Spillback Modelling in Dynamic Traffic Assignment Using Implicit Path Enumeration

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1 Introduction

In a recent paper [Bellei, Gentile and Papola, 2002] we have proposed a new continuous implicit path formulation of Dynamic Traffic Assignment (DTA), where a user equilibrium is expressed as a fixed point problem in terms of arc flow temporal profiles. There, it is shown that, by extending to the dynamic case the concept of Network Loading Map, is no more needed to introduce the Continuous Dynamic Network Loading [see for instance Xu *et al.*, 1999] as a sub-problem of the DTA in order to ensure the temporal consistency of the supply model. On this basis it is possible to devise efficient assignment algorithms, whose complexity is equal to the one resulting in the static case multiplied by the number of time intervals in which the period of analysis is divided. With specific reference to a Logit path choice model, an implicit path enumeration network loading procedure is also obtained as an extension of Dial's algorithm; then, the fixed point problem is solved through a Method of Successive Averages. Another important feature of the proposed method lays in the fact that it does not exploit the intrinsic acyclic graph characterizing the corresponding discrete version of the network loading map. Specifically we do not introduce the hypothesis that the largest time interval must be shortest than the smallest free flow speed arc travel time. This way, at the algorithm level it is possible to define "long time intervals" (5-10 min) and this allows to solve large instances of the problem in reasonable computing time [Gentile and Meschini, 2003].

The fixed point problem is formalized by combining the network loading map and an arc performance function capable of representing explicitly the formation and dispersion of vehicle queues, but allowing the queue length to overcome the arc length (vertical queues).

The aim of this paper is to extend this formulation in order to take into account the interaction among network links upstream and downstream road intersections deriving from time varying entering and exiting arc capacities due to vehicle queue spillovers, which is equivalent to introducing constraints on the queue lengths.

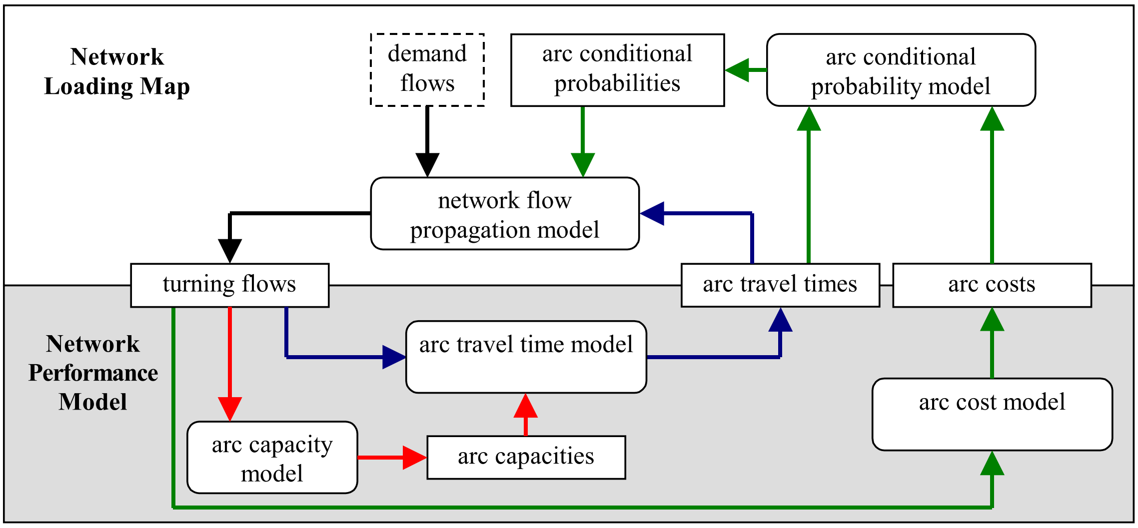
This approach is consistent with the definition of the *spillback* phenomenon provided by [Adamo *et al.*, 1999] as a hypercritic flow state, either propagating backward from the final section of an arc and reaching its initial section, or originating on the latter, that reduces the exiting capacities of the arcs belonging to its backward star and eventually influences their flow states. Specifically, it shall be pointed out that this paper aims at representing a "polite" spillback, where users that cannot enter a given arc due to the presence of a queue spillover do not occupy the intersection, but wait on the upstream arc until downstream the space necessary to their entrance becomes available.

