Road Pricing Problem Including Route Choice and Elastic Demand – a Game Theory Approach

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1 Introduction

In recent years, researchers have become increasingly interested in the effects of introducing different road pricing measures on transportation networks. Who is involved in decision-making and how should decisions be made? How will travelers change their travel behavior after the introduction of road pricing? How will travelers interact with each other and how can the road authority influence or even control travel behavior of travelers? To answer such questions we need to establish a flexible and generic framework for analyzing the behavior of travelers as well as the road authority.

Game theory provides such a framework for modeling decision-making processes in which multiple players are involved with different objectives, rules of the game and assumptions. Yang and Yagar (1995) formulated the control-assignment problem using game theory. The integrated traffic control problem and the dynamic traffic assignment problem as a non-cooperative game between traffic authority and highway users is presented in the work of Chen and Ben-Akiva (1998). Tolling at a frontier and an application of game theory and queuing analysis to develop micro-formulations of congestion can be found in Levinson (1988, 2003). Considering the problem of designing optimal tolls on the network, there is a need for better insights into the interactions between travelers and the road authority, the nature and the consequences of this interaction. In this paper we aim at analyzing a very simple route choice problem with elastic demand where road pricing is introduced in a game theoretic framework. First, the road pricing problem is formulated using game theory notions, and different games are described. After that, a game-theoretic approach is applied to formulate the road pricing game as monopoly, Stackelberg, and Cournot game, respectively. The main purpose of the experiment reported here is to show the outcomes of different games established for the optimal design toll problem.

2 Game theory applied to road pricing

In the road pricing problem, we are dealing with an N+1-player game, where there are N players (travelers) making a travel choice decision, and one player (the road manager) making a control or design decision (in this case, setting road tolls). In fact, there are two games played in conjunction with each other. The first game is a non-cooperative game where all N travelers aim to maximize their own utility by choosing the best travel strategy (e.g. route choice), taking into account all other travelers' strategies. The second game is between the travelers and the road manager, where the road manager aims to maximize some network performance by choosing a control strategy, taking into account that travelers respond to the control strategy by adapting their travel strategies. The definitions and notation used here are adapted from Altman et al. (2003).

Consider first the N-player game of the travelers, where S_i is the set of available alternatives for each traveler i, $i \in \{1,\ldots,N\}$. The strategy $s_i \in S_i$ that each traveler i will play depends on the control strategy set by the road manager, denoted by vector θ, and on the strategies of all other players, denoted by $s_{-i} \equiv (s_1,\ldots,s_{i-1},s_{i+1},\ldots,s_N)$. We assume that each traveler makes his decision independently and unilaterally seeks the maximum utility payoff, taking into account the possible rational choices of the other travelers. Let $J_i\left(s_{-i}(\theta),s_i(\theta),\theta\right)$ denote the utility payoff for traveler i for a given control strategy θ. The utility payoff can include all kinds of travel utilities and travel costs. If all other travelers play strategies s^*_{-i}, then traveler i will play the strategy that maximizes his payoff utility, i.e.

$$s_i^*(\theta) = \arg\max_{s_i \in S_i} J_i\left(s^*_{-i}(\theta),s_i^*(\theta),\theta\right). \tag{1}$$

If Equation (1) holds for all travelers $i \in \{1,\ldots,N\}$, then $s^*(\theta) \equiv (s^*_{-i}(\theta),s_i^*(\theta))$ is called a *Nash equilibrium* for the control strategy θ. In this equilibrium, no traveler can improve his utility payoff by unilateral deviation. Note that this coincides with the concept of a *Wardrop user-equilibrium*.

Now consider the complete N+1-player game where the road manager faces the N travelers. The set Θ describes the alternative strategies available to the road manager. Suppose he chooses strategy $\theta \in \Theta$. Depending on this strategy and on the strategies chosen by the travelers, $s^*(\theta)$, his utility payoff is denoted by $R(s^*(\theta),\theta),$ and may represent e.g. the total system utility or to the total profits made. The road manager chooses the strategy θ^* in which he aims to maximize his utility payoff, depending on the responses of the travelers:

$$\theta^* = \arg\max_{\theta\in\Theta} R\left(s^*(\theta),\theta\right). \tag{2}$$

If Equations (1) and (2) are satisfied for all N+1 players, where $\theta = \theta^*$ in Equation (1), then this is a Nash equilibrium in which no player can be better off by unilaterally playing another strategy. Although all equilibriums use the concept of Nash, depending on the influence each of the players has in the game, a different equilibrium or game type can be defined in the N+1-player game. We can distinguish the following games:

(a) *Monopoly game* – The road manager not only sets its own control, but also controls the strategies that the travelers will play. In other words, the road manager sets θ^* as well as s^*. This will lead to a so-called system optimum.
(b) *Stackleberg game* – The road manager is the ‘leader’ by setting the control, thereby directly influencing the travelers which are ‘followers’. The travelers only indirectly influence the road manager by making travel decisions based on the control. It is assumed that the road manager has complete knowledge of how travelers respond to control measures. The road manager sets θ^* and the travelers follow by playing $s^*(\theta^*)$.
(c) *Cournot game* – In contrast to the Stackleberg game, the travelers are now assumed to have a direct influence on the road manager, having complete knowledge of the responses of road manager to their decisions. The road manager sets $\theta^*(s^*)$, depending on the travelers strategies $s^*(\theta^*)$.

