Robust and Flexible Vehicle Routing in Practical Situations

Kenneth Sörensen* Marc Sevaux†

*University of Antwerp
Faculty of Applied Economics
Prinsstraat 13, 2000 Antwerp, Belgium
kenneth.sorensen@ua.ac.be

†LAMIH-ROI, Valenciennes University
Le Mont-Houy
59313 Valenciennes, France
marc.sevaux@univ-valenciennes.fr

1 Introduction

The objective of vehicle routing problems is to determine the order in which to visit a spatially distributed set of customers. Mainly due to their practical importance and the difficult challenge they pose, routing problems have received an enormous amount of research attention. In this paper we focus on the *capacitated, distance-constrained* vehicle routing problem, although the proposed method for robust/flexible optimisation is applicable to any vehicle routing formulation.

The computational complexity of most routing problems has made them an important candidate for solution using metaheuristics. Indeed, for most routing problems, metaheuristics dominate the list of best-performing solution methods. The literature on metaheuristics for the vehicle routing problem has been thoroughly surveyed by Gendreau et al. (1998). A comprehensive comparison of the computational results reported by several approaches, finds that tabu search approaches dominate the list of successful algorithms, especially the approach due to Taillard (1993). Recently, Prins (2004) has shown that genetic algorithms can achieve similar performance as tabu search, when combined with extensive local search.

Standard vehicle routing formulations however, assume that all data concerning customer demand, travel costs, etc. are known with perfect certainty at design time. For many reasons, e.g. the uncertainty related to traffic conditions, these assumptions are unwarranted in a large number of practical situations. As a result, a number of *stochastic vehicle routing* formulations have been developed. Common examples of stochastic vehicle routing problems have either stochastic travel times (e.g. Lambert et al. (1993)) or stochastic demands (e.g. Stewart and Golden (1983)). In some cases, the list of customers to be visited is stochastic in the sense that each customer has a certain probability of requiring service and the actual routes can only be developed at design time. An approach called *a priori routing* is developed by Bertsimas (1992); Bertsimas and Simchi-Levi (1996). Algorithms for stochastic vehicle routing problems are considerably more intricate than their deterministic counterparts, and the optimal solution can be found within a reasonable amount of time for very small problems only. For a review of stochastic vehicle routing formulations and problems, we refer to Gendreau et al. (1996). Vehicle routing problems in which not all the data are known at planning time are called *dynamic vehicle routing problems*. For a recent survey, we refer to Gendreau and Potvin (1998).

In this paper we develop and apply a framework to find robust and flexible optimisation of stochastic variants of vehicle routing problems. This framework combines metaheuristic optimisation with monte-carlo sampling of the stochastic parameters of the problem. The approach and its advantages over more traditional methods based on stochastic programming are discussed in section 2. An example application is given in section 3.

2 General framework for robust and flexible optimisation

A minimisation problem can be written as

$$f(x^*; p) = \min_{x \in X(p)} f(x; p), \tag{1}$$

where f is the objective function and p is the set of parameters for the given problem instance. The optimal solution x^* should belong to the set of all feasible solutions for this problem instance (the domain $X(p)$). If the set of problem data contains some uncertain elements, we represent this set by π. For a given solution x, the objective function value $f(x; \pi)$ now is a random variable that cannot be minimised. Also, the set of feasible solutions $X(\pi)$ is stochastic, and as a result the feasibility of a given solution x depends on the realisations of the stochastic parameters.

A *robust* solution is characterised by the fact that it has a high quality across the set of potential realisations of the stochastic parameters of the problem. It is however important to note that the preferred formalisation of robustness can differ between decision makers. A *flexible* solution is one that can be easily adapted to the realisations of the stochastic parameters. Flexibility implies the existence of some procedure to adapt a solution to the specific outcomes of the stochastic parameters. We call such a procedure a *repair procedure* and require of it that it is several orders of magnitude faster (in the computational sense) than the optimisation procedure used to find the solution.

