Planning Lines with Minimal Transfers - A Dantzig-Wolfe Approach

Anita Schöbel* Susanne Scholl*

*University of Kaiserslautern
Postfach 3049
67653 Kaiserslautern, Germany
{schoebel,sscholl}@mathematik.uni-kl.de

1 Motivation and related literature

In the strategic planning process of a public transportation company one important step is to find a suitable line concept, i.e. to define the routes of the bus or railway lines.

Given a public transportation network $PTN = (S, E)$ with its set of stations S and its set of direct connections E, a *line* is defined as a path in this network. The *line concept* is the set of lines offered by the public transportation company.

The line planning problem has been well studied in the literature. For an early contribution we refer to Dienst (1978). The more recent models can be classified roughly into two types of models. In a *cost-oriented approach* the goal is to find a line concept serving all customers and minimizing the costs for the public transportation company. The basic cost model has been suggested in Claessens et al.(1998) and Goosens et al.(2001); recently Goosens et al.(2002) takes also into account different types of vehicles simultaneously.

A new approach is to take into account that the behavior of the customers depends on the design of the lines. A first model including such demand changes was treated with simulated annealing in cooperation with *Deutsche Bahn*, see Klingele (2000) and Schmidt (2001).

On the other hand, in the *direct travelers approach* by Bussieck et al.(1996) and Bussieck (1998) the goal is to maximize the number of direct customers (i.e. customers that need not change the line to reach their destination), given upper and lower bounds on the allowed frequencies on each edge.

Although the latter model is a customer-oriented approach it maximizes the amount of one group of customers but without considering the remaining ones which might have very many transfers on their trips. It also does not take into account the travel times for the customers: Sometimes it is preferable to have a transfer but reach the destination earlier instead of sitting in the same line for the whole trip but having a large detour. In this paper we develop a new model taking into account these points. Our model allows to consider the sum over all travel times over all customers including penalties for the transfers needed.

The remainder of the paper is organized as follows. In Section 2 we introduce our model and point out variations of the objective. Then we describe our solution approach which is based on a Dantzig-Wolfe decomposition in Section 3 and indicate the real-world application on which we are applying our model.

2 The model

A *public transportation network* is a finite, undirected graph $PTN = (S, E)$ with a node set S representing stops or stations, and an edge set E, where each edge $\{u, v\}$ indicates that there exists a direct ride from station u to station v (i.e. a ride that does not pass any other station in between).

We assume the PTN as already given and fixed. We further assume that a *line pool* $\mathcal{L}$ is given, consisting of a set of paths in the PTN. Each line $l \in \mathcal{L}$ is specified by a sequence of stations. Given a station $s \in S$ we furthermore define

$$\mathcal{L}(s) = \{l \in \mathcal{L} : s \in l\}$$

as the set of all lines passing through s.

The line planning problem is to choose a "good" subset of lines $L \in \mathcal{L}$, which is then called the *line concept.*
Let $\mathcal{R} \subseteq S \times S$ denote the set of all origin-destination (OD) pairs (s, t), where w_{st} is the number of customers wishing to travel from station s to station t.

For line planning we use the PTN to construct a directed graph, the so-called *change&go-network* $G_{CG} = (\mathcal{V}, \mathcal{E})$ as follows: We extend the set S of stations to a set $\mathcal{V}$ of nodes with nodes representing either station-line-pairs (change&go nodes: $\mathcal{V}_{CG}$) or the start and end points of the customers (origin-destination nodes: $\mathcal{V}_{OD}$), i.e. $\mathcal{V} := \mathcal{V}_{CG} \cup \mathcal{V}_{OD}$ with

- $\mathcal{V}_{CG} := \{(s, l) \in S \times \mathcal{L} : l \in \mathcal{L}(s)\}$ (set of all station-line-pairs)
- $\mathcal{V}_{OD} := \{(s, 0) : (s, t) \in \mathcal{R} \text{or} (t, s) \in \mathcal{R}\}$ (origin-destination-nodes)

The new set of edges $\mathcal{E}$ consists of edges between nodes of the same stations (representing getting in or out of a vehicle or changing a line) and edges between nodes of the same line (representing driving on a line):

$\mathcal{E} := \mathcal{E}_{change} \cup \mathcal{E}_{OD} \cup \mathcal{E}_{go}$ with

- $\mathcal{E}_{change} := \{((s, l_1), (s, l_2)) \in \mathcal{V}_{CG} \times \mathcal{V}_{CG}\}$ (changing edges)
- $\mathcal{E}_l := \{((s, l), (s', l) \in \mathcal{V}_{CG} \times \mathcal{V}_{CG} : (s, s') \in \mathcal{E}\}$ (driving edges of line $l \in \mathcal{L}$)
- $\mathcal{E}_{go} := \bigcup_{l \in \mathcal{L}} \mathcal{E}_l$ (driving edges)

