Updating O/D Matrices and Calibrating Link Cost Functions Jointly from Traffic Counts and Time Measurements

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In this paper we analyse the problem of joint updating of the demand and link cost parameters from traffic counts and travel time measures. Considering an infrastructure network with a defined topology, *assignment* models (Fig. 1) simulate how origin-destination demand flows affect link flows in a transportation network, and resulting performance. Traffic assignment models (Wardrop, 1952; Cantarella and Cascetta, 1995) have as input a supply model and a demand model and give as output link *flows* and link performance in term of *costs*.

A variant of the assignment problem consists in the *demand update from traffic counts* (Fig. 1). In this problem, with respect to the assignment problem, the demand (in terms of level and model parameters) is an output and the traffic flows are an input. Various models in terms of their theoretical approach have been proposed, differing in relation to the demand or to the model demand parameters update. As regards the demand update, it differs in relation to the link cost functions and to the users' behaviour hypothesis and in the elements to update: level of trips for different origins, destinations and demand parameters. Two different formulations can be considered for congested or uncongested networks. In an uncongested network the following problems were proposed: based on the maximum entropy method (Van Zuyle and Willumsen, 1980); based on the statistical method like generalized minimum least square (Cascetta, 1984), Bayesian (Maher, 1983), multicriteria (Brenninger-Gothe *et al.*, 1989); based on a combination of the previous methods (McNeil, 1983; Nguyen, 1984; Ben Akiva *et al.*, 1985; Cascetta and Nguyen, 1988). In a congested network the problem was proposed: an optimization problem on two levels (Fisk, 1988); a single-level optimization problem (Bell *et al.*, 1996); as a fixed point (Cascetta and Postorino, 2001). For congested networks the convexity of the objective function and solution uniqueness are not guaranteed. For each model different procedures were proposed and are generally based on heuristics. An analysis for these models is reported in Cascetta (2001). Many models proposed in recent years deal with the use of traffic counts to estimate Origin/Destination (O/D) trip matrices under different assumptions on the type of "a-priori" information available on demand (surveys, outdated estimates, models, etc.) and the type of network and assignment mapping (see Cascetta and Nguyen 1988). In relation to the model demand parameters update, in comparison to the demand update, it has received less attention in literature (Hogberg, 1976; Willumsen, 1981; Willumsen and Tamin, 1989, Cascetta and Russo, 1997). On the other hand, the practical implication in the calibration of model demand parameters deriving from traffic counts is notable.

Another variant of the assignment problem consists in the *travel time calibration for Origin-Destination (O/D) pairs* (Fig. 1). In this problem, with respect to the assignment problem, the link cost parameters are an output and the costs are an input. In this case the literature can be divided into two periods: in the first period link cost functions and link flow functions are calibrated in order to define the relation between the link flow or density and link cost or speed (see for example the publication of the Bureau of Public Roads, 1964, the TRRL, 1980a, 1980b, 1980c, Transportation Research Board, 1985); in the second period travel time and cost are measured for O/D pairs with floating cars and the proposed works can be divided by uncongested and congested networks. In uncongested networks the following problems were proposed: travel time measurement and flow, density and speed estimation (Bolla *et al.*, 2000); estimation of the number of floating cars and the percentage of links to monitor (Srinivasan and Jovanis, 1996). In congested networks the following problems were proposed: travel time measurement on the links and travel time on the Origin/Destination estimation (Lehmann and Kwella 1998).

