A Probabilistic Two-Day Delivery Vehicle Routing Problem

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1 Introduction

This talk has been motivated by the study of a real-world application on blood delivery. The Austrian Red Cross (ARC), a non-profit organization, is in charge of delivering blood to hospitals on their request. In current operations, the ARC is obliged to fulfill any order within the following day. This policy leads to high delivery costs. Quite often, the ARC has to pay extra working hours to its drivers in order to fulfill all the orders. Even solving a Vehicle Routing Problem (VRP) every day of operations will not ameliorate the current situation. The ARC is interested in changing policy in order to reduce costs through higher flexibility.

A possible solution is to have control on hospitals' blood inventory. However, this is quite complex to implement for several legal and responsibility reasons. The ARC is investigating the possibility of providing two different types of service: one which delivers the blood within one day and the other within two days. Obviously, these services must have different prices. Under such a policy, the ARC will be confronted with two different types of requests each day t: requests from hospitals that want blood delivered within the same day H_0^t, and requests from a set, say H_1^t, of hospitals that allow blood delivering within the following day. The ARC has to decide if to serve hospitals belonging to the set H_1^t immediately or the day after. These decisions should depend on the hospitals that have to be served and the foreseen blood orders for the following days. The goal is to minimize the total expected delivery costs.

This problem can be modelled as a probabilistic two-day delivery VRP. The features of our real application suggest the following hypotheses on the model:

- blood orders are random variables,
- there are no time windows for delivering blood during the day,
- there are no capacity constraints on vehicles.

Network routing problems have been deeply investigated both in a deterministic [5] and in a stochastic [2] setting. However, to the best of our knowledge, the considered problem is new. A similar version has been studied only in a deterministic setting [1].

In section 2 we present a mathematical formulation of the problem. The algorithm is reported in section 3. Expected results and conclusions follow in section 4.

2 Mathematical model formulation

Under mild assumptions the described problem is a Markov decision process. The state of the system depends exclusively on the state and the decision taken in the previous stage.

Denoting the random variables with:

H_0^t = the set of orders, placed on day t, to be served immediately, (during the first day),
H_1^t = the set of orders, placed on day t, to be served within two days;

the decision variable at time (stage) $t \in \{0, \dots, T\}$ is:

$B^t \subseteq H_1^t$, subset of hospitals of the set H_1^t that should be served immediately.

The problem can now be formalized as follows:

$$\text{Min } c\left(B^0 \cup H_0^0\right) + E\left[\sum_{t=1}^{T} c\left(B^t \cup H(t)\right)\right]$$

$$\text{s. t.} \quad \begin{array}{ll} H(t) = H_0^t \cup \left(H_1^{t-1} \setminus B^{t-1}\right) & \forall t = 1, \dots, T. \\ B^t \subseteq H_1^t & \forall t = 1, \dots, T. \end{array} \tag{1}$$

where $c(A)$ denotes the delivery costs of the solution of serving hospitals belonging to the set A (optimal solution of the VRP). Constraints (1) represent the transition of the system from one stage to the next.

However, optimization methods for Markov decision processes cannot be applied to real instances with a fairly large set of hospitals (customers), since the number of states is $2^{|\mathcal{H}|}$ where $\mathcal{H}$ is the set of hospitals. Furthermore, finding the optimal cost solution requires solving several VRP problems and therefore is itself a *NP*-hard problem.

3 Solution procedure

The problem is a finite horizon sequential decision process under uncertainty. The developed solution procedure uses information about the probability that a customer orders blood on any given day in the future. To this aim, historical data on orders of blood are analyzed to get an estimate of the probability distribution for the orders from the customers. To solve the problem, we apply a rolling horizon approach, solving a two-day problem at each stage. The reduced problem, in terms of number of stages considered, is solved using an algorithm based on the Savings based Ant System (SbAS) [3, 4].

Key element in our solution approach is the estimation of the expected delivery costs for the following days. Since our algorithm is envisioned in a rolling horizon approach, future demand is the demand of tomorrow. We compute the cost for each scenario and then evaluate the expected costs (using the scenario probability). However, the scenario costs depend on the decision taken and on the next day demand and so on. Hence, here we apply a bootstrap-like technique to approximate these costs.

Therefore our optimization procedure can be divided into two main phases:

- In the first phase of our algorithm, we determine a number of customers who will be served within 24-hours, B^t, out of the customers who expect their service within 48-hours, H_1^t, according to a randomized heuristic rule. Then, we generate tours for this day by using SbAS.
- We compute the expected delivery costs for the remaining periods. We sample on the possible scenarios for the demand on day $t+1$, and for each sampled scenario we compute the next day delivery costs applying the Savings heuristic. Moreover, we add a penalty cost for delaying customers to day $t+2$, otherwise, the local search procedure would always serve only the minimum possible number of customers on day $t+1$. This penalty cost is computed multiplying the number of delayed customers by an estimate of the average additional costs incurred by such a customer. This estimate can be based on previous stages for which costs are already known (for details, see below). We also use a second estimate based on distances and take a weighted mean.

Our decision on customers to be served immediately on day t, B^t, is modified by local search as well, and our optimization procedure is repeated.

Let us now outline how, for the penalty term computation, an estimate c_t of the additional costs incurred by a delayed customer is computed. We start with an arbitrary initial value c_0 and modify this value in each iteration by formula (2). When the stochastic process approaches its steady state, it can be expected that c_t approaches the true value of the cost parameter under consideration.

$$c_{t+1} := \rho \cdot c_t + (1-\rho) \cdot \frac{c\left(H_0^t \cup B^t \cup \left(H_1^{t-1} \setminus B^{t-1}\right)\right) - c\left(H_0^t \cup B^t\right)}{\left|H_1^{t-1} \setminus B^{t-1}\right|}. \tag{2}$$

In the second estimation approach, we consider the sum of distances from the depot to the delayed customers instead of the number of these customers.

4 Conclusions

This work is the first step towards the development of a decision support system for the Austrian Red Cross blood bank in order to reduce the delivery costs. A comparison of the solution quality with the current real-world solution is planned.

Appendix: Pseudo code for the solution procedure

procedure `SolveATwoDayVrp` {
Initialization: $t = 0$; $c = c_0$; $\bar{c} = \bar{c}_0$;
estimate the probability distribution of the demand by
using historical data on orders of blood deliveries;
Step 0: select $B^t \subseteq H_1^t$;
while termination criterion is not met { // *Iterationloop 0*
Step 1: solve the VRP for the customers of the set
$H_0^t \cup B^t \cup (H_1^{t-1} \setminus B^{t-1})$ by using the Savings based Ant System;
for a certain number of iterations { // *Iterationloop 1*
Step 2: sample one instance for the sets
H_0^{t+1} and H_1^{t+1} by using the estimated distribution of the demand;
Step 3: select $B^{t+1} \subseteq H_1^{t+1}$;
while termination criterion is not met { // *Iterationloop 2*
$G = H_0^{t+1} \cup B^{t+1} \cup \left(H_1^t \setminus B^t\right)$; // *customers chosen for day* $t+1$
compute the tours and delivery costs $f(G)$ by using a Savings heuristic;
with $\varphi(A) =$ sum of distances from the depot to the customers in A,
$totalCosts := f(G) + \lambda \cdot c_t \cdot \left|H_1^{t+1} \setminus B^{t+1}\right| + (1-\lambda) \cdot \bar{c}_t \cdot \varphi\left(H_1^{t+1} \setminus B^{t+1}\right)$;
actualize best value $minTotalCosts$ of $totