Modeling and Simulation of Multilane and Multiclass Traffic Flow at On and Off Ramps

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1 Introduction

Simulation of traffic flow is important for testing and assessing the effect of Intelligent Transportation Systems (ITS), and for model-based predictive control. It provides planners and policy-makers with the ability to test traffic management schemes and their effects on network performance. Many traffic phenomena such as the formation of traffic jams, stop and go waves, etc., can be reproduced by traffic modeling and simulation. In this paper, we will focus only on macroscopic models due to the relatively small number of model parameters and the resulting simple calibration. In this type of models, traffic is viewed as a continuum. Lighthill & Whiham were the pioneers in constructing those types of models. Their model (LW model) has the form of first order extended and improved partial differential equation. Inspired by this model, many other higher-order models have been developed based either on principle of car-following model, such as Payne (1971), Phillips (1979) or recently on gas-kinetic theory, such as Helbing (1996), Hoogendoorn (1999).

The model developed in this paper aims at describing correctly the traffic dynamics at on and off ramps by taking explicitly into account lane-changing process (diverging and merging). We obtain such kind of model from a gas-kinetic theory that describes the evolution of the phase-space density of vehicles on a freeway via a partial differential equation. In this equation, the left hand side describes the continuous dynamics of the phase-space density function due to the motions of traffic flow and the right hand side describes the discontinuous changes of this function due to the events such as lane-changing, breaking, etc. The lane changing processes from on ramps to main lanes or from main lanes to off ramps are described by the mandatory lane-changing rate. This transition rate is then derived from the microscopic driving behavior at the on and off ramps based on the so-called Markov renewal process. Finally, the resulting macroscopic traffic model is derived by the method of moments.

The content of this paper is outlined as follows. We start with brief description of the gas-kinetic model for multilane and multiclass traffic flow in section 2. Section 3 illustrates the development of a model for lane-changing process at on and off ramps. In section 4, the resulting macroscopic traffic model for multilane and multiclass traffic flow is presented. Section 5 shows the performance of the developed and calibrated model. Finally, we conclude the paper in section 6.

2 The gas-kinetic multilane model for multiclass users

This section presents the multilane gas-kinetic equations of the phase-space density for heterogeneous traffic operations. These equations are a generalization of the single gas-kinetic equations, which were first introduced by Prigogine et al (1971) and then by Paveri-Fontan (1975). Let $\rho_{u,i}(x,v,t)$ be the so-called reduced multilane phase-space density (PSD) of vehicle class u on lane i. Helbing et al (1999) proposed the following equation

$$\frac{\partial \rho_{u,i}}{\partial t} + \underbrace{v\frac{\partial \rho_{u,i}}{\partial x}}_{convection} + \underbrace{\frac{\partial}{\partial v}\left(\rho_{u,i}\frac{V_{u,i}^{0}-v}{\tau_{u,i}}\right)}_{relaxation} = \underbrace{\left(\frac{\partial \rho_{u,i}}{\partial t}\right)_{\text{int}}}_{\text{int}eraction} + \underbrace{\left(\frac{\partial \rho_{u,i}}{\partial t}\right)_{lc}}_{lane-changing} \qquad (2.1)$$

- The *convection* term reflect the changes of PSD due to the motion of vehicles flowing into or out of the road cell [x,x+dx).
- The *relaxation* term reflects the changes of PSD due to the tendency of vehicles to accelerate towards the desired velocity
- The *interaction* term describes the changes of PSD due to the interaction between the slow vehicles and the faster ones.
- The *lane-changing* term describes the changes of PSD due to the vehicles changing from and to current lane.

In equation (2.1), the lane-changing term consists of three types of lane-changing behavior

- Lane-changing due to interactions between faster vehicles and slower ones in order to avoid collisions
- Spontaneous lane-changing due to the effects of traffic regulations and the preferences of drivers to use the more comfortable lanes.
- Mandatory lane-changing due to sudden changes of the road layout, such as on-off ramps, lane close, etc.