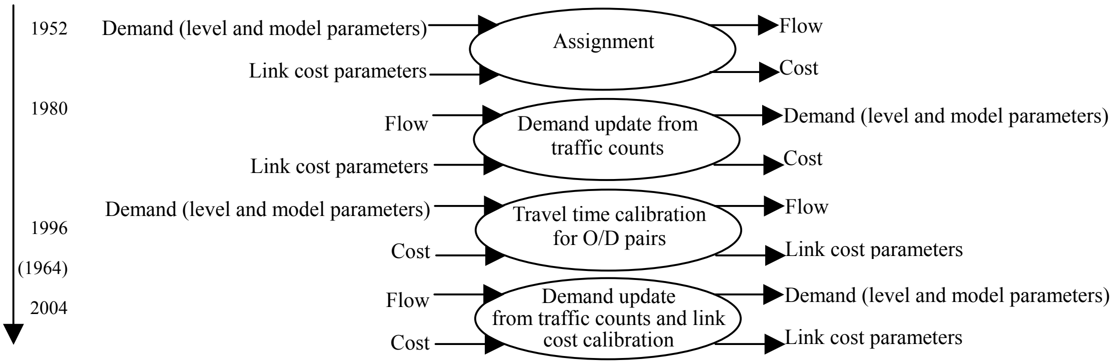


Fig. 1 - Demand/Supply interaction problems proposed in the literature

From state-of-the-art analysis it can be observed that the two problems of updating the demand and/or calibrating model demand parameters and updating or estimating the link cost parameters are studied separately. Link cost functions are calibrated separately from the demand matrix and during the system simulation steps are aggregated within the same procedure. Congruence between the two calibrations is not guaranteed. The two problems studied within the same method are equivalent to a *reverse assignment* problem (Fig. 1) where the inputs are the link flows counted and the link costs measured and the outputs are the demand in terms of level and/or model parameters and link cost parameters. The joint study of the two problems is important for strategic, tactical and operative planning. For strategic and tactical planning it is important to have a correct congruence between demand, link cost functions, and simulated flows in the current configuration and to simulate the design configurations. For operative planning it is important for the application in the ITS (Intelligent Transportation System) area especially for transit system in urban area. In this case information about the system state derives from: flow measurement instrument (like loops) and travel time measures (like GPS installed on buses). The information is used separately for demand update or for link cost update. For the correct simulation of the system a reverse assignment problem has to be applied.

The demand update from traffic counts problem in congested system is in some case an undetermined problem because the solution uniqueness is not guarantee. By including the link cost calibration another set of coefficients are considered and it makes in literature the problem even more underdermined. In this paper a model and a procedure for calibrating link cost functions and updating O/D matrices from traffic counts are proposed within the same method. The method can also be applied to calibrate link cost functions and a demand model (for example at route choice or modal split level). Input of the model consists of the initial O/D matrix, travel times measured on different links on the network and in different time slices, the link cost functions for each link category and a simulation model for user behaviour. The output of the model consists in the optimal O/D matrix, the parameters calibrated on link cost functions and the optimal flow on the links.

In order to verify if the method could be used for obtaining valid results, the method was applied in a test system. An initial true O/D matrix and BPR link cost functions were assumed. For different sets of parameters the global method was applied and the optimal results were compared. The model is formulated with an optimization model where the minimization of the distances between initial and optimal O/D matrix, simulated and counted flows, simulated and measured travel times are considered. For congested networks the convexity of the objective function is not guaranteed and some hypotheses are reported on the link cost functions that can be used. In the optimization approach the solution is obtained by the specification of an objective function and in its minimization. The objective functions in the two different formulation are:

$(\mathbf{d}^*, \alpha^*) = \arg\min_{\mathrm{d}^\circ \in \mathrm{Sd},\, \alpha \in \mathrm{S}\alpha} \Gamma_1(\boldsymbol{d}, \mathbf{d}^{\sim}, \boldsymbol{f}, \mathbf{f}^{\wedge}, \boldsymbol{c}, \mathbf{c}^{\wedge})$

or

$(\boldsymbol{\beta}^*, \alpha^*) = \arg\min_{\beta \in \mathrm{S}\beta,\, \alpha \in \mathrm{S}\alpha} \Gamma_2(\boldsymbol{f}, \mathbf{f}^{\wedge}, \boldsymbol{\beta}, \boldsymbol{\beta}^{\sim}, \boldsymbol{c}, \mathbf{c}^{\wedge})$

where

- $\mathbf{d}^*$, β^*, α^* are respectively the optimal values for demand, demand model parameters and link cost model parameters;

- $\boldsymbol{c} = \gamma(\boldsymbol{f}, \alpha)$ a vector of link cost functions obtained from the estimated flow $\boldsymbol{f}$ and link cost parameters α;
- $\boldsymbol{f} = \boldsymbol{\nu}(\boldsymbol{c}, \boldsymbol{d})$ an assignment model obtained from the vector of link cost functions ($\boldsymbol{c}=\gamma(\boldsymbol{f}, \alpha)$) and the demand vector to estimate $\boldsymbol{d}$;
- $\mathbf{d} = \omega(\beta)$ a demand models vector;
- $\Gamma_1(\boldsymbol{d}, \mathbf{d}^{\sim}, \boldsymbol{f}, \mathbf{f}^{\wedge}, \boldsymbol{c}, \mathbf{c}^{\wedge}) = z_1(\boldsymbol{d}, \mathbf{d}^{\sim}) + z_2(\boldsymbol{f}, \mathbf{f}^{\wedge}) + z_4(\boldsymbol{c}, \mathbf{c}^{\wedge})$;
- $\Gamma_2(\boldsymbol{f}, \mathbf{f}^{\wedge}, \boldsymbol{\beta}, \boldsymbol{\beta}^{\sim}, \boldsymbol{c}, \mathbf{c}^{\wedge}) = z_2(\boldsymbol{f}, \mathbf{f}^{\wedge}) + z_3(\beta, \beta^{\sim}) + z_4(\boldsymbol{c}, \mathbf{c}^{\wedge})$;
- $z_1(\boldsymbol{d}, \mathbf{d}^{\sim})$ a distance index between the demand vector to estimate $\boldsymbol{d}$ and the *a priori* demand vector $\mathbf{d}^{\sim}$;
- $z_2(\boldsymbol{f}, \mathbf{f}^{\wedge})$ a distance index between the flow $\boldsymbol{f} = \boldsymbol{\nu}(\gamma(\boldsymbol{f}, \alpha), \boldsymbol{d})$ obtained from the estimated flow $\boldsymbol{f}$, demand $\boldsymbol{d}$ and link cost parameters α values and the a priori link flow vector counted $\mathbf{f}^{\wedge}$;
- $z_3(\beta, \beta^{\sim})$ a distance index between the parameter demand vector to estimate β and the a priori parameter demand vector $\beta^{\sim}$;
- $z_4(\boldsymbol{c}, \mathbf{c}^{\wedge})$ a distance index between the cost $\boldsymbol{c} = \gamma(\boldsymbol{f}, \alpha)$ obtained from the estimated flow $\boldsymbol{f}$ and link cost parameters α and the a priori link cost vector measured $\mathbf{c}^{\wedge}$.