The different game concepts will be illustrated in the next section. It should be pointed out that the Stackleberg game is the most realistic game approach. This is a dynamic game and can be solved using the backwards induction method, see e.g. Basar and Olsder (1995). For more complex games, mathematical bi-level problem formulations can be used for solving these games, see e.g. Joksimovic et *al*. (2004).

3 A few experiments

In this section we will look at the following simple problem to illustrate how the road pricing problem can be analyzed using game theory. Suppose there are two travelers traveling from origin A to destination B. There are two alternative routes available to go to B. The first route is tolled (toll is equal to θ), the second route is untolled. Depending on the toll, the travelers can decide to take either route 1 or route 2, or not to travel at all. The third choice is represented by a third virtual route, such that we can consider three route alternatives as available strategies to each traveler, i.e. $S_i = \{1,2,3\}$ for traveler i = 1,2. Figure 1 illustrates the problem.

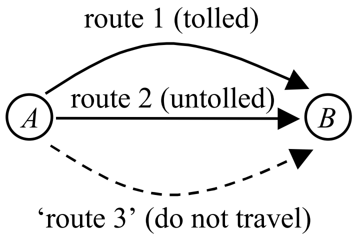

Figure 1: Network description of the road pricing problem

Each strategy yields a different utility, depending on the utility to make the trip, the travel time on the route (that increases whenever more travelers use it) and a possible route toll. We assume that each traveler i aims to maximize his/her individual travel utility (payoff,) given by

$$J_i(s_1(\theta), s_2(\theta)) = \begin{cases} \bar{U} - \alpha\tau_1(s_1(\theta), s_2(\theta)) - \theta, & \text{if } s_i(\theta) = 1, \\ \bar{U} - \alpha\tau_2(s_1(\theta), s_2(\theta)), & \text{if } s_i(\theta) = 2, \\ 0, & \text{if } s_i(\theta) = 3. \end{cases} \tag{3}$$

In Equation (3), $\bar{U}$ represents the trip utility when making the trip to destination B (we assume that $\bar{U} = 210$), $\tau_r(\cdot)$ denotes the route travel time for route r depending on the chosen strategies, and α represents the value of time (we assume that $\alpha = 6$ for all travelers). Note that negative net utilities on route 1 and 2 means that one will choose not travel, i.e. if the cost (disutility) of making the trip is larger than the utility of the trip itself. The route travel times are given as a function of the chosen strategies in the sense that the more travelers use a certain route, the higher the travel time:

$$\tau_1(s_1(\theta), s_2(\theta)) = \begin{cases} 10, & \text{if } s_1(\theta) = 1 \text{ or } s_2(\theta) = 1 \text{ (e.g. route flow on route 1 is 1)}, \\ 18, & \text{if } s_1(\theta) = 1 \text{ and } s_2(\theta) = 1 \text{ (e.g. route flow on route 1 is 2)}, \end{cases} \tag{4}$$

and

$$\tau_1(s_1(\theta), s_2(\theta)) = \begin{cases} 20, & \text{if } s_1(\theta) = 2 \text{ or } s_2(\theta) = 2 \text{ (e.g. route flow on route 2 is 1)}, \\ 40, & \text{if } s_1(\theta) = 2 \text{ and } s_2(\theta) = 2 \text{ (e.g. route flow on route 2 is 2)}. \end{cases} \tag{5}$$

Solving the game between the two travelers for a Nash equilibrium corresponds to a Wardrop equilibrium with elastic demand, in which no traveler can improve his/her utility by unilaterally changing route or deciding not to travel. For the sake of clarity we will only look at pure strategies[1] in this example, but it may be extended to mixed strategies. The utility payoff table, depending on the toll θ, is given below in Table 1 for the two travelers, where the values between brackets are the payoffs for travelers 1 and 2, respectively.

		Traveler 2		
		Route 1	Route 2	Route 3
Traveler 1	Route 1	$(102-\theta, 102-\theta)$	$(150-\theta, 90)$	$(150-\theta, 0)$
	Route 2	$(90, 150-\theta)$	$(-30, -30)$	$(90, 0)$
	Route 3	$(0, 150-\theta)$	$(0, 90)$	$(0, 0)$

Table 1: Utility payoff table for travelers

For example, if traveler 1 chooses route 1 and traveler 2 choose route 2, then the travel utility for traveler 1 is $J_1(1,2) = 210 - 6\cdot 10 - \theta = 150 - \theta$. Now, let us add the road manager as a player, assuming that he tries to maximize the total system utility, i.e.

$$R\left(s^*(\theta), \theta\right) = J_1(s^*(\theta)) + J_2(s^*(\theta)). \tag{6}$$

The strategy set of the road manager is assumed to be $\Theta = \{\theta \mid \theta \geq 0\}$. Depending on the strategy $\theta \in \Theta$ that the road manager plays and depending on the strategies the travelers play, the payoffs for the road manager are presented in Table 2.

