Hierarchical Nonlinear Model-Predictive Ramp Metering Control for Freeway Networks

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1 Introduction

Ramp metering control is one of the most efficient freeway network traffic control measures that, if properly applied, may result in significant amelioration of traffic conditions. Ramp metering control strategies are divided into local and coordinated strategies. Local on-ramp metering is based on information collected from sensors located in the vicinity of the controlled on-ramp. Coordinated strategies take under consideration measurements taken from a region covering the entire network, and control decisions for each metered on-ramp are coordinated towards a common overall goal. A more powerful design approach is based on the combination of the coordinated and local strategies within a hierarchical control structure, as has been suggested in (Papageorgiou, 1984). This paper presents the combination of the optimal coordinated model-based ramp metering control strategy AMOC (Advanced Motorway Optimal Control), see (Kotsialos *et al.*, 2002b), with the well-known local feedback ramp metering strategy ALINEA and a variation of it, see (Smaragdis and Papageorgiou, 2003). The AMOC strategy considers the coordinated ramp metering problem as a finite horizon constrained nonlinear discrete-time optimal control problem. The ALINEA strategy and its variation flow-based ALINEA, are local feedback regulators based on downstream occupancy or density and flow measurements, respectively. Based on the general receding-horizon model predictive control approach, AMOC, ALINEA and flow-based ALINEA are combined in a single hierarchical control structure. In this paper this structure is described, and results from its simulated application to the Amsterdam ring-road are presented.

2 Local ramp metering feedback regulators

In this section the local ramp metering regulators ALINEA and flow-based ALINEA will be described briefly, see (Smaragdis and Papageorgiou, 2003) for details. Figure 1 depicts a

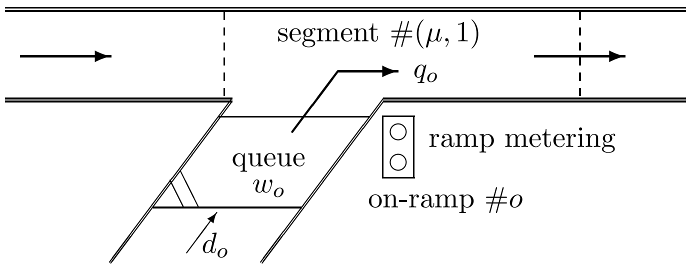


Figure 1: A controlled on-ramp.

controlled on-ramp o receiving traffic demand d_o which is added to the queue w_o. This queue feeds the freeway with a flow q_o, which is controlled by the traffic light adjacent to the on-ramp.

The downstream freeway link μ is characterized by its critical density $\rho_{\mu,cr}$ measured in vehicles/km/lane. ALINEA aims at driving and maintaining the density $\rho_{\mu,1}$ of the first segment $(\mu,1)$ of link μ at near the predetermined set-point $\tilde{\rho}_{\mu,1}$, by regulating the on-ramp's inflow to the freeway q_o. This is done according to the following feedback control law,

$$q_o^r(k_c) = q_o^r(k_c-1) + K_A\left[\tilde{\rho}_{\mu,1} - \rho_{\mu,1}(k_c-1)\right] \tag{1}$$

where K_A is the feedback gain and $k_c = 0,1,\ldots$ is the discrete time index of control application related to the control sample time T_c (i.e., we have $t = k_c \cdot T_c$ for the time t).

Flow-based ALINEA aims at driving the flow of segment $(\mu,1)$ at the predetermined set-point $\tilde{q}_{\mu,1}$ by regulating the on-ramp's inflow to the freeway according to the feedback control law

$$q_o^r(k_c) = q_o^r(k_c-1) + K_F\left[\tilde{q}_{\mu,1} - q_{\mu,1}(k_c-1)\right] \tag{2}$$

where K_F the feedback gain.

In order to avoid the creation of large queues, a queue control policy is employed in conjuction with either of the two local metering strategies. Let $w_{o,\max}$ denote the maximum allowed queue in origin o. Then the queue control law takes the form

$$q_o^w(k_c) = -\frac{1}{T_c}\left[w_{o,\max} - w(k_c)\right] + d_o(k_c-1). \tag{3}$$

The on-ramp outflow is then $q_o(k_c) := \max\left\{q_o^r(k_c), q_o^w(k_c)\right\}$.