- $\mathcal{E}_{OD} := \{((s,0),(s,l)) \in \mathcal{V}_{OD} \times \mathcal{V}_{CG} \text{and} ((t,l),(t.0)) \in \mathcal{V}_{CG} \times \mathcal{V}_{OD} : (s,t) \in \mathcal{R}\}$ (origin-destination-edges)

We define weights on all edges $e \in \mathcal{E}$ of the change&go-network representing the inconvenience customers have by using the edge. Then, for a single OD-pair we can determine the lines the customer is likely to use by calculating a shortest path in the change&go network. Therefore the choice of the edge costs c_e is very important.
Some examples:

1. Customers only count transfers:

$$c_e = \begin{cases} 1 & : \quad e \in \mathcal{E}_{change} \\ 0 & : \quad \text{else} \end{cases}$$

   Note that in this case, it is possible to shrink the change&go-network to a network with $|\mathcal{L}| + |S|$ nodes and $|\mathcal{E}_{change}| + |\mathcal{E}_{OD}|$ edges.

2. Real travel time:

$$c_e = \begin{cases} 0 & : \quad e \in \mathcal{E}_{OD} \\ \text{travel time in minutes} & : \quad e \in \mathcal{E}_{go} \\ \text{time needed for changing platform} & : \quad e \in \mathcal{E}_{change} \end{cases}$$

It often is reasonable to make transfers more inconvenient by increasing c_e for all $e \in \mathcal{E}_{change}$ in the real travel time model.
Other combinations and variations are possible.

Since we assume that customers prefer shortest paths according to the weights c_e we need an implicit calculation of shortest paths within our model. This is obtained by solving the following network flow problem for each origin-destination pair $(s,t) \in \mathcal{R}$.

$$\theta x_{st} = b_{st},$$

where $\theta \in \mathbb{Z}^{|\mathcal{V}| \times |\mathcal{E}|}$ is the node-arc-incidence matrix of the G_{CG}, $b_{st} \in \mathbb{Z}^{|\mathcal{V}|}$ is defined by

$$b_{st}^i = \begin{cases} 1 & : \quad i = (s,0) \\ -1 & : \quad i = (t,0) \\ 0 & : \quad \text{else} \end{cases}$$

and a variable $x_{st}^e = 1$ if and only if edge e is used on a shortest path from node $(s,0)$ to $(t,0)$ in G_{CG}. To specify the lines in the line concept we introduce for each line $l \in \mathcal{L}$ a variable $y_l \in \mathbb{B} = \{0,1\}$ which is set to 1 if and only if line l is chosen to be in the line concept. Our model, *Line Planning with Minimal Travel Times* (LPMT) can now be presented.

(LPMT)

$$\min \sum_{(s,t)\in\mathcal{R}} \sum_{e\in\mathcal{E}} w_{st}\ c_e\ x_{st}^e$$

$$
\begin{array}{lll}
s.t. & \sum_{(s,t)\in\mathcal{R}} \sum_{e\in\mathcal{E}_l} x_{st}^e \leq y_l M_l & \forall\, l \in \mathcal{L} \quad (1)\\
& \theta x_{st} = b_{st} & \forall\, (s,t) \in \mathcal{R} \quad (2)\\
& \sum_{l\in\mathcal{L}} C_l y_l \leq B & \quad (3)\\
& x_{st}^e, y_l \in \mathbb{B} & \forall\, (s,t)\in\mathcal{R}, e\in\mathcal{E}, l\in\mathcal{L} \quad (4)
\end{array}
$$

Constraint (1) makes sure that a line must be included in the line concept if the line is used by some OD-pair, where M_l is a sufficiently large number, at least bigger than the number of edges of the line l. Constraint (2) forces that customers use shortest paths according to the weights c_e.

Note that so far the best line concept from a customer-oriented point of view would be to introduce all lines of the line pool. This is certainly no option for a public transportation company, since running a line is costly. Let C_l be an estimation of the costs which occur if line l is chosen and let B be the budget the public transportation company is willing to spend. Then the budget constraint (3) takes the economic aspects into account.

The objective function we use is customer-oriented. We allow to specify some edge cost c_e for each edge in the change&go- network and for each OD-pair $(s,t) \in \mathcal{R}$ we sum up the costs $\sum_{e\in\mathcal{E}} w_{st}\ c_e\ x_{st}^e$ of a shortest path from s to t. Adding over all $(s,t) \in \mathcal{R}$ means that we minimize the average costs of the customers. There are various possibilities to choose c_e. Some of them have been mentioned above.