The following equation describes these lane-changing processes:

$$\left(\frac{\partial \rho_{u,i}}{\partial t}\right)_{lc} = \underbrace{\left(\frac{\partial \rho_{u,i}}{\partial t}\right)_{lc}^{\text{int}}}_{\text{int}\,eraction} + \underbrace{\left(\frac{\partial \rho_{u,i}}{\partial t}\right)_{lc}^{spon}}_{spon\tan eous} + \underbrace{\left(\frac{\partial \rho_{u,i}}{\partial t}\right)_{lc}^{man}}_{mandatory} \tag{2.2}$$

In equation (2.2), the first two terms have been explicitly determined by Helbing et al (1999), Hoogendoorn (2000) with success in application to uninterrupted traffic flow operations. In case of the interrupted flow at on and off ramps, the third term needs to be determined. This will be described in the next section.

3 Lane changing process at on and off ramps

Considering a merging process from lane 0 (on ramp) to lane 1 (adjacent main lane) as described in Figure 3.1. The decision to make a lane-change is based on the distance h_r between the subject vehicle and the vehicle at the rear (lag-gap) and the distance h_f between it and the vehicle in front of it (lead-gap) on the main lane. When both gaps suffice, the lane-changing maneuver will be performed, hence, the probability that a gap on the adjacent main lane is accepted depends on the joint probability distribution of lag-gap and lead-gap on that lane. These gaps suffice if the space between the subject vehicle and the one behind it as well as the one in front of it is larger than certain threshold values.
Let $h_{u,1}$ and $h_{u,2}$ denote the threshold value of lag-gap and lead-gap for lane-changing manoeuvre, respectively. Assuming that the vehicle only reacts to the one in front $h_u = h_0 + T_u v$. Where v is the velocity of the following vehicle, h_0 is the minimal distance reflecting the safety-margin acceptance of the drivers on the target lane, T_u is the reaction time of vehicle class u. Since approaching the end of the ramps, all drivers are willing to accept really smaller lag gap, therefore, they disturb traffic on the main lane significantly. Hence, with respect to the lag gap threshold value, $h_{u,1} = h_0 + \frac{x_{end} - x}{x_{end} - x_{start}} T_u w = h_0 + \alpha T_u w$

Now let us assume that the mandatory lane-changing rate is proportional to the number of vehicles stemming from on ramp or exiting at off ramp and that all those vehicles are forced to change their lane at the end of the ramp (at location x_{end}) as below:

$$\left(\frac{\partial \rho_{u,i}}{\partial t}\right)_{lc}^{man} = \frac{P_{u,i}\rho_{u,i}v}{x_{end} - x} \tag{2.3}$$

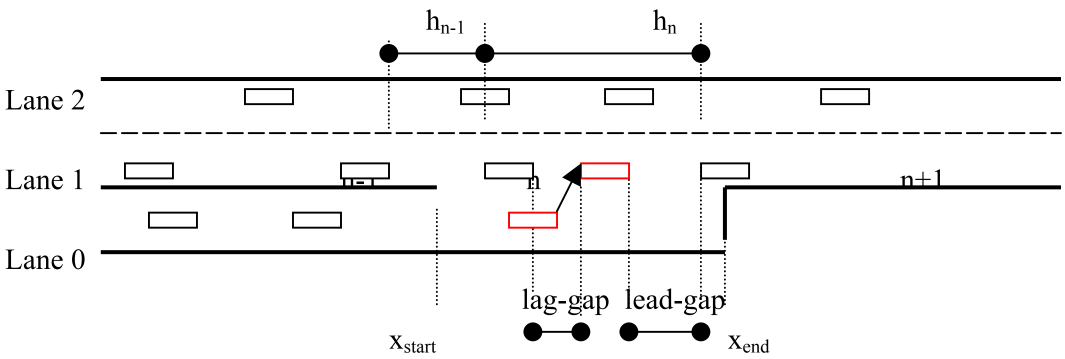


Figure 3.1 Lane- changing behavior at on ramp

In equation (2.3) $P_{u,i}$ denotes the desired lane-changing probability of vehicle class u from lane i to lane i+1 (i-1 in case of off ramp). Assuming that the gap between each vehicle pair only depends on the nearest-neighbor ones.