Overview Our framework for robust and flexible optimisation can be summarised as follows:

1. Use an optimisation procedure that generates many diverse solutions. Most metaheuristics will do this.
2. Evaluate each solution generated with a *robust evaluation function*. Perform the following steps n_e times and combine the evaluations into a measure of robustness/flexibility.
   (a) Sample the stochastic parameters of the problem for the given solution.

(b) (Only for flexible solutions) Use the repair procedure to improve the solution.

(c) Calculate the quality of the (repaired) solution

3. Pick the solution that performs best with respect to this robust evaluation function.

Robust evaluation function A typical robust evaluation function has the following form.

$$f^*(x) = \frac{1}{n_e} \sum_{i=1}^{n_e} f(x; \mathcal{S}_i(\pi)) \tag{2}$$

The *sampling function* $\mathcal{S}$ generates a potential outcome of the stochastic data π of the problem. $\mathcal{S}_i(\pi)$ is the i-th sampling of the stochastic problem data. The solution x is evaluated on n_e samples and these evaluations are then combined into $f^*(x)$—a measure of the robustness of x—by taking the average.

If a repair function is used, and a flexible solution is sought, a robust evaluation function is

$$f^*(x) = \frac{1}{n_e} \sum_{i=1}^{n_e} f(\mathcal{R}(x; (\mathcal{S}_i(\pi))) \tag{3}$$

I.e., a solution is first repaired using the repair function, before it is evaluated.

Incorporating the decision maker's risk-averseness If the decision maker is relative risk-averse, he might prefer to evaluate the worst-case performance of the solution. The robust evaluation function then becomes

$$f^*(x) = \max_{i=1}^{n_e} f(x; \mathcal{S}_i(\pi)).$$

A measure of the risk associated with a given solution is given by the standard deviation of the evaluations of the derived solutions, given by

$$\sigma^*(x) = \sqrt{\frac{1}{n_e - 1} \sum_{i=1}^{n_e} [f(x; \mathcal{S}_i(\pi)) - f^*(x)]}.$$

Several of these measures of robustness can be calculated for each solution and a solution can be chosen using a multi-objective decision making process. Other, even more complex expressions of robustness can be considered, such as the probability that the quality of the solution falls below a certain threshold.

Penalty functions Some solutions might become infeasible for some realisations of the stochastic parameters. The decision maker might therefore decide to only allow solutions that are feasible across all potential realisations. This would imply that a solution that is infeasible in at least one of the n_e cases, would receive an infinitely large robust evaluation function value. A less drastic approach is to allow for some infeasibility through the use of *penalty functions*. A robust evaluation function that incorporates penalty functions is of the form

$$f^*(x) = \frac{1}{n_e} \sum_{i=1}^{n_e} [f(x; \mathcal{S}_i(\pi)) + \mathcal{P}(x; \mathcal{S}_i(\pi))],$$

where $\mathcal{P}$ is a penalty function that should reflect the "severity" of the constraint violation of solution x under the realisation $\mathcal{S}_i(\pi)$ of the stochastic parameters.

Advantages The framework provides an answer to several problems that cannot be adequately dealt with by more traditional methods based on stochastic programming. First, since the framework requires only to adapt the objective function of a metaheuristic for a deterministic problem, it is very easy to use. Secondly, the framework is in principle applicable to any stochastic optimisation problem. There is no limit on the number and type of stochastic parameters that can be entered into the problem formulation. E.g. scenarios can be used if desired. Thirdly, the framework can easily be extended to—for example—include penalty functions for infeasible solutions or to take the risk preference of the decision maker into account. Fourthly, finding the optimum of large-scale stochastic problems is more often than not intractable. As indicated by Birge (1997), the complexity of stochastic programs grows proportionally to the number of possible realisations of the stochastic parameters, which in turn grows exponentially with the number of stochastic parameters. Because in realistic cases this number is usually very large or even infinite (in the case of continuous distributions), only very small problems can be solved to "optimality", however this is defined. Fifthly, robustness and flexibility are terms that can be used to express a number of different properties of a solution, of which average-case and worst-case performance are probably the most widely used, but many others can be considered. The framework does not force an unnecessary choice of objective onto the decision maker. Using the robust evaluation function, decision makers can determine their preferred type of robustness or flexibility and find the best solution according to their preferences. Different expressions of the concepts of robustness can be evaluated simultaneously