3 Dantzig-Wolfe Decomposition

The line planning problem introduced in Section 2 is NP-hard and moreover in real-world instances, gets huge (see Section 4). But fortunately (LPMT) has an easy block structure with only a few coupling constraints and all blocks (except the single budget constraint) totally unimodular. This structure can be utilized to set up a Dantzig-Wolfe decomposition.

We now present the formulation of the Master LP and of the corresponding subproblems. For further details on the algorithm the reader is referred to Dantzig and Wolfe(1960).

The block structure of the model is shown in the following reformulation.

$\min \sum_{(s,t)\in\mathcal{R}} \sum_{e\in\mathcal{E}} w_{st} c_e x_{st}^e$

$\boxed{\sum_{(s,t)\in\mathcal{R}} \sum_{e\in l} x_{st}^e \leq y_l M}$ coupling constraints

$\boxed{X^{s_1,t_1}}$

$\boxed{X^{s_2,t_2}}$

$\ddots$

$\boxed{X^{s_r,t_r}}$

$\boxed{Y}$

with $|\mathcal{R}| + 1$ blocks:

$X^{st} := \{x_{st} \in \mathbb{Z}^{|\mathcal{E}|} : \theta x_{st} = b_{st}, 0 \leq x_{st}^e \leq 1, \ \forall\ e \in \mathcal{E}\}$

$Y := \{y \in \mathbb{Z}^{|\mathcal{L}|} : C^T y \leq B, 0 \leq y_l \leq 1, \ \forall\ l \in \mathcal{L}\}$
and $|\mathcal{L}|$ coupling constraints:

$$\sum_{(s,t)\in\mathcal{R}} A_X \ x_{st} - A_Y \ y \leq 0$$

with coefficient matrix $(A_X|\ldots|A_X|A_Y)$ of the coupling constraints where A_X is an $|\mathcal{L}| \times |\mathcal{E}|$ matrix given by elements $a_{le} = 1$, if $e \in \mathcal{E}_l$, zero otherwise. It is equal for each OD-pair.

A_Y is an $|\mathcal{L}| \times |\mathcal{L}|$ diagonal matrix containing M_l as its lth diagonal element.

With the weight-cost-constants $c_{st}^e := w_{st} c_e$ and the $|\mathcal{L}|$-vector v as slack variable we get the following master LP:

(Master)

$$z = \min \sum_{(s,t)\in\mathcal{R}} \sum_i (c_{st}\ x_{st}^{(i)})\alpha_{st}^i$$

$$\begin{array}{llr} s.t. & \sum_{(s,t)\in\mathcal{R}} \sum_i (A_X\ x_{st}^{(i)})\alpha_{st}^i - \sum_i (A_Y\ y^{(i)})\beta^i + Iv = 0 & (1) \\ & \sum_i \alpha_{st}^i = 1 & \forall\ (s,t) \in \mathcal{R} \quad (2) \\ & \sum_i \beta^i = 1 & (3) \\ & \alpha_{st}^i, \beta^i \leq 1 & \forall\ (s,t) \in \mathcal{R} \quad (4) \\ & v_l, \alpha_{st}^i, \beta^i \geq 0 & \end{array}$$

where the $x_{st}^{(i)}, y^{(i)}$ are the extreme points of X^{st} and Y.
The subproblems of the X^{st}-blocks are

$$z = \min(c_{st} - \pi A_X) x_{st} - \mu_{st}$$

$$\text{s.t.} \quad x_{st} \in X^{st}$$

and the subproblem of the Y-block is

$$z = \min(-\pi A_Y) y - \mu_{00}$$

$$\text{s.t.} \quad y_l \in Y$$

where $\{\pi_i\}_{i\in\mathcal{L}}$ are the dual variables of the coupling constraints, $\{\mu_{st}\}_{(s,t)\in\mathcal{R}}$ are the dual variables of the alpha convexity constraints and μ_{00} is the dual variable of the beta convexity constraint.

4 Real-world application

Our approach is currently tested on instances of the long distance trains of the German railway network. The line pool we use was generated by German railway (DB). The given PTN consists of a line pool of 423 lines, 35322 OD-pairs, 233 stations and 319 tracks.

This leads to a change&go-network with 6705 nodes, 343271 edges, $2.42 \cdot 10^{10}$ variables and 236834434 constraints. We implemented our model using Xpress MP 2003. Numerical results will be presented.

In the line planning models known in the literature lines are mainly computed together with frequencies. The inclusion of frequencies into the (LPMT) is current research.

5 Literature

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