In order to update the O/D matrix and calibrate the link cost parameters from traffic counts and travel time measures, in the Fig. 2 a scheme of the whole procedure is reported where: **Y**, **Z**, **W**, **H** the variance covariance matrixes for the distance index respectively for z_1, z_2, z_3, z_4; **M** the assignment matrix obtained as $\mathbf{M} = \Delta\, \pi(\Delta^{T}\, \boldsymbol{c}, \boldsymbol{\beta})$ where is Δ the link-path incidence matrix and π the matrix of route choice probability on all O/D pairs; Ψ the objective function for one of the two model proposed.

In order to verify the possibility of applying the method for obtaining valid results, in terms of its procedure, on a transportation system and for testing the convergence to the optimal point, the method of reverse assignment for O/D update and link cost parameters calibration is applied in the test system reported in Fig. 3. The system has four origin nodes (1, 2, 3 and 4), four destination nodes (1, 2, 3 and 4), seven bi-directional real links. For the application initial true values for demand and link cost parameters are considered. Considering the true values of the demand and the link cost parameters the counted flows and the measured costs were obtained in 8 links with a DUE assignment (link selected with || in Fig. 3).

For applying the method the true values of the demand and link cost parameters are modified with a random variation, with a prefixed percentage of variation, to obtain the starting values of the demand and link cost parameters. These starting values simulate the demand values and the link cost parameters that are used in practical applications and that differ from the true values which are not known. To eliminate the effect of random variation in the final results 10 different initial values for demand and link cost parameters were generated and the results reported are the averages of the 10 different optimizations.

As regards comparison of different procedures, interesting results can be obtained with the results reported in Tab. 1 and partially graphically in Fig. 4. The results reported in (A) and (B) concern the traditional procedure of the demand update from traffic counts. (A) refers to the optimization considering true initial values in the link cost parameters and it is just a joker since knowledge of the true cost parameters values is assumed. If the procedure is applied with true values of the link cost parameters, the true flows on the network and the costs are reproduced also with high variation in the initial values of the demand. This is the case in which all the parameters are known. Hence, if the application of the traditional procedure is developed with un-knowledge in the initial link cost parameters (B) the true link cost parameters are not reproduced and for large errors in the initial demand also the true demand is not reproduced. From the results reported in (C) it emerges clearly that if the demand and the link cost parameters are jointly updated from traffic counts and time measures, the final demand and cost are better reproduced with respect to the traditional procedure.

In relation to the optimal objective function value (in cases A and B sum of the OD and Flow columns in MSE optimal-true; in case C sum of the Cost, OD and Flow columns in MSE optimal-true), in the optimal point it is not greatly influenced by the coefficients of variation but it is greatly affected on the random variation level of the true demand or cost fixing the other parameters. These results show that the final result depends considerably on the error present in the initial value of the demand and the link cost parameters and on the error present in the counted flows.

The work proposed in this paper opens up new research prospects for researchers mainly interested in demand together with supply and assignment. The main research perspectives are: model for defining a fixed point problem instead of an optimization problem with solution uniqueness; procedure for generating the local optimal and convergence test; a procedure to compare local and global optimal solutions; experimentation on real systems; extension to the dynamic approach; calibration of demand model parameters instead of demand values.

