The Period Vehicle Routing Problem with Service Level Choice

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The period vehicle routing problem (PVRP) is a variation of the classic vehicle routing problem (VRP) in which routes must be constructed over a period of time (for example, one week) to visit customers. Customers must be visited a pre-set number of times over the period with a schedule that is chosen from a menu of schedule options. The PVRP is stated as follows:

Given: a depot ($i = 0$) and a set of nodes ($i = 1..N$) each with daily demand d_i and a menu of service options; a set of arcs A between all nodes with travel times t_{ij}; and a fleet of K vehicles each with fixed capacity C and driver work shift $\mathcal{T}$.

Find: a set of daily tours for each vehicle over the period that minimizes travel time while satisfying operational constraints (capacity, work shifts, visit requirements).

The first problem motivating the PVRP was introduced in Beltrami and Bodin (1974) for assigning hoist compactor trucks in municipal waste collection. The PVRP was formally defined in Russell and Igo (1979) as an "Assignment Routing Problem," and first formulated in Christofides and Beasley (1984). Solution methods have focused on VRP-type heuristics; see Tan and Beasley (1984), Russell and Gribbin (1991), and Chao *et al.* (1995). Cordeau *et al.* (1997) implement a tabu search algorithm for the PVRP.

In these references, it is assumed that each customer is visited with a pre-set frequency. Each node may be served from a node-specific set of schedule options with a fixed number of visits per week. For example, if a node is to be visited twice a week, the schedule choices may be $\{(Mon, Tues); (Mon, Wed); (Wed, Fri)\}$. However, determining schedule choices for a node independent of routing decisions may lead to routing inefficiencies that can be resolved by expanding the service options for that node, possibly with different frequencies. This, however, increases the size of the problem. The PVRP, even without service level choice, is computationally complex since it is a generalization of the standard VRP which is NP-hard. Moreover, the frequency choice impacts the volume accumulated at a node, as well as the stopping time, as explained later. We consider a special case of disjoint schedule choices: $s = \{M, T\}$ where $M = (Mon, Wed, Fri)$ and $T = (Tue, Thur)$. The visit frequency γ^s is 3 for $s = M$ and 2 for $s = T$. Nodes are defined as daily nodes ($i \in \mathcal{N}^+$) which are visited by both schedules or infrequent nodes ($i \in \mathcal{N}^-$) which are served on a less frequent basis that needs to be decided (either M or T). This paper explores modeling issues that arise when frequency choice is introduced and suggests efficient solution methods.

To ensure that vehicle capacities are not exceeded, one must know the volume collected and distributed at a node. As service frequency decreases, more items are demanded and/or accumulate between visits, represented by a demand adjustment factor β_i^s. This term is set to one for daily nodes, but is proportionally higher for lower visit frequencies. In the applications of the PVRP with service level choice under consideration, stopping time at a node increases with the amount of material collected and distributed which is again a direct function of visit frequency. The stopping time parameter τ_i^s is a function of the demand at node i and the frequency of schedule s. For a node i in $\mathcal{N}^+$, $\tau_i^M = \tau_i^T$, and for a node in $\mathcal{N}^-$, $\tau_i^M < \tau_i^T$. The PVRP with service level choice is:

Given: a depot ($i = 0$) and a set of nodes ($i = 1..N$) each with daily demand d_i, adjustment factor β_i^s, stopping time τ_i^s, and a minimum visit frequencies f_i; a set of arcs A between all nodes with travel times t_{ij}; and a fleet of K vehicles each with fixed capacity C and driver work shift $\mathcal{T}$.

Find: a set of daily tours for each vehicle over the period that optimizes an objective of minimum total travel time and maximum service level weighted by α_s while satisfying operational constraints (capacity, work shifts, and visit minimums).

The following formulation for the PVRP with service level choice is an extension of the VRP formulation in Fisher and Jaikumar (1981).

Decision variables

$$y_{ik}^s = \begin{cases} 1 & \text{if node } i \text{ is visited by vehicle } k \text{ on schedule } s \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ijk}^s = \begin{cases} 1 & \text{if vehicle } k \text{ traverses the arc between nodes } i \text{ and } j \text{ on schedule } s \\ 0 & \text{otherwise} \end{cases}$$

$$\min \sum_{s=M,T} \sum_{k=1..K} \left[\sum_{ij \in A} (t_{ij} + \tau_i^s) x_{ijk}^s - \sum_{i=1..N} d_i \alpha^s y_{ik}^s \right] \tag{1a}$$

subject to

$$\sum_{s=M,T} \sum_{k=1..K} \gamma^s y_{ik}^s \geq f_i \qquad i = 1..N \tag{1b}$$

$$\sum_{k=1..K} y_{ik}^s \geq 1 \qquad \forall i \in \mathcal{N}^+; s = M,T \tag{1c}$$

$$\sum_{i=1..N} d_i \beta_i^s y_{ik}^s \leq C y_{0k}^s \qquad k = 1..K; s = M,T \tag{1d}$$