To achieve this extension we introduce a sequence of models describing, for given turning flow temporal profiles, the dynamic of the network nodes and arcs, and capable of representing the propagation of congestion among contiguous road links. Specifically, the Network Performance Model will be articulated in a sequence of three models, namely the *arc entering capacity model*, the *arc exiting capacity model*, and the *arc travel time model for time varying exiting capacity*. Fixed point formulation of DTA with spillback

Limiting our attention to macroscopic flow modelling and thus neglecting the microscopic approaches to the problem, very few works in the literature achieve in representing spillback within DTA; among these [Daganzo, 1994 and 1995; Lebacque, 1996 and Adamo *et al.*, 1999]. The latter can be applied to networks with many origins and destinations, although it requires a thick "event based" time discretization, which results in a demanding algorithm applicable to limited size networks.

In this paper we extend to the representation of queue spillovers the model proposed in [Bellei, Gentile and Papola, 2002], where the DTA is formalized and solved as a fixed point problem in terms of the arc inflow temporal profiles. But here, in order to model correctly the spillback phenomenon, we consider the turning flows (that is, the inflows disaggregated by maneuver) as current variable of the fixed point problem, as these have a role in how the available capacity at a node is split among the upstream arcs.

By extending the concept of network loading map, yielding turning flows for given demand flows consistently with certain arc performances (travel times and costs), the equilibrium is formulated by combining it with the network performance model, yielding the arc performances for given turning flows as depicted in the figure below.



Scheme of the fixed point formulation of DTA with spillback.

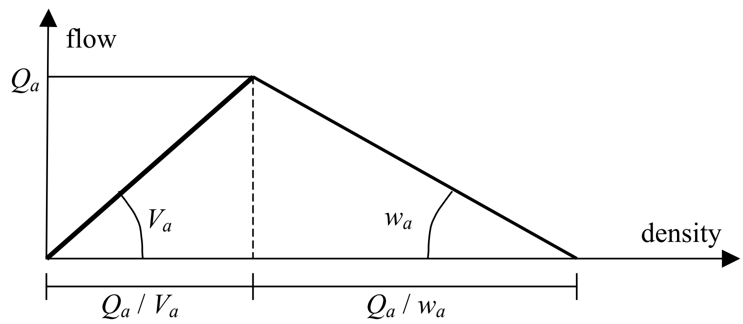
2 Network performance model

The generic arc is modelled here in three parts: an *initial bottleneck*, a *final bottleneck* and a *running link*.

The time varying capacity of the initial bottleneck takes into account the physical capacity of the link and the effect of queues propagating backward on the arc itself that can reach the initial section and can thus induce spillback conditions. Specifically, it maintains the inflow on the running link below the arc capacity and it ensures that at any time the number of users on the link does not exceed its storage capacity. Thus, the initial bottleneck guarantees the consistency of the network performance model in the context of DTA, where the arc inflow may assume any non-negative value. Clearly, at the equilibrium the inflows do not exceed the time varying arc entering capacities as the implicit constraints on the queue lengths are satisfied; however this is not true in general at a given step of the solving procedure.

The time varying capacity of the final bottleneck takes into account the hypercritic flow states, referred to as *queue*, that are generated either by a constant capacity reduction at the end of the arc due to the average effect of a road intersection, or by spillback conditions on the downstream arcs.

The running link models the congestion due to the interaction along the arc of vehicles travelling in hypocritical conditions; it consists of a homogeneous channel where flow states are determined on the basis of the Simplified Kinematic Wave theory assuming a certain *fundamental diagram*. For simplicity, in the following we assume that the fundamental diagram of the generic arc a with length L_a has a triangular shape, as depicted below:



Fundamental diagram.

where Q_a is the arc physical capacity, V_a is the free flow speed, and w_a is the hypercritical wave speed. However, the extension to more complex flow models can be achieved based on the methods proposed in [Gentile, Meschini, Papola, 2003].

2.1 Arc entering capacity model

This model represents the effect on the arc entering capacity of queues generated on the arc final section due to a limited exiting capacity. Specifically, it determines the maximum inflow that maintains the queue length not higher than the arc length, and by comparison with the actual inflow it identifies the time intervals when the spillback phenomenon occurs.