3 The AMOC strategy

The AMOC control strategy, (Kotsialos *et al.*, 2002b), (Kotsialos and Papageorgiou, 2001), considers the problem of optimal coordinated ramp metering control for a freeway network as a constrained discrete-time nonlinear optimal control problem. In general terms such a problem is formulated as follows,

$$J = \vartheta\left[K\right] + \sum_{k=0}^{K-1} \varphi\left[\mathbf{x}(k), \mathbf{u}(k), \mathbf{d}(k)\right] \tag{4}$$

subject to

$$\mathbf{x}(k+1) = \mathbf{f}\left[\mathbf{x}(k), \mathbf{u}(k), \mathbf{d}(k)\right], \ \mathbf{x}(0) = \mathbf{x}_0 \tag{5}$$

$$u_{i,\min} \leq u_i(k) \leq u_{i,\max} \ \forall i = 1, \ldots, m \tag{6}$$

where k is the model discrete time index (related to the model time-step T), K is the considered time horizon, $\mathbf{x} \in \Re^n$ is the state vector, $\mathbf{u} \in \Re^m$ is the vector of control variables, $\mathbf{d}$ is the vector of external disturbances and ϑ, φ, $\mathbf{f}$ are arbitrary, twice differentiable, nonlinear functions.

For the modeling of the traffic flow process, the freeway network is divided into links with homogeneous characteristics and each link is divided into segments. The traffic conditions in segment i of link m at time $t = kT$ are macroscopically described by the traffic density $\rho_{m,i}(k)$ (veh/km/lane), the mean speed $v_{m,i}(k)$ (km/h) and the traffic flow or volume $q_{m,i}(k)$ (veh/h). The traffic conditions at each origin (normal on-ramps and motorway ramps) o, are described by the origin queues $w_o(k)$ (veh). Links may merge or diverge at network nodes which are modelled by use of suitable static equations reflecting the corresponding traffic flow interactions. As a result, the state vector of the freeway network traffic flow process consists of the density and mean speed of every freeway segment and the queue of every origin, i.e., $\mathbf{x} = \left[\rho_{1,1} v_{1,1} \ldots \rho_{1,N_1} v_{1,N_1} \ldots \rho_{M,1} v_{M,1} \ldots \rho_{M,N_M} v_{M,N_M} w_1 \ldots w_O\right]^T$, where N_m is the number of segments of freeway link m, M is the total number of freeway links in the network and O is the number of origins.

The control vector consists of the inflows q_o (veh/hour), $o = 1, \ldots, O_c$, $O_c \leq O$, of every controlled origin. Actually, the control is modeled through the notion of the ramp metering rate, but for the sake of simplicity and without losing anything substantial in our exposition, we consider that q_o is the control variable. As a result, the control vector takes the form $\mathbf{u} = \left[q_1 \ldots q_{O_c}\right]^T$, see (Kotsialos and Papageorgiou, 2001) for details. Equation (6) corresponds to the minimum on-ramp outflow, chosen by the site operators, and the maximum possible ramp outflow.

The disturbance vector $\mathbf{d}$ consists of the traffic demand d_o at every origin o and the turning rates β_n^m at every node n, $n = 1, \ldots, B$, of the freeway with an off-ramp, or freeway bifurcation, with m considered as the main outlink of n by convention. β_n^m is the percentage of the flow at junction n that chooses to follow the main outlink m. This means that the disturbance vector is organized as $\mathbf{d} = \left[d_1 \ldots d_O \beta_1^{m_1} \ldots \beta_B^{m_B}\right]^T$. Additional information could be included in the disturbance vector, such as the weather conditions and their effect on the traffic process parameters or the existence of an incident with its characteristics (location, severity, etc.). These disturbances can be accommodated appropriately by changing certain model parameters.

The process model (5) used for AMOC is the METANET model, see (Messmer and Papageorgiou, 1990), which has the required form (5).