Let $A_u(v|x,t)$ be the event that a vehicle of class *u* driving with velocity ***v*** merges into the main lane at (x,t), and B_u be the event that a vehicle merging is of class *u*. Then

$$P_{u,i} = P(B_u \,\&\, A_u) = P(A_u \mid B_u)P(B_u) \tag{2.4}$$

Obviously $P\left(B_u\right) = \dfrac{r_u^i}{\sum_{s=1}^{U} r_s^i} = \dfrac{r_u^i}{r_i}$ (2.5)

and the following relation is applied to the lane-changing probability at on and off ramps

$$P\left(A_u \mid B_u\right) = P\left(h_r \geq h_{s,1}(w), h_f \geq h_{u,2}(v)\right) = \sum_{s=1}^{U}\left[1-\left\langle F_{rear}\left(h_{s,1}(w)\right)\right\rangle\right]\left[1-\left\langle F_{lead}\left(h_{u,2}(v)\right)\right\rangle\right]$$

(2.6)

In equation (2.6), F(.) denotes the cumulative gap distribution function and <.> denotes the mean value. By definition for any function *ϕ(x)* one gets $\left\langle \phi(x) \right\rangle = \int_0^{\infty} \phi(x) f(x)dx$, where f(x) is probability density function.

Let $f_{lead}(h)$ be the gap density distribution function of the leader and, for simplicity, assuming that it is exponentially distributed as $f_{lead}(h) = \dfrac{1}{E(h)} e^{-\frac{h}{E(h)}}$ (2.7)

In expression (2.7), $E(h) = \int_0^{\infty} hf(h)dh = 1/r$ denotes the mean distance gap. By definition,

$$F_{lead}(h) = \int_0^h f(z)dz = 1 - e^{-rh} \tag{2.8}$$

Let $f_{rear}(h)$ be the lag gap density distribution function and considering two situations. First, the nearest following vehicle moves into lane 2 in order to give way to merging one with probability p then the space for merging is h_n+h_{n-1}, and the lag gap distribution is denoted by $f_{rear}^1(h)$. This distribution is determined based on convolution theorem. Secondly, if the nearest following vehicle is unable to change her lane with probability (1- p) then the space for merging is h_n, and the lag gap distribution is denoted by $f_{rear}^2(h)$. Based on the renewal process the distribution of lag gap is determined as follows:

$$f_{rear}^1(h) = \frac{1 - F_{rear}^1(h)}{E(h_n + h_{n-1})} = \frac{1 - \int\limits_0^h f(z) \int\limits_0^{h-z} f(x)dxdz}{2E(h)} = \frac{re^{-rh}\left(1 + rh\right)}{2}$$

and $f_{rear}^2(h) = \dfrac{1 - F_{rear}^2(h)}{E(h_n)} = \dfrac{1 - \int\limits_0^h f(x)dx}{E(h)} = re^{-rh}$

hence, $F_{rear}(h) = p_{1,2}\int\limits_0^h f_{rear}^1(x)dx + \left(1 - p_{1,2}\right)\int\limits_0^h f_{rear}^2(x)dx = 1 - e^{-rh} - \dfrac{p_{1,2}rh}{2}e^{-rh}$ (2.9)

Substituting (2.8) and (2.9) into (2.6) we obtain the desired lane-changing probability of vehicles class u from on-ramps, irrespective of vehicle types on the main lanes

$$P_i\left(A_u \mid B_u\right) = \int\limits_0^\infty e^{-r_{u,i+1}h_{u,2}(v)} g_{u,i}(v)dv \sum_{s=1}^{U} \int\limits_0^\infty e^{-r_{s,i+1}h_{u,1}(w)} \left(1 + 0.5pr_{s,i+1}h_{u,1}(w)\right) g_{s,i+1}(w)dw$$

$$P\left(A_u \mid B_u\right)_i = e^{-r_{u,i+1}h_0} \int_0^\infty e^{-\alpha r_{u,i+1}T_u v} g_{u,i}(v)dv \sum_{s=1}^{U} e^{-r_{s,i+1}h_0} \int_0^\infty e^{-\alpha r_{s,i+1}T_u w} \left(1 + 0.5pr_{s,i+1}\left(h_0 + \alpha T_s w\right)\right) g_{s,i+1}(w)dw$$

$$P\left(A_u \mid B_u\right)_i = e^{-r_{u,i+1}h_0} \int\limits_0^\infty e^{-\alpha r_{u,i+1}T_u v} g_{u,i}(v)dv$$

$$\sum_{s=1}^{U} e^{-r_{s,i+1}h_0} \left[\left(1 + 0.5pr_{s,i+1}h_0\right) \int\limits_0^\infty e^{-\alpha r_{s,i+1}T_u w} g_{s,i+1}(w)dw + 0.5p\alpha r_{s,i+1}T_s \int\limits_0^\infty we^{-\alpha r_{s,i+1}T_u w} g_{s,i+1}(w)dw \right]$$