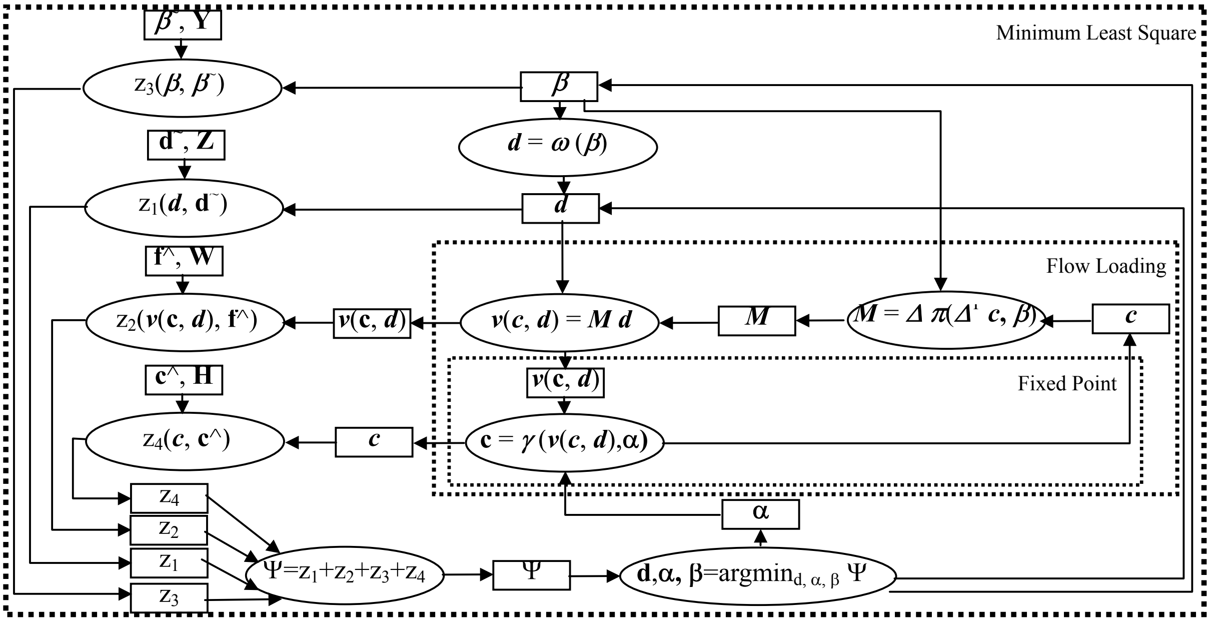


Fig. 2 - Procedure for O/D updating and link cost parameters calibration from traffic counts and travel time measurement: reverse assignment

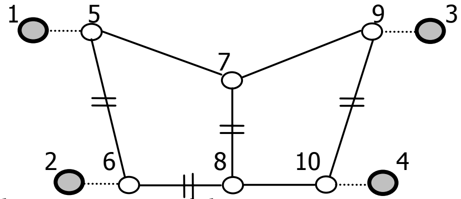
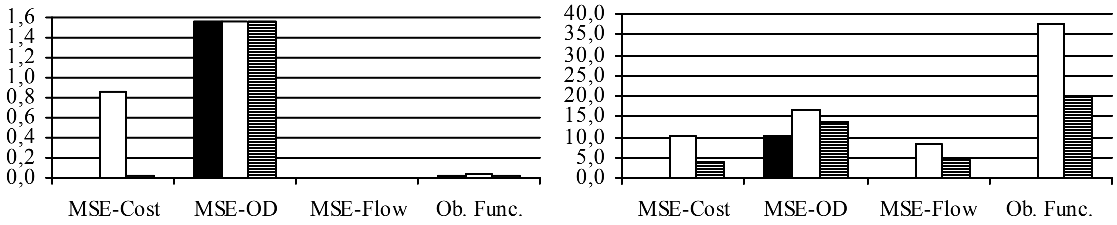


Fig. 3 - Characteristics of the transportation system

Tab. 1 - Result obtained from optimization

variation %		Optimization only on the demand												Joint optimization					
		(A) Random variation in demand						(B) Random variation in demand and cost						(C) Random variation in demand and cost					
		MSE begin-true			MSE optimal-true			MSE begin-true			MSE optimal-true			MSE begin-true			MSE optimal-true		
α_1, α_2	OD	Cost	OD	Flow	Cost	OD	Flow	Cost	OD	Flow	Cost	OD	Flow	Cost	OD	Flow	Cost	OD	Flow
10	10	2.8	6.6	6.0	0.0	1.6	0.0	3.9	6.6	6.0	0.9	1.6	0.0	3.9	6.6	6.0	0.0	1.6	0.0
20	20	10.9	26.0	36.3	0.0	10.1	0.0	16.5	26.0	30.9	10.1	16.4	8.5	16.5	26.0	30.9	3.8	13.8	4.5

Assignment DUE - C_V = coefficient of variation for variance in objective function (cost=0.2, OD=1, Flow=0.01) - α_1, α_2 = parameters in BPR link cost function



Random variation link cost parameters 10%, OD 10%   Random variation link cost parameters 20%, OD 20%

O/D optimization (■) without and (☐) with initial random variation on the cost; Joint optimization (▤)

Fig. 4 – Comparison between indicators with different initial parameters

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