The chosen cost criterion J is the Total Time Spent (TTS) including additional terms that penalize large oscillations of the control variables and deviations from possible maximum ramp queue constraints. The minimization of the TTS increases the system's throughput, sustains its operation at a high level near capacity, and results in fair ramp metering policies. A feasible-direction algorithm is used for the numerical solution of the formulated problem, see (Papageorgiou and Marinaki, 1995). This algorithm is known to converge under relatively mild conditions, see (Fletcher, 2000).

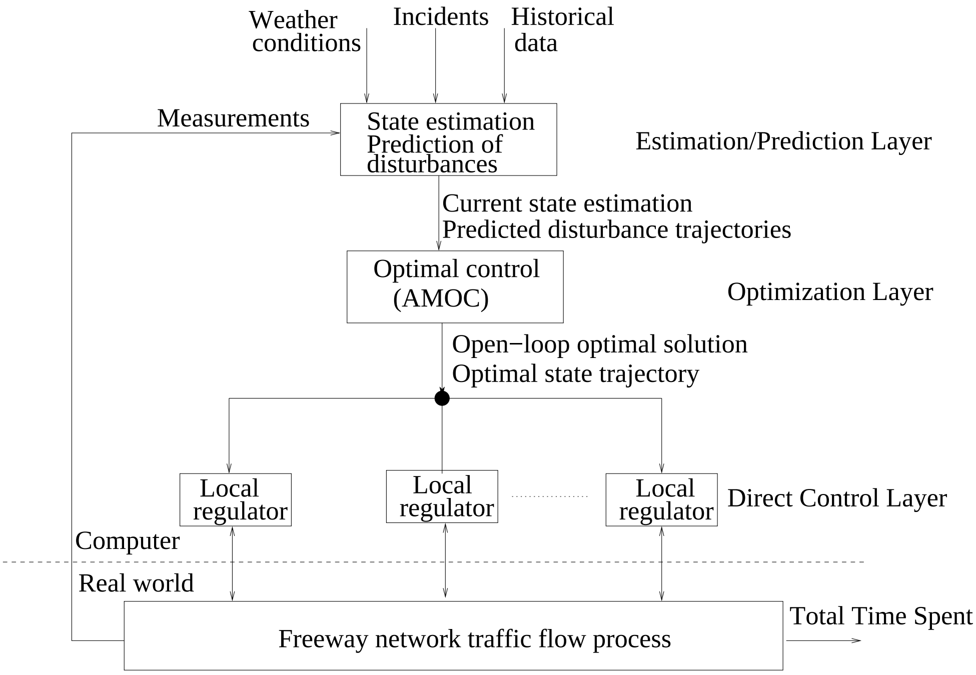


Figure 2: Hierarchical control structure.

4 Hierarchical control

The fact that the solution delivered from AMOC is an open-loop optimal control trajectory, means that errors in the estimation of the initial state $\mathbf{x}_0$ and in the prediction of the future disturbances as well as model inaccuracies will drive the system away from the expected behavior. Since estimation, modeling and prediction errors are inevitable, a receding horizon approach is employed to address the problem of mismatch between the predicted and actual system behavior. For the problem of coordinated ramp metering in freeway networks, this approach may take the form of a hierarchical control system, see figure 2.

The hierarchical control structure consists of three layers. The highest layer is the Estimation/Prediction Layer. It receives as input historical data, information about incidents and weather conditions, and real-time measurements from sensors installed in the freeway network. All this information is processed in order to provide the current state estimation and future predictions of the disturbances to the next layer. The Optimization Layer (AMOC) considers the current time as $t = 0$ and uses the current state estimate as initial condition $\mathbf{x}_0$. Given the predictions $\mathbf{d}(k)$, $k = 0, \ldots, K-1$ the optimal control problem (4)–(6) is solved. The solution of the optimal control problem is the optimal control trajectory and the corresponding optimal state trajectory. These trajectories are forwarded as input to the decentralized Direct Control Layer, that has the task to realize the suggested policy.