$$\sum_{j=0..N} x_{ijk}^s = y_{ik}^s \qquad i = 0..N; k = 1..K; s = M,T \tag{1e}$$

$$\sum_{ij \in A} (t_{ij} + \tau_i^s) x_{ijk}^s \leq \mathcal{T} \qquad k = 1..K; s = M,T \tag{1f}$$

$$\sum_{j=0..N} x_{ijk}^s = \sum_{j=0..N} x_{jik}^s \qquad i = 0..N; k = 1..K; s = M,T \tag{1g}$$

$$\sum_{i,j \in S} x_{ijk}^s \leq |S| - 1 \qquad \forall S \subset \{1..N\}; k = 1..K; s = M,T \tag{1h}$$

$$y_{ik}^s \in \{0,1\} \qquad i = 0..N; k = 1..K; s = M,T \tag{1i}$$

$$x_{ijk}^s \in \{0,1\} \qquad \forall i,j \in A; k = 1..K; s = M,T \tag{1j}$$

The objective function (1a) balances travel time and service level. The first term is the duration of each route, including arc travel times and node stopping times. The second term represents a demand-weighted service level term which provides incentive to offer more frequent service to higher demand nodes. The parameter α^s represents the value of service relative to travel time. Since more frequent service is more desirable, $\alpha^M > \alpha^T$. Setting $\alpha^s = 0$ is equivalent to considering only routing in the objective and specifying large values of α^s solves the problem of considering only demands when choosing service level and ignoring routing. We use a parametric analysis to determine appropriate values of α^s.

Constraints (1b) enforce the minimum frequency of visits for each node. Constraints (1c) guarantee that nodes in $\mathcal{N}^+$ receive service from both schedules.[1] Constraints (1d) represent physical capacity constraints in the vehicle since the amount of material collected and distributed at a node is a function of the service level. It is assumed that the demand collected is equal to the demand distributed. These constraints also ensure that all tours contain a visit to the depot. Constraints (1e) link the x and y variables. Constraints (1f) represent driver work shift limits. Constraints (1g) ensure flow conservation at each node. Constraints (1h) are the subtour elimination constraints and constraints (1i) and (1j) define the binary variables for assignment and routing. The structure of this problem suggests that a Lagrangian relaxation approach may be efficient, as relaxing constraints (1e) decomposes the problem by the y (assignment) and x (routing) variables.

[1] Without this constraint, a node in $\mathcal{N}^+$ could instead receive service from two separate vehicles on MWF and still satisfy (1c).

We have experimented with several Lagrangian relaxations of formulation (1), as well as variations of (1) with different representations of the capacity constraints. For example, in the driver work shift constraints (1f), travel time t_{ij} is an arc-specific parameter, but stopping time τ_i^s is a function only of a node and the service level at the node. Therefore, stopping time may be expressed as a function of x or y. Vehicle capacity constraints (1d) can also be written either in terms of x or y. These choices then impact decomposition methods for solving the PVRP with service level choice and the resulting subproblems that need to be solved. Preliminary results from small test problems suggest that formulation (1) outperforms variations in which stopping time is written in y both in terms of time to convergence and number of iterations to convergence. We are testing if these results hold for larger problems. Currently we solve each subproblem with CPLEX; for larger problems, we are replacing the CPLEX solution methods for the routing subproblem in x with more efficient methods. These subproblems are difficult because one must solve a prize-collecting traveling salesman problem with time constraints for each vehicle and schedule combination.

We note that our problem is somewhat related to the inventory routing problem (IRP); however, there are several important differences. An advantage of the IRP in regards to decomposition methods, exploited in Federgruen and Zipkin (1984), is that any allocation solution derived from a feasible routing solution is feasible in terms of capacity constraints. This is true because (a) one can distribute nothing to a node when it is visited thereby satisfying vehicle capacity constraints[2] and (b) there are no time constraints on tours in the general IRP. The introduction of these two factors complicates solution methods for the PVRP with service level choice. Another key difference between the IRP and the PVRP with service level choice is that the IRP maximizes the service level of an *item* while minimizing routing costs whereas the PVRP with service level choice maximizes the service level of a *node* while minimizing routing costs. This impacts the formulation of the objective function: the objective function is nonlinear for the IRP and linear for the PVRP with service level choice.

Our interest in the PVRP problem with service level choice is motivated by an application of an inter-library loan operation in the north suburbs of Chicago. Books and related materials are collected and delivered to hundreds of libraries, using four vehicles, which start and end their route at the library system's headquarters. While our formulation is largely affected by this application, it describes well the practice of numerous other pickup and/or delivery situations. Our first contribution is in the formulation aspect, allowing for more flexibility by introducing service level choice as well as explicitly incorporating its implications (on the quantities and on stopping times). Second, although the results are still preliminary, we believe that by implementing Lagrangian relaxation to this problem we can solve problems of realistic size, including the inter-library loan application described above.

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[2]In the PVRP, it is assumed that all items must be distributed to and collected from a node at each visit and one cannot hold extra items at the depot.

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