orders simultaneously the transportation cost incurred is

$$K_0 + \sum_{i \in S} K_i \qquad (1)$$

Demands at the retailers are assumed to be deterministic at a constant rate. In addition to the transportation costs, the 3PL provider is paid for holding stocks at depots in the retailers' sites.

The optimization problem associated with the above model is to determine when to place orders for the various retailers, and what are the quantities to ship when replenishments take place. The goal is to minimize the total average-time transportation and storage costs. Had $K_0 = 0$, the problem would be in fact of n independent EOQ (*Economic Order Quantity*) problems. The case where $K_0 > 0$ makes the optimization problem more intriguing, as it calls for coordination of the timing of the various transportation activities for the sake of placing joint replenishments. The model considered here with joint transportation setup cost of the form (1), is known in the literature as the *first order interaction model*, see Federgruen and Zheng (1995). More involved cases are considered in the literature. For example, see Federgruen and Zheng (1992), and Federgruen et al. (1992). We also like to mention Meca et al. (2003) which deal with the special case where the minor transportation setup costs are zero for all retailers in N.

The first order interaction model described above is the simplest model that involves cooperation among the retailers. In spite of its relative simplicity, the structure of optimal policies for this problem is yet unknown, except for the Zero-Inventory-Ordering (ZIO) property, which insures that under any optimal replenishment policy, each retailer orders only when its inventory level is zero. In view of its complexity, practitioners resort to suboptimal policies which are efficient in terms of the computational effort and which have some guaranteed deviation from the optimal average-time total cost. In particular, we refer here to policies in which ordering of retailer i takes place in equi-distant time intervals of length $2^{m_i} B$ for some integer m_i, $1 \le i \le n$, and for some base time-unit B. Such policies are called *power-of-two* (POT) policies. They are known, see Jackson et al. (1985), to yield an average-time total cost which is at most 6% higher than the optimal average-time total cost. By optimizing over B, $B \in [1,2)$, the worst-case gap can be reduced to 2%, see Roundy (1985). Our cost allocation results hold for any fixed value of B, $B \in [1,2)$, and in particular for the optimal one. For the sake of simplicity we assume that time units are scaled so that $B = 1$.

The natural question to be asked after solving for an optimal POT is how the retailers should split the total costs among themselves. Many options exist here. For example, a naïve allocation can be that all pay their holding costs and whenever an order is placed, all ordering retailers pay their minor transportation costs, and share evenly the major transportation cost. This scheme has the advantage of being simple, aesthetic and maybe easy to be argued for on non-theoretical grounds. Yet it is possible that some retailers may feel that they pay more than others towards the common goal of minimizing the total social costs. In fact, they may end up subsidizing the others. Thus, a more systematic approach is needed. We define a cooperative transferable utility game representing the above posed allocation problem and suggest the application of a game-based cost-sharing rule, the core. We show that the resulting game is concave, and we give an example of a core allocation. We also prove that the core contains infinitely many allocations.

Section 2 of this extended abstract contains some notation and preliminaries. Section 3 covers our main results. The proofs can be found in the working paper, see reference.