We distinguish between two different scenarios for the Direct Control Layer. In the first case, the optimal control trajectories received from the Optimization Layer are directly applied to the traffic process. In the second case, the optimal control trajectories are discarded and only the optimal state trajectories are used. In this case, ALINEA and flow-based ALINEA are employed as local regulators while the optimal state trajectory is used to determine the set-points for each particular on-ramp. For each on-ramp o with merging segment $(\mu, 1)$ a local regulator is applied with control sample time $k_c = z_c \cdot k$, $z_c \in \mathcal{N}$. We define the average quantities $\bar{\rho}^*_{\mu,1}(k_c) = \sum_{z=k}^{k+z_c} \rho^*_{\mu,1}(z)/z_c$ and $\bar{q}^*_{\mu,1}(k_c) = \sum_{z=k}^{k+z_c} q^*_{\mu,1}(z)/z_c$, where the $*$-index denotes optimal values resulting from AMOC. If $\bar{\rho}^*_{\mu,1}(k_c) \geq \rho_{cr,\mu}$ then ALINEA is applied during the period k_c with set-point $\tilde{\rho}_{\mu,1} = \bar{\rho}^*_{\mu,1}(k_c)$. Otherwise, if $\bar{q}^*_{\mu,1}(k_c) \leq \alpha \cdot q_{\mu,cap}$, where $q_{\mu,cap}$ is the downstream capacity and α a threshold parameter, then flow-based ALINEA is applied for the period k_c with set-point $\tilde{q}_{\mu,1} = \bar{q}^*_{\mu,1}(k_c)$. A typical value for α is 0.9. In any other case, ALINEA is applied with $\tilde{\rho}_{\mu,1} = \rho_{\mu,cr}$. Note that the ramp queue control (3) is applied in both scenarios to guarantee that the maximum queues are not exceeded as well as to establish comparable conditions.

The local regulators operate with the optimal state trajectories for a period $K_P \leq K$, where K_P is the application horizon (as opposed to the optimization horizon), after which the whole process is repeated, thereby closing the control loop of AMOC. The state estimation and the disturbance predictions are updated, a new optimization is performed and the resulting solution is used to determine the new local feedback regulators' set-points.

5 Application to the Amsterdam ring-road

The control structure described in section 4 has been applied to the Amsterdam ring-road (counter-clockwise direction only). This is a 32 km network with 21 on-ramps and 20 off-ramps. The traffic flow process is simulated using the validated METANET modeling tool, see (Kotsialos *et al.*, 2002a). During the simulation horizon, the traffic demand and the turning rates of METANET are provided according to real measurements. These measurements, however, are not given as input to AMOC. Instead we assume that the Estimation/Prediction Layer provides AMOC with smooth trajectories of the actual demand trajectories, i.e. that a very efficient demand predictor is available. Furthermore, we assume a perfect model and a perfect state-estimator which is able to provide AMOC with complete information about the current state of the system. Finally, we assume that a good predictor of the turning rates is present, able to provide the (constant) mean value of every time-varying turning rate. In figure 3a) an example depicting the measured and the predicted demand trajectories for a given on-ramp is given, and in 3b) a similar example for a turning rate trajectory is given.

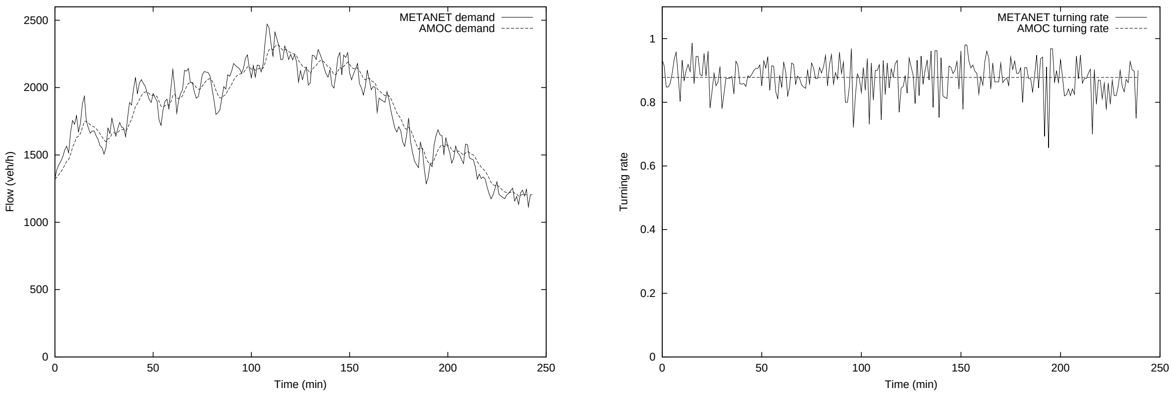
For all control scenarios, all the origins are controlled whereby the urban on-ramps have a maximum queue equal to 100 vehicles, while motorway on-ramps have a maximum queue equal to 200 vehicles.

