Integrated Bus and Driver Scheduling

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1 Introduction

For several decades, Operations Research has been successful in solving a wide variety of optimization problems in public transit (see Desaulniers and Hickman, 2003). Several commercial software packages based on mathematical programming techniques have been designed for and used by transit agencies to help in planning and running their operations. Among the problems faced by such agencies, operational planning deals with how the operations should be conducted to offer the proposed service at minimum cost. They include problems such as bus scheduling, driver scheduling, bus parking and dispatching in garages, and maintenance scheduling.

In general, bus scheduling is performed before driver scheduling in the operational planning process of a public transit agency. On the one hand, since driver relief opportunities are numerous in most contexts, an efficient driver schedule can often be obtained from a near-optimal bus schedule to yield an overall high-quality solution. On the other hand, when these relief opportunities are rare, as for a line-by-line scheduling process, a very efficient bus schedule may lead to a poor or even an infeasible driver schedule. Integrating bus and driver scheduling is therefore essential in these situations, and research on this topic is presented in this paper.

2 Problem Description

Let T be a set on *n* timetabled trips where trip $i \in T$ starts at time s_i and ends at time e_i. These trips are qualified as *active* since passengers travel along them. Denote by τ_{ij} the travel time between the end location of trip *i* and the start location of trip *j* and assume that this travel time is the same for all buses. Two trips *i* and *j* are said to be compatible if and only if they can be covered consecutively by the same bus (*j* immediately following *i*), that is, if and only if $e_i + \tau_{ij} \leq s_j$. The travel between two such trips is called a *deadhead trip* since there are no passengers onboard.

Let *K* be the set of *m* depots housing the buses that must be assigned to cover the active trips. Depot $k \in K$ manages v^k identical buses which must start and end their schedule at this depot. A bus leaving a depot to reach the start location of an active trip is said to be performing a *pull-out trip*, while it performs a *pull-in trip* when it returns to the depot from the end location of an active trip. A feasible schedule for a bus housed at depot *k* is composed of a pull-out trip starting at *k*, a sequence of active trips separated by deadhead trips, and a pull-in trip ending at *k*. A deadhead trip that involves a long waiting time before the start of the next active trip is often replaced by a pull-in trip, an idle period at the depot, and a pull-out trip. The bus schedules are seen as sequences of *vehicle blocks*, where each block consists of a sequence of trips that starts and ends at the same depot without returning to it in the middle of the sequence. Given that a cost is incurred each time that a bus performs an activity, the bus scheduling problem can be defined as the problem of finding a set of feasible bus schedules such that each active trip $i \in T$ is covered by exactly one schedule, at most v^k buses are available at each depot $k \in K$, and the sum of the schedule costs is minimized. Note that the active trips bear no cost since they represent a fixed quantity for any feasible solution. Note also that a fixed cost can be added to the pull-out or the pull-in trip costs.

The driver scheduling problem is separable by depot and consists of determining the work days, also called *duties*, of the drivers based at a depot in order to cover all the vehicle blocks assigned to this depot. Since a driver exchange can occur at various points along a vehicle block, all blocks are divided into a sequence of *segments* according to these *relief points*. The consecutive segments along a block assigned to the same driver are collectively called a *piece of work*. Duties are therefore composed of pieces of work that are usually separated by breaks. Different duty types can be considered. These may be dissimilar, for instance, in terms of the number of pieces of work they can contain and their possible starting times and durations. In particular, there exist straight duties that contain a single piece of work, and split duties containing two pieces of work.

Driver duties are subject to a wide variety of safety regulations and collective agreements rules such as a maximum duty spread, a maximum duration of a piece of work, and a predefined time interval in which a break must be awarded. These rules vary according to the duty type. In general, one first seeks to minimize the total number of duties and second the total number of worked hours. In summary, the driver scheduling problem can be stated as follows: Given the segments of a set of vehicle blocks, find a set of valid duties that covers all these segments and minimizes total cost.

Considering the above two problems together we obtain the *integrated bus and driver scheduling problem* which can be stated as follows: Given a set of timetabled trips and a fleet of buses assigned to several depots, find minimum-cost blocks and valid driver duties such that each active trip is covered by one block, each active trip segment is covered by one duty, and each inactive trip (e.g., deadhead, pull-in, and pull-out trips) used in the bus schedule is also covered by one duty. Each block must start and end at the same depot and driver duties must comply with a set of work rules.

3 Solution Approach

Haase, Desaulniers, and Desrosiers (2001) proposed a set partitioning formulation of the above integrated problem where the objective function minimizes the total number of duties and buses. The model involves only duty variables and one bus counter variable. Bus-count constraints, similar to the plane-count constraints of Klabjan et al. (2002) are considered in the model. These constraints provide lower bounds on the number of buses required at specific times of the horizon, namely each time that a bus can leave the depot to reach the beginning location of an active trip just in time. Solving this model provides optimal duties and ensures that an optimal bus schedule can be obtained *a posteriori* using a simple polynomial-time procedure.

In this paper we consider an objective function that solely minimizes the total number of duties. Furthermore, constraints for counting the number of buses are not invoked. Nevertheless, as buses may idle in a parking lot in the middle of the day, one must ensure that that they will always return to their depot at the end of the day. Hence, constraints similar to those used to count the buses are imposed.

For this version of the problem, we present computational experiments for the single depot version of the above formulation. We use a column generation solution approach to solve the linear relaxation of the model. We observe that the two-step procedure proposed by Freling, Wagelmans and Paixão (1999) and Freling, Huisman and Wagelmans (2003) to generate the duty columns is much more efficient than the direct approach. At any column generation iteration, a constrained shortest path subproblem is solved as follows: First, pieces of work are generated by solving an all-pairs shortest path problem. Second, these pieces of work are combined to form valid duties. In the first step, a time discretization is used to eliminate the resource constraints even though this increases the number of shortest path problems to solve. A partial pricing strategy is used for overcoming this drawback (see Desaluniers, Desrosiers, and Solomon, 2002). Finally, we explore recent stabilization strategies that incorporate dual information at the beginning of the process to accelerate the overall solution approach (see du Merle, Villeneuve, Desrosiers, and Hansen, 1999). Overall, the approach is heuristic since we minimize the tailing-off effect of column generation and use a heuristic branching strategy. We report on tests conducted on data provided by GIRO Inc. for a single bus line of the City of Madrid. The largest of these problems involves 463 trips and 926 trip segments.

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