# **Analysis of Congested Networks**

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## 1 Background

Transport systems are full of interactions between the demand for travel and the supply costs. As the demand increases, congestion builds up in the network and the costs of travel increases. In response to increased costs of travel, people may alter their demand for travel in terms of change of route, departure-time, mode, or choose not to travel. This demand/supply interaction enables an economic analysis to be undertaken of any proposed intervention, either in the way the demand arises or in the supply that the network offers, so as to reduce the impact of congestion.

Traditional modelling techniques, based on speed-flow relationships, have been criticised for failing to represent the consequences of flow-breakdown on the network and the behavioural responses of drivers to the resulting congestion (May et al, 2000). They specify demand and supply in terms of the vehicles per hour actually occurring on the network rather than the trips they represent being accomplished over a given period, the costs of which will depend upon the degree of congestion that they encounter.

This paper presents findings from a recent study funded by UK Department for Transport which took a fresh look at how demand and supply should be characterised in relation to travel on congested urban networks, including the units in which they should be measures, and to assess the extent to which micro-simulation might offer advantages over more conventional aggregated models in representing the complex demand/supply interactions (Hill et al, 2001). The initial study is focused mainly on the supply-side issues.

## 2 Characterisation of congestion

This section gives a summary description of the ways in which congestions builds up in single links and in networks. This has been used to identify the important characteristics of both demand and supply which ideally need to be reflected in a model, using micro-simulation techniques.

In single links queuing occurs when inflow exceeds capacity, and may lead to a loss of throughput if friction occurs at the point which determines capacity. If inflow in excess of capacity continues for long enough, queues may form to enter the link. In response to increased costs, demand may change by drivers deciding no longer to travel, or to use other modes, or to share vehicles, or to reschedule their journeys. Re-routeing is not feasible.

In networks the situation is complicated by the interaction between these effects on different single links. Queues may block upstream junctions, leading to localised reductions in throughput for certain movements. These effects will be influenced by the physical layout of the network and traffic control at intersections. In addition they will be influenced by the spatial and temporal shapes of the O-D matrix. For example, movements which are longer will have a greater impact on the network. Similarly, delays resulting from earlier periods of high demand will have a continuing impact on later movements. In response to increased costs, demand may change in all of the ways described for single links. In addition, as well as re-routeing, all of which will in turn change the impact of the O-D matrix on the network.

These considerations suggest that a micro-simulation of congested networks should ideally reflect the following supply side attributes:

- link types
- numbers of lanes
- network shape
- type of junction control
- parking availability

and the following on the behavioural side:

- spatial distribution of the O-D matrix
- temporal distribution of the O-D matrix
- rescheduling in response to costs
- re-routeing in response to costs
- other demand responses, at least in aggregate.

### **3** Distinction between performance and supply curves

Performance curves relate the parameters of the traffic in a network at a given time. They are based on measurements of vehicle-km and vehicle-hr in a network in a given time period, and can be used to estimate network equivalents of speed-flow curves. These parameters can be measured, and have been used to describe the way in which costs of using the network rise as usage increases.

Supply curves reflect the costs experienced by a driver at a given level of demand, which will be affected by the journey length and the route taken, as well as by the impacts of other demands on the network both at the same time and in earlier time periods. Supply curves cannot be readily observed in the way that performance curves can.

The space-time domains used to measure performance curves and supply curves are different, as shown in Figure 1. The supply curves are generated through a vehicle-tracking approach (May et al, 2000), whereby individual vehicles' journey time and distance from origin to destination are used to give a generalised cost. The resulting generalised costs are then summarised by vehicles' departure-times to give supply costs for the departure time-period of interest. (A summary by vehicles' arrival-times gives the supply costs for the arrival time-period.) The supply curves are then represented as generalised costs per trip vs. the trips demanded over a given time period.



Fig. 1 Space-time domains.

The performance measures are generated through a time-slice approach, whereby vehicle trajectories (time and distance) within a specific time-period are used to get the generalised cost for the period. There is no distinction as to the departure- or arrival-time of individual vehicles.

## 4 Simulation tests and data analysis

The steady-state equilibrium assignment model SATURN (van Vliet, 1982) and a dynamic microscopic simulation model, DRACULA, are used in the study.

The dynamic network model DRACULA (Dynamic Route Assignment Combining User Learning and microsimulation) has been developed at University of Leeds since 1993 (Liu et al 1995). It adapted a new approach to modelling road traffic networks whereby the emphasis is on the "microsimulation" of individual trip makers' choices and individual vehicles' movements. A day-to-day choice model simulates for each potential traveller decisions on whether to travel, if to travel, the route to be taken, and the preferred departure time. This information is then passed to a traffic micro-simulation model which follows individual vehicles through the network along the pre-defined fixed routes and simulates their en-route interactions with other traffic and with traffic controls. At the end of the "day" (the study period), drivers learn from their experience and update their perception of the network which will in turn influence their future travel decisions.