The simulation horizon is 4 hours and in absence of control measures the resulting TTS is 14,168 veh·hours. In the case of perfect disturbance prediction, the application of the AMOC open-loop optimal control trajectories results in TTS equal to 6,974 veh· hours, an improvement of 51%. This is the best that can be achieved which, however, cannot be realised due to prediction and estimation errors.

Table 1 summarizes the results of the application of the hierarchical control when the optimization horizon is 1 hour and the application horizon is 30, 20 and 10 minutes. The achieved TTS for each scenario is shown together with the percentage of the improvement over the no-control case and the difference with the optimal open-loop control case with perfect pre-

Table 1: TTS for optimization horizon of 1 hour.

Application	AMOC with local feedback control			Direct application of AMOC		
horizon	TTS (veh·hours)	Improvement over the no-control case	Difference with the optimal open-loop case	TTS (veh·hours)	Improvement over the no-control case	Difference with the optimal open-loop case
30 min	8,472	40.4%	21.5%	8,928	37.0%	28.0%
20 min	8,317	41.5%	19.3%	8,987	36.7%	28.7%
10 min	8,211	42.2%	17.7%	8,534	39.8%	22.4%



(a) Demand example.

(b) Turning rate example.

Figure 3: Examples of measured and predicted disturbance trajectories.

diction and estimation. It may be seen that the use of the local feedback controllers results in superior performance for all cases. The most efficient application horizon, for both cases of Direct Control Layer implementation is 10 minutes. This is due to the fact that, more frequent updating (feedback!) of the initial state and of the disturbances rejects all past modeling/prediction/estimation errors and thereby improves the solutions provided by AMOC. This is because the errors in the predictions of the disturbances make the state of the network, as predicted by AMOC, to deviate from the actual state, as simulated by METANET. The short application horizon helps AMOC to reduce this mismatch, which directly affects its efficiency.

6 Conclusion

In this paper a hierarchical coordinated ramp metering control strategy based on the nonlinear model-predictive approach was presented. A constrained nonlinear discrete-time optimal control problem is combined with local feedback control, in order to render the optimal control solution more efficient in presence of various sources of modeling/prediction/estimation mismatch. Further research needs to be conducted for different scenarios with respect to disturbance and modeling errors, and the extent of the optimization and application horizons.

References

R. Fletcher, *Practical Methods of Optimization,* John Wiley and Sons Ltd, Second edition, (2000).

A. Kotsialos and M. Papageorgiou, "Optimal coordinated ramp metering with Advanced Motorway Optimal Control", *Transportation Research Record* 1748, 55–71 (2001).

A. Kotsialos, M. Papageorgiou, C. Diakaki, Y. Pavlis and F. Middleham, "Traffic flow modeling of large-scale motorway networks using the macroscopic modeling tool METANET", *IEEE Transactions on Intelligent Transportation Systems* 3(4), 282–292 (2002a).

A. Kotsialos, M. Papageorgiou, M. Mangeas and H. Haj-Salem, "Coordinated and integrated control of motorway networks via nonlinear optimal control", *Transportation Research C* 10(1), 65–84 (2002b).

A. Messmer and M. Papageorgiou, "METANET: A macroscopic simulation program for motorway networks", *Traffic Engineering and Control* 31(8/9), 466–470;549 (1990).

M. Papageorgiou, "Multilayer control system design applied to freeway traffic", *IEEE Transactions on Automatic Control* AC-29(6), 482–490 (1984).

M. Papageorgiou and M. Marinaki, *A Feasible Direction Algorithm for the Numerical Solution of Optimal Control Problems*, Chania, Greece, Dynamic Systems and Simulation Laboratory, Technical University of Crete (1995).

E. Smaragdis and M. Papageorgiou, "A series of new local ramp metering strategies", presented at the 82nd TRB Annual Meeting, Washington, D.C., USA (2003).