Clearly, if the queue was rigid (only one hypercritical density) any hypercritical flow state occurring at the arc final section would propagate instantaneously to the arc initial section and thus for any instant when the queue exceeds the length of the arc, we would have that the entering capacity is equal to the outflow. However, in reality hypercritical flow states may occur at different densities (stop and go) and the propagation speed of the flow states is finite. This can be taken into account avoiding the explicit evaluation of the queue temporal profile, which is cumbersome since the speed and density of vehicles in the queue vary accordingly with the outflow. Instead, exploiting the analytical solution of the simplified kinematic wave theory based on cumulative flows proposed by [Newell, 1993], the model identifies the time instants when the outflow states assumed hypercritical (the potential queue), propagating backward along the arc, would reach the initial section.

Let $G_a(\tau)$ be the backward propagation of the outflow temporal profile $E_a(\tau)$ assumed hypercritical, representing the maximum cumulative flow temporal profile that could have entered the arc without violating the constraint on the queue length consistently with the current outflow pattern. We have:

$$G_a(\tau) = E_a(\tau - \frac{L_a}{w_a}) + L_a \cdot Q_a \left(\frac{1}{w_a} + \frac{1}{V_a} \right). \tag{1}$$

The *Newell-Luke minimum principle* [Daganzo, 1997; Newell, 1993] states that when more than one kinematic wave reaches a point at a same time, the flow state yielding the minimum cumulative flow dominates the others. On this basis, the cumulative entering capacity temporal profile can be set as the lower envelope of the cumulative inflow temporal profiles $F_a(\tau)$ and of $G_a(\tau)$, so that we have:

$$\mu_a(\tau) = \min\{Q_a , \begin{cases} \dfrac{dG_a(\tau)}{d\tau}, & \text{if } G_a(\tau) < F_a(\tau); \\ f_a(\tau), & \text{otherwise} \end{cases} \}, \tag{2}$$

where $f_a(\tau)$ is the inflow temporal profile and $\mu_a(\tau)$ is the entering capacity temporal profile.

2.2 Arc exiting capacity model

This model determines, with reference to a given node, the time varying exiting capacities of the upstream arcs on the basis of the entering capacities of the downstream arcs and of the local turning flows. The resulting model is spatially non separable, because the exiting capacities of all the arcs belonging to the backward star of the node are determined jointly; it is indeed the model that propagates the spillback congestion through the network.

In order to simplify the presentation we here consider only two typology of nodes: *merging* and *diversions*; and assume that a more general graph is redefined accordingly.

When considering a merging, the problem is how to split the available capacity $\mu_a(\tau)$ of the single arc a exiting its initial node $TL(a)$, among the outflows $e_b(\tau)$ of the arcs $b \in BS(TL(a))$ of its backward star. If the spillback phenomenon is active, that is $\sum_b e_b(\tau) > \mu_a(\tau)$, in order to satisfy the outflow capacity constraint of node $TL(a)$ we have to limit its inflow capacity. In doing this we assume that all the available outflow capacity is split proportionally to the physical capacities, if the share of capacity this way assigned to the approach is actually utilized by its outflow, otherwise it is equal to the latter. Thus we have:

$$\xi_b(\tau) = \begin{cases} \dfrac{C_b}{\sum\limits_{c\in B} C_c} \cdot \left[\mu_a(\tau) - \sum\limits_{c\in BS(TL(a))\setminus B} e_c(\tau) \right], & \text{if } c \in B; \\ C_b, & \text{otherwise;} \end{cases} \tag{3}$$

$$B = \{ b \in BS(TL(a)) : e_b(\tau) \geq \xi_b(\tau) \} , \tag{4}$$

where $C_a \leq Q_a$ is the physical capacity of the arc opportunely reduced by average effect of the intersection. The separation set $B \subseteq BS(TL(a))$ satisfying jointly (3) and (4) can be obtained, starting from $BS(TL(a))$, by subtracting iteratively from the current set the arcs for which the entering capacity determined through (3) is higher than the outflow.