In DRACULA, the simulation tracks individual vehicles along pre-specified routes to their destinations, rather than using junction-by-junction turning percentages (which can lead to implausible, cyclic routes). The simulation runs over a variable period by the end of which all vehicles from a given departure-time period have reached their destinations, rather than over a fixed time period. The DRACULA model is thus able to measure directly the supply costs as defined in Section 3.

In the study, the supply-side issues were investigated using two "idealised" networks with a steadily increasing level of travel demand to determine how these two networks would perform and how supply costs could be estimated using different modelling approaches. The first network is a single O-D, two-route network, used to enable the underlying principles to be understood. The second is a broadly symmetrical ring-radial network, based loosely on conditions in Cambridge.

The first, simple network tests were used to compare the supply costs as determined by SATURN with those determined by DRACULA, as the demand in trips is creased. It was also used to investigate the effect of using a peaked temporal distribution of demand relative to a flat distribution of demand.

Figure 2 shows the supply curves from SATURN and DRACULA simulation of the simple network with a flat and a peaked distribution of demand. It can be seen that DRACULA simulated costs (test 2 curve) are higher than those from SATURN at medium to high demand levels (test 1).

The peaked distribution of demand induced higher costs: the supply costs from SATURN multiple time-period modelling (test 5 curve) are much higher than those from a single time-period assignment model (test 1). However, they are still not as high as the supply costs estimated from a DRACULA run (test 4) which allows route choice to be dynamically estimated, separated for the different time-periods.



Supply relationships

Fig. 2 Simulation results on the simple network. Test 1 (shown in triangles) is a SATURN single time-period equilibrium runs on a uniform (flat) distribution of demand. Test 2 (squires) is a DRACULA (single-day) simulation with flat demand and fixed route choices as obtained from test 1. Test 4 (circles) is DRACULA day-to-day simulation with a peaked distribution of demand and dynamic route choice, whilst test 5 (diamonds) a SATURN multiple time-period modelling of a peaked demand.

The second network was used to investigate the added dimension of different spatial distribution of demand and vehicle interactions within a network. The base matrix of the network has been calibrated to traffic conditions in Cambridge. In addition, three more matrix shapes are tested, one with increased inbound movements compared to the base case, one with increased crosstown movements which need to use both radials and orbitals, and one with increased through traffic, which can choose between radial and orbital routes.

The changes in matrix shape were used to provide approximate estimates of the marginal cost of trips on different O-D pairs, for differing levels of demand. This is done by dividing the increase in total cost from the base matrix by the number of extra trips at each level of the matrix factor:

$$C_{nm}^{marginal} = \frac{\partial (\sum_{i,j} (T_{ij} \times C_{ij}))}{\partial T_{nm}}$$
(eq. 1)

where  $\partial T_{nm}$  is the increase in trips for O-D pair (n,m) and  $C_{nm}^{marginal}$  the marginal cost for the O-D pair. A similar method can be used to calculate the marginal cost of the average trip in a matrix from comparison of total costs as the scaling factor F is increased in a given matrix :

$$C_{T_F}^{marginal} = \frac{\partial (\sum_{i,j} (T_{ij} \times C_{ij}))}{\partial T_F}$$
(eq. 2)

where  $\partial T_F$  is the increase in trips for matrix **T** as the scaling factor F is increased and  $C_T^{marginal}$  the marginal cost of the average trip for matrix **T** at demand level F.



Figure 3: Movement-specific marginal cost functions as simulated by SATURN.

Figure 3 shows the marginal costs plotted against demand factor F, for three main O-D movements. It can be seen that the marginal cost of a through-town trip is higher than that for the other movements at lower demand levels. The difference could be due to the fact that through-town trips have longer trip length. As demand increases, the effect of congestion means that cross-town movements become more costly, hence the more rapidly increased marginal cost curve of the cross-town movement. Such an analysis could be used, with appropriate supply curves, to estimate the appropriate congestion charge for each movement, and the movements on which demand management should first focus.

## 5 Summary

This paper presents a fresh look at how demand and supply should be characterised in relation to travel on congested urban networks. A clear distinction between network performance measures and supply costs is made, and method developed to generate performance and supply costs in network micro-simulation models.

Simulation tests were carried out on a simple two-route network and a more complex network based on Cambridge using an equilibrium assignment model SATURN and a dynamic micro-simulation model DRACULA. The results suggest that the congestion effect is better represented with a dynamic micro-simulation model; that it is important to ensure that temporal variations in demand are taken into account in estimating the cost of congestion; and that marginal cost curves for different O-D pairs can vary substantially which has important implications for the way in which demand management measures are applied to them.

Further research will include more extensive examination of departure-time profiles and the influence of arrival constraints, investigation of higher levels of spatial distribution and the implications for O-D based supply curves, and the inclusion of trip-specific costs (e.g. parking charges) and distance-related variable costs and their implications on demands.

## References

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