		Traveler 2		
		Route 1	Route 2	Route 3
Traveler 1	Route 1	$204-2\theta$	$240-\theta$	$150-\theta$
	Route 2	$240-\theta$	-60	90
	Route 3	$150-\theta$	90	0

Table 2: Utility payoff table for the road manager

Let us solve the previous defined payoff tables for different game concepts and different values of tolls. First, we discuss the monopoly game, then the Stackleberg game and finally the Cournot game.

[1] In pure strategies, each player chooses only one strategy, whereas in mixed strategies, there are probabilities for choosing each strategy. In terms of a Wardrop user-equilibrium, we are looking at discrete flows instead of continuous flows. Wardrop's first principle that all travel utilities are equal for all used alternatives may no longer hold in this case. In fact, the more general equilibrium rule applies in which the travelers aim to maximize the minimum travel utility for all players.

Monopoly game

In the monopoly game, the road manager sets the toll as well as the travel decisions of the travelers such that his utility is maximized. Note that the utility always decreases as θ increases, hence $\theta^* = 0$. In this case, the maximum utility can be obtained if the travelers distribute themselves between routes 1 and 2, i.e. $s^* = \{(1,2),(2,1)\}$. Hence, in this system optimum, the total travel utility in the system is 240. Note that this optimum would not occur if travelers have free choice, since $\theta = 0$ yields a Nash-Wardrop equilibrium for both travelers to choose route 1.

Stackleberg game

Now the travelers will maximize individually their own travel utility, depending on the toll set by the road manager. Figure 2 illustrates the total system utility for different values of θ with the corresponding optimal strategies played by the travelers. When $0 \le \theta < 12$, travelers will both choose route 1. If $12 \le \theta < 150$, travelers distribute themselves between route 1 and 2, while for $\theta \ge 150$ one traveler will take route 2 and another traveler will not travel at all. Clearly, the optimum for the road manager is $\theta^* = 12$, yielding a total system utility of 228.

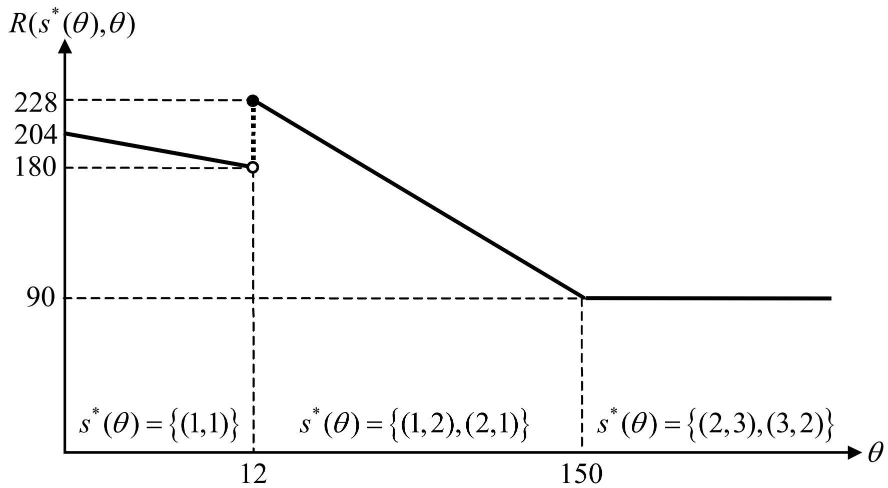

Figure 2: *Total system utility depending on toll value*

Cournot game

It can be shown that when the travelers and the road manager have equal influence on each others strategies, that there are multiple Cournot solutions. There is however one dominating strategy, being that the travelers both take route 1 and that the road manager sets zero tolls, yielding a total system utility of 204.

The following table summarizes the outcomes for the different games.

Game	θ^*	$s_i^*(\theta)$	R	J_i
Monopoly	0	$\{(1,2),(2,1)\}$	240	$\{(90,50),(50,90)\}$
Stackelberg	12	$\{(1,2),(2,1)\}$	228	$\{(78,50),(50,78)\}$
Cournot	0	$\{(1,1)\}$	204	$\{(102,102)\}$

Table 3: Comparison between outcomes of different games

4 Conclusions

The purpose of the paper was to gain more insight into the road pricing problem using concepts from game theory. The theory presented here can be extended to include e.g. departure time choice, heterogeneous travelers and imperfect information.

5 References

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