2 Notation and Preliminaries

Let $N = \{1,2,...,n\}$ the set of retailers. K_0 is the major transportation setup cost. Retailer i, $1 \le i \le n$, is associated with the following parameters: K_i - its minor transportation setup cost; h_i - its holding cost rate, and d_i - its demand rate; We also let $g_i = h_i d_i / 2$ to be its holding cost parameter. Finally, we assume zero lead times. Identical lead times can be handled similarly. W.l.o.g. we assume that the retailers in N are indexed in a non-decreasing order of K_i / g_i, i.e., $K_1 / g_1 \leq K_2 / g_2 \leq ... \leq K_n / g_n$. For ease of notation let $i_0 = 0$, $g_0 = 0$ and $K_{n+1} / g_{n+1} = \infty$. For any retailer i, $1 \leq i \leq n+1$, let $\tau_i^{'} = \sqrt{K_i / g_i}$ and $\tau_i = \sqrt{(K_0 + K_i) / g_i}$. Thus, the sequence $\tau_i^{'}$ is non-decreasing in i. Let also, $T_i^{'}$ and T_i be the POT rounding-off of $\tau_i^{'}$ and τ_i, respectively. For a subset $S = \{i_1, i_2, ..., i_s\} \subseteq N$ of s retailers, we let $K^0(S) = K_0 + \sum_{i \in S} K_i$, and $G(S) = \sum_{i \in S} g_i$. Also let $i^*(S) = \arg\max \left\{ k, 1 \leq k \leq s \;\middle|\; \frac{\Sigma 0 \leq j \leq k \, K_{i_j}}{\Sigma 0 \leq j \leq k \, g_{i_j}} \geq \frac{K_{i_k}}{g_{i_k}} \right\}$. Note that $i^*(S) \geq 1$, and let $S^0 = \{i_1, ..., i^*(S)\}$. From Jackson et al. (1985) we learn that the optimal POT policy for the retailers in S is as follows: The retailers in S^0 order simultaneously every 2^{m_0} time units, where 2^{m_0} is the integer POT closest to $\tau_{\min}(S)$, where $\tau_{\min}(S) = \sqrt{K^0(S^0) / G(S^0)}$, i.e., m_0 is the unique integer that satisfies the inequality $2^{m_0 - 0.5} \leq \tau_{\min}(S) < 2^{m_0 + 0.5}$. We denote the POT reorder interval of S^0, namely 2^{m_0} by $T_{\min}(S)$. If $i^*(S) < i_s$, then each of the retailers $i_j \in S \setminus S^0$ orders at most as frequently as the set S^0; Indeed, each time such a retailer orders, also the set S^0 orders (but not the other way around). In fact, a retailer i_j, $i_j \in S \setminus S^0$ orders at times as prescribed by its individual EOQ model, i.e., at $\tau_{i_j}^{'}$, rounded to the closest integer POT, named $T_{i_j}^{'}$. The optimal average-time total costs of S, under the restriction to POT policies, namely $\nu(S)$, equals the optimal objective value of the following integer program named $(JRPPT(S))$: $Min \sum_{j=0}^{s} (\frac{K_{i_j}}{t_{i_j}} + g_{i_j} t_{i_j})$ subject to the following constraints that should hold for $0 \le j \le s$: $t_{i_j} = 2^{m_{i_j}}$, $m_{i_j} \ge m_0$ and m_{i_j} is integer. $(JRPPT(S))$ is solved in Jackson et al. (1985) and shown to have an optimal objective value of the form $\nu(S) = \frac{K^0(S^0)}{T_{\min}(S)} + T_{\min}(S)G(S^0) + \sum_{i_j \in S \setminus S^0} (\frac{K_{i_j}}{T'_{i_j}} + g_{i_j} T'_{i_j})$. As $\nu(\Phi) = 0$, and $\nu(S)$ is a real number, the pair (N, ν) defines a cooperative game with transferable utility. Moreover, as for any coalition S, $\Phi \subseteq S \subseteq N$, $\nu(S) + \nu(N \setminus S) \ge \nu(N)$, the formation of the grand coalition is a natural outcome from a bargaining process. The question we pose is how to allocate the cost $\nu(N)$ among the retailers. In other words, we look for a cost sharing (or Pareto efficient) solution concept. For this sake we refer to the following definitions. A game (N, ν) is said to be *concave* if the following property holds:

$$\nu(R \cup \{\ell\}) - \nu(R) \ge \nu(S \cup \{\ell\}) - \nu(S), \quad \text{for any} \quad R \subset S \subset N, \quad \ell \in N \setminus S. \qquad (2)$$

A vector $x \in R^n$ is said to be a *core allocation* for the game (N, ν), if $\sum_{i \in N} x_i = \nu(N)$ and if for any set of retailers S with $\Phi \subseteq S \subseteq N$, $\sum_{i \in S} x_i \le \nu(S)$. A game is called *balanced* if its core is not empty and it is called *totally balanced* if all the games with the same characteristic function but restricted to subsets of players, are balanced too. It is well known that a concave game is totally balanced. In this research we show that (N, ν) is a concave game and we present a core allocation. We also prove that the core contains infinitely many allocations.

3 Our results

Theorem 1: *the transferable utility cooperative game with* N *as its set of players and with* $v(S)$, $\Phi \subseteq S \subseteq N$, *as its characteristic function, is concave. In particular it is totally balanced.*

We next describe a core allocation, which we find appealing. Establishing that the suggested allocation is indeed a core allocation can be considered as an alternative proof for the balancedness of the game. Yet recall that Theorem 1 says more than just balancedness. Let

$$\tau^2 = \tau^2_{\min}(N) = \frac{K^0(N^0)}{G(N^0)} \qquad (3)$$

and define θ_j as follows: $\theta_j = \dfrac{g_j \tau^2 - K_j}{K_0}$ for $j \in N^0$, and $\theta_j = 0$ for $j \notin N^0$. (4)

The proposed cost allocation x_j for $j \in N$ is the following:

$$x_j = \frac{\theta_j K_0 + K_j}{T_{\min}(N)} + g_j T_{\min}(N) \text{ for } j \in N^0 \textit{ and } x_j = \frac{K_j}{T_j^{'}} + g_j T_j^{'} \text{ for } j \notin N^0 \quad (5)$$

Theorem 2: *The cost allocation specified in (3)-(5) is a core allocation, under which all retailers pay their own minor transportation setup cost and holding cost and each retailer in* N^0, *pays part of the major transportations setup cost. Moreover, the allocation of the major transportation setup cost is non-increasing in* j, $j \in N$. *In particular, retailers in* N^0 *do not pay anything towards the major transportation setup cost.*

Theorem 3: *There are infinitely many core allocations.*

On the face of it, the core allocation proposed in (3)-(5) and the ones constructed in the proof of Theorem 3, see paper, suffer from two drawbacks. The first is that the set of retailers $N \setminus N^0$ seems not to pay its fair-share of the major transportations setup cost. In fact, as it pays (almost) nothing against K_0, it can be looked at as a set of free riders. The second possible drawback is the fact that each retailer pays the direct holding cost it inflicts under the prescribed policy. This can be seen as unfair by the retailers in N^0 since their actual replenishment interval $T_{\min}(N)$ might be significantly larger than their unconstrained interval. As a result the retailers in N^0 may pay a greater holding cost than what they would have paid had $K_0 = 0$.

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