(2.10)

Using the expansion $e^{x} = \sum_{0}^{\infty} \frac{x^{n}}{n!}$ equation (2.10) becomes

$$P_i\left(A_u \mid B_u\right) = e^{-r_{u,i+1}h_0} \sum_0^\infty \int_0^\infty \frac{\left(-r_{u,i+1}T_u v\right)^n}{n!} g_{u,i}(v)dv$$

$$\sum_{s=1}^{U} e^{-r_{s,i+1}h_0}\left[\left(1+0.5pr_{s,i+1}h_0\right)\sum_0^\infty\int_0^\infty \frac{\left(-\alpha r_{s,i+1}T_u w\right)^n}{n!} g_{s,i+1}(w)dw + 0.5p\alpha r_{s,i+1}T_s \sum_0^\infty \int_0^\infty w\frac{\left(-\alpha r_{s,i+1}T_u w\right)^n}{n!} g_{s,i+1}(w)dw\right]$$

After a lengthy algebra calculation we end up with the following equation for mandatory lane-changing probability from on ramp to the adjacent main lane

$$P_{u,i} = \frac{r_u^i}{r_i} e^{-r_{u,i+1}\left(h_0+T_uV_{u,i}\right)}\left[1+\frac{\Theta_{u,i}\left(r_{u,i+1}T_u\right)^2}{2}\right]\sum_{s=1}^{U} e^{-r_{s,i+1}\left(h_0+\alpha T_sV_{s,i+1}\right)}$$

$$\left[\left(1+0.5pr_{s,i+1}h_0\right)\left(1+\frac{\Theta_{s,i+1}\left(\alpha r_{s,i+1}T_s\right)^2}{2}\right)+0.5p\alpha r_{s,i+1}T_s\left(1-\frac{\alpha r_{s,i+1}T_s\left(2-\alpha r_{s,i+1}T_sV_{s,i+1}\right)\Theta_{s,i+1}}{2}\right)\right]$$

(2.11)

From equation (2.11) it can be seen that the mandatory lane-changing probability depends on a lot of variables such as density on the main lane, the velocity of merging traffic and main traffic as well as the velocity variance, etc. Besides, it is also dependent on the safety margin that reflects the willingness of the subject drivers to accept smaller gaps when approaching the end of the ramps.

For the case of off ramp, it is easy to show that the probability of diverging flow is dependent only on the vehicles in front. Therefore, we have the following expression for determining the probability of diverging

$$P_{u,i} = \frac{r_u^i}{r_i} e^{-r_{u,i+1}\left(h_0+T_uV_{u,i}\right)}\left[1+\frac{\Theta_{u,i}\left(r_{u,i+1}T_u\right)^2}{2}\right]\sum_{s=1}^{U} e^{-r_{s,i-1}\left(h_0+T_sV_{s,i-1}\right)}\left[1+\frac{\Theta_{s,i-1}\left(r_{s,i-1}T_s\right)^2}{2}\right] \quad (2.12)$$

4 Multilane and multilcass macroscopic traffic flow model

To derive the macroscopic equations from equation (2.1), one multiples both sides with v^k (k=0,1,2,..) then integrate them over the velocity v ∈ (0,∝), this is the so-called *method of moments*. In this paper, we only consider the lane-changing behavior between freeway and on-off ramps. After a lengthy algebra calculation, the resulting macroscopic traffic model for the evolution of density r(x,t), mean velocity v(x,t) and flow rate q(x,t) in merging/diverging zone is obtained as below:

- Conservation law

$$\frac{\partial r_{u,i}}{\partial t} + \underbrace{\frac{\partial q_{u,i}}{\partial x}}_{convection} = \underbrace{\left(V_{u,i+1}\Delta_{u,i+1} - V_{u,i}\Delta_{u,i}\right)}_{mandatory\ \ lane\_changing} \tag{3.1}$$