Considering a *diversion* from arc a, when the spillback phenomenon is active form an arc $b \in FS(HD(a))$, that is $f_b(\tau) > \mu_b(\tau)$, in order to satisfy the outflow capacity constraint of node $HD(a)$ based on the FIFO rule we have to limit its inflow capacity, in such a way that the ratio between the entering capacity of arc b and the exiting capacity of arc a is equal to the ratio between the turning flow on arc b and the outflow from arc a:

$$\frac{\mu_b(\tau)}{\xi_a(\tau)} = \frac{f_b(\tau)}{e_a(\tau)} . \tag{4}$$

Clearly if the spillback is active on more that one arc $b \in FS(HD(a))$, the exiting capacity is consistent with the most penalizing approach. If no spillback is active, then the exiting capacity equals the reduced physical capacity. In general we have:

$$\xi_a(\tau) = \min\{ C_a \ , \ \mu_b(\tau) \cdot \frac{e_a(\tau)}{f_b(\tau)} \ \forall b \in FS(HD(a)) : f_{ab}(\tau) > \mu_b(\tau) \} . \tag{5}$$

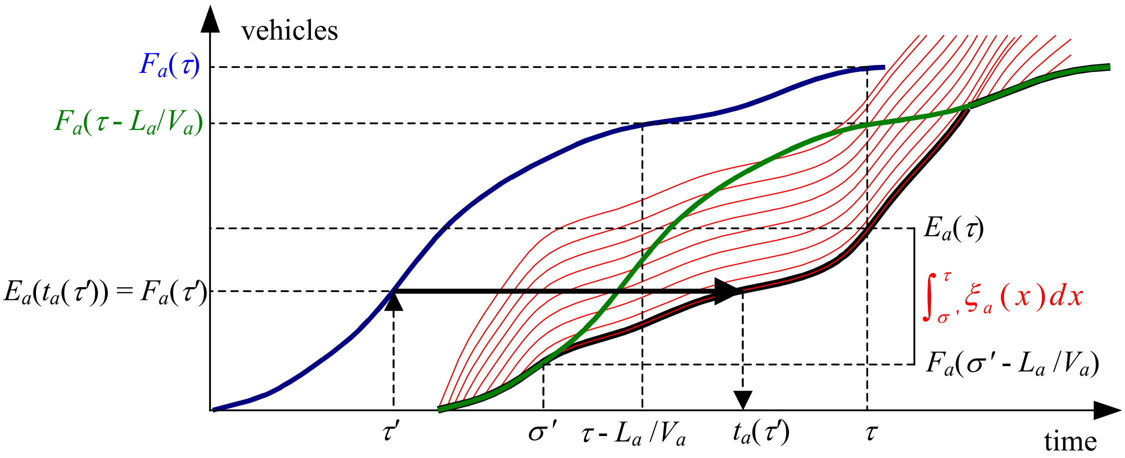
2.3 Arc travel time model for time varying exiting capacity

Once the temporal profile of the exiting capacity of each arc has been determined, it is possible to utilize the following link based macroscopic arc performance model to determine the arc travel time temporal profiles.

We are interested in determining, with reference to a bottleneck with time varying capacity $\xi_a(\tau)$ and constant running time L_a / V_a , the travel time temporal profile $t_a(\tau)$ resulting from a given inflow $f_a(\tau)$. Knowing the cumulative inflow temporal profile $F_a(\tau)$, based on the Newell-Luke minimum principle, the cumulative outflow temporal profile is given by:

$$E_a(\tau) = \min\{F_a(\sigma - \frac{L_a}{V_a}) + \int_{\sigma}^{\tau} \xi_a(x)\, dx \;\forall \sigma \le \tau\}. \qquad (6)$$

Then, based on the FIFO rule, the travel time temporal profile can be determined solving the following implicit equation:
$E_a(t_a(\tau)) = F_a(\tau)$.
Note that, based on the monotonicity of the three profiles involved, the numerical computation of $t_a(\tau)$ starting from $E_a(\tau)$ and $F_a(\tau)$ is trivial.



Bottleneck with time varying capacity.

3 Conclusions

The proposed model has been applied to several elementary networks in order to verify the correspondence with the results obtained by other authors [Daganzo, 1998]. Particularly, interesting is the comparison of the DTA model with spillback and the previous model without it. The model has been then tested on a real case (the highway belt around Venice); the results obtained were compared with those obtained on the same network with the Cell Transmission Model by Daganzo (1994), (1995) and are satisfactory. More work is ongoing to improve the convergence of the solution algorithm.

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