3 An example experiment

Due to lack of space, only one small experiment is discussed. The problem is a capacitated, distance-constrained vehicle routing problem with stochastic travel times and stochastic customer demand. The problem is defined on an undirected graph $G = (V, E)$ with a set of nodes $V = \{0, 1, \ldots, n\}$. Node 0 corresponds to the depot, that has a set of identical vehicles of capacity Q and maximal travel cost C. Nodes 1 to n represent a set of spatially distributed customers, the demand of which is given by q_i. The travel cost between customer i and customer j is given by c_{ij}, the weight of the edge between node i and node j. The objective of the deterministic VRP is to find a set of minimum total cost routes that have the following properties: (1) each route begins and ends at the depot, (2) each customer is visited exactly once, (3) the capacity and maximal travel cost of the vehicles is not exceeded.

To solve this problem, we have developed a GA|PM, or genetic algorithm with population management. A GA|PM is a memetic algorithm (a GA hybridised with local search) that uses distance measures to measure and control the diversity of a small population. For a complete description of GA|PM, we refer to Sörensen (2003).

In the stochastic problem, travel costs c_{ij} and customer demand q_i are assumed to be stochastic. The demand of customer i is uniformly distributed between $0.75\bar{q}_i$ and $1.25\bar{q}_i$, where $\bar{q}_i$ is the average demand of customer i. We assume that the average demand of a customer is equal to the demand in the deterministic case. Travel costs between customers i and j are uniformly distributed between $0.8\bar{c}_{ij}$ and $1.2\bar{c}_{ij}$. If the maximum capacity of a vehicle is exceeded, a penalty cost of $\alpha_2 = 500$ units per unit of exceeded capacity is incurred. If the travel cost in a route is greater than the maximum travel cost, a penalty of $\alpha_1 = 100$ units per cost unit is incurred. These values are deliberately set to a relatively high level to increase the need for robust solutions.

The dispatcher is interested in a solution that has a good average performance nd a good worst-case performance. Therefore, two objective functions are calculated for each solution encountered by the VRP. f_1^* measures the average performance of the solution. Let $f(x;p)$ represent the sum of travel costs in all the routes of solution x for a given set of demand and travel costs p.

$$f_1^*(x) = \frac{1}{n_e} \sum_k [f(x; \mathcal{S}_k(\pi)) + \mathcal{P}(x; \mathcal{S}_k(\pi))] \tag{4}$$

The second robust evaluation function $f_2^*(x)$ measures the worst-case performance of a given solution.

$$f_1^*(x) = \max_k [f(x; \mathcal{S}_k(\pi)) + \mathcal{P}(x; \mathcal{S}_k(\pi))] \tag{5}$$

The GA|PM with robust evaluation functions f_1^* and f_2^* is applied to 14 vehicle routing problems. A small extract containing the results of a single experiment, is given in table 1.

Data file	$\boldsymbol{n}$	**Criterion**	$\boldsymbol{f}$	$\boldsymbol{f_1^*}$	$\boldsymbol{\sigma^*}$	**best case**	$\boldsymbol{f_2^*}$
vrpnc01	50	f	549.76	3211.60	2891.00	523.73	17779.56
		f_1^*	604.76	605.01	13.21	567.72	876.66
		f_2^*	629.12	629.28	10.61	595.26	661.48

Table 1: Vehicle routing with stochastic demand and cost, example result

Table 1 lists results for data file vrpnc01 having 50 customers. The column *criterion* indicates which objective function has been minimised by the solution in the row. The solution in the first row minimises the ordinary objective function value f. It can be seen that this solution has a relatively low ordinary evaluation function value, but scores rather badly with respect to the first and second robust evaluation function and moreover, has a very high standard deviation. Although this solution scores very well in the deterministic case, it is clearly not robust. The solution in row 2 minimises f_1^*. This solution obviously also has a very good worst-case value and a very good standard deviation, indicating that it is *considerably* more robust than the solution in row 1. The solution in row 3 is the most conservative one, optimising its worst-case performance.