- Momentum dynamics

$$\frac{\partial q_{u,i}}{\partial t} + \underbrace{\frac{\partial E_{u,i}}{\partial x}}_{convection} = \underbrace{\frac{r_{u,i}\left(V_{u,i}^{0} - V_{u,i}\right)}{\tau_{u,i}}}_{relaxation} - \underbrace{(1-p)B_{u,i}}_{braking} + \underbrace{\left(E_{u,i+1}\Delta_{u,i+1} - E_{u,i}\Delta_{u,i}\right)}_{mandatory\ \ lane\_changing} \tag{3.2}$$

In equation (3.1) and (3.2), $E_{u,i} = r_{u,i}\left(V_{u,i}^2 + \Theta_{u,i}\right)$ denotes the term *traffic energy*, where $\Theta_{u,i} = \Theta\left(r_{u,i}, V_{u,i}\right)$ is velocity variance. The *convection* term describes the changes in traffic variables at a very small cell [x,x+dx) due to the motion of vehicles along the road. The *relaxation* term reflects the tendency of vehicles to relax to equilibrium situation. The *braking* term describes the changes of traffic variables due to interactions between fast vehicles and slow ones. The *mandatory lane-changing* terms describes the changes of traffic variables due to mandatory lane-changing process at on and off ramps.

$$\Delta_{u,i} = \frac{P_{u,i} r_{u,i}}{x_{end} - x} \tag{3.3}$$

The Boltzmann braking function $B_{u,i} = \gamma_{u,i} r_{u,i}^2 \Theta_{u,i}$, where $\gamma_{u,i}$ is a factor taking into account the space requirement by the finite dimension of vehicles, and is determined as follows:

$$\gamma_{u,i} = {}^{1}\!\Big/_{1 - r_{u,i}\left(l_u + T_u V_{u,i}\right)} \tag{3.4}$$

where l_u is the length of vehicle class *u*

Solving the set of equations (3.1) and (3.2) allows us to predict the evolution of traffic variables of vehicle class *u* on lane *i* of the freeway. This is presented in the next section.

5 Simulation and calibration results

The developed model is simulated and calibrated with data obtained from freeway A1 in The Netherlands. The data used in this paper is obtained from the Dutch Ministry of Transport, Public Works and Water Management. This data contains the time-dependent (every 5 minutes) traffic flow rate and mean velocity at each detector on freeway from KM 86.0 to KM108, and a part of this freeway from KM 86.0 to KM 89.0 consisting a weaving section during time period 14h00 to 19h00, *22th October 2002* is used as shown in Figure 5.1. The high demand from on ramp (KM 88.2) results in congestion at this on ramp, and then propagates upstream and blocks the traffic at the off ramp (km86.6)

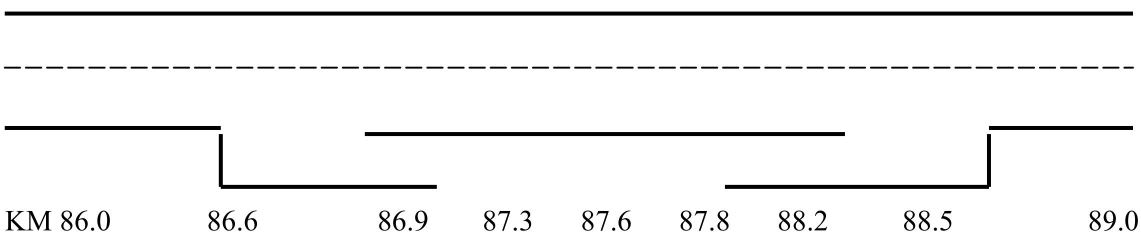


Figure 5.1 Layout of roadway for simulation

The model is simulated using the HLLE numerical scheme (Ngoduy et al, 2004) and then calibrated by an automated calibration procedure using the Nelder-Mead algorithm (Ngoduy et al, 2003). The length of the cells equals 50 m, while the time-step is equal to 1s. The data used to feed the model is given at KM86.0 and KM 89.9 of main stream, KM86.6 of off-ramp and KM88.2 of on-ramp (boundary conditions) and the objective function is calculated with the data at the remaining detectors. The optimal parameters are found with success. The outputs of simulation are shown in comparison with real data in Figure 5.2 and Figure 5.3.