An interesting result is that both robust solutions have a relatively good score on the ordinary evaluation function. This is not surprising as the definition of a robust solution is one that has a good quality across the potential realisations of the stochastic parameters of the problem. The reverse however is not true. This can be clearly seen in figure 1. This figure shows the result of 200 generations (400 solutions) for data file vrpnc01. All solutions are evaluated using the ordinary evaluation function and the robust evaluation function f_1^*. Almost all solutions in this chart exhibit a relatively good ordinary evaluation function value, but many of them have a very bad robust evaluation function value. The opposite however is not true: all solutions that have a good robust evaluation function value, also have a good ordinary evaluation function value.

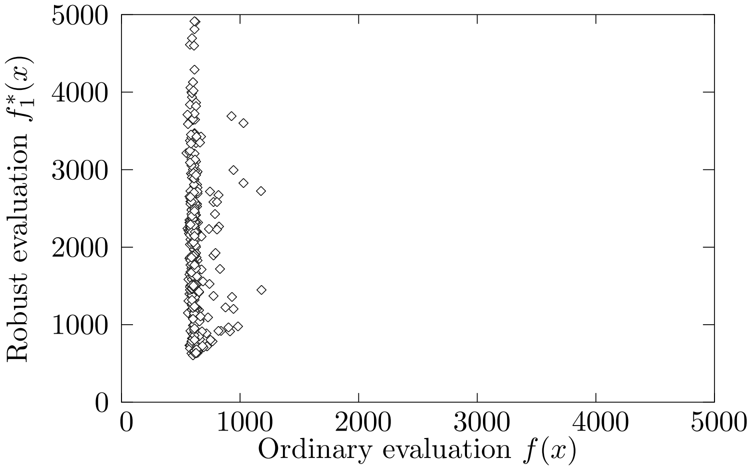

Figure 1: Ordinary evaluation and robust evaluation for vrpnc01

4 Conclusions

In this paper we have shown how a metaheuristics-based framework for robust and flexible optimisation can be succesfully applied to vehicle routing problems. The proposed method was shown to have several advantages over more traditional methods based on stochastic programming.

References

D. J. Bertsimas and D. Simchi-Levi. "A new generation of vehicle routing research: robust algorithms, addressing uncertainty", *Operations Research* 44, 286–304 (1996).

D.J. Bertsimas. "A vehicle routing problem with stochastic demand", *Operations Research* 40, 574–585 (1992).

J.R. Birge. "Stochastic programming computation and applications", *INFORMS Journal on Computing* 9, 111–133 (1997).

M. Gendreau, G. Laporte, and J.-Y. Potvin. "Metaheuristics for the vehicle routing problem", Technical Report G–98–52, GERAD (1998).

M. Gendreau, G. Laporte, and R. Séguin. "Stochastic vehicle routing", *European Journal of Operational Research* 88, 3–12 (1996).

M. Gendreau and J.-Y. Potvin. "Dynamic vehicle routing and dispatching", In T.G. Crainic and G. Laporte (editors), *Fleet Management and Logistics*, pages 115–126. Kluwer Academic Publishers, Boston (1998).

V. Lambert, G. Laporte, and F.V. Louveaux. "Designing collection routes through bank branches", *Computers and Operations Research* 20, 783–791 (1993).

C. Prins. "A simple and effective evolutionary algorithm for the vehicle routing problem", *Computers and Operations Research* (2004). To appear, available online since 24 May 2003.

K. Sörensen. *A framework for robust and flexible optimisation using metaheuristics with applications in supply chain design*,, Ph.D. thesis, University of Antwerp, Belgium (2003).

W.R. Stewart and B.L. Golden. "Stochastic vehicle routing: A comprehensive approach", *European Journal of Operational Research* 14, 371–385 (1983).

É. D. Taillard. "Parallel iterative search methods for vehicle routing problems", *Networks* 23, 661–673 (1993).