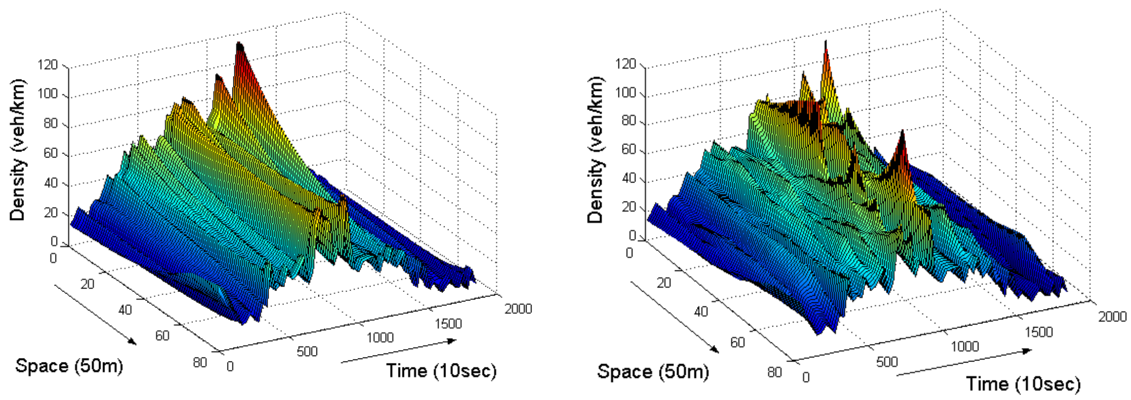
Figure 5.2 shows the time-space evolution of predicted density (5.3a) in comparison with the measured density (5.3b). Figure 5.3 describes the time-space evolution of predicted velocity (5.3a) in comparison with the measured velocity (5.3b). It can be seen from both Figure 5.2 and Figure 5.3 that the developed model is able to predict correctly the evolution of mean velocity and density in the test case freeway A1, especially their fluctuations during congestion period due to the high demand of the on ramp. These results support the accuracy of the model for determining explicitly the mandatory lane- changing rate at on/off ramps.

6 Conclusions

In this paper, we have developed a model for determining the mandatory lane-changing rate between on/off ramps and freeway using the Markov renewal process. The macroscopic traffic model for multiple user class at merging/diverging zones has subsequently been derived from the gas-kinetic theory for multilane and multiclass traffic flow based on the method of moment.

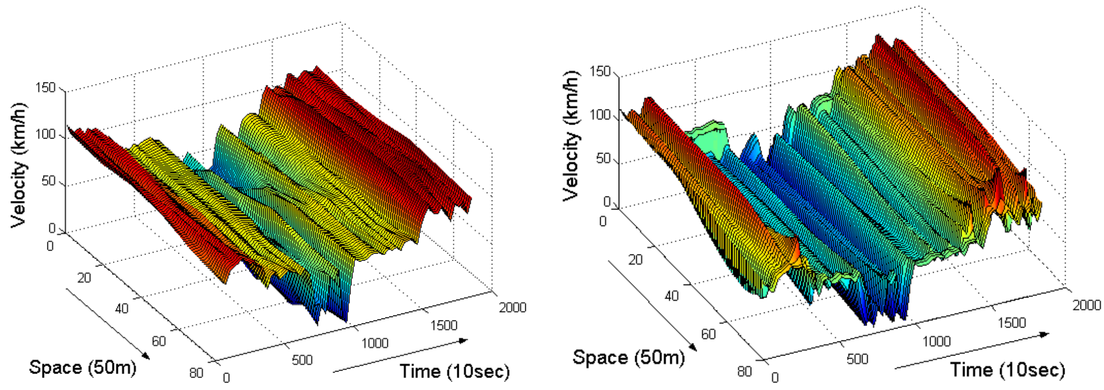
We have found that the lane-changing probability depends on a lot of factors such as density, velocity and velocity variance of both on/off ramps and the main lane, which have not been taken explicitly into account before. It is also dependent on the safety margin reflecting the willingness of drivers to accept a smaller gap when approaching the end of the ramps. The results of calibrated model with real data during congested period in freeway A1-The Netherlands show very good agreement. This successful implementation paves the way for the application of relevant control measures at the weaving sections on freeways so as to prevent congestion from spilling back.

Further work will be the implementation of this model into HELENA traffic network prediction model (Hoogendoorn et al, 2002) then compares the performance of HELENA with current macroscopic network models such as METANET.



a) b)

Figure 5.2 Predicted density (a) vs. Real density (b)



a) b)

Figure 5.3 Predicted velocity (a) vs. Real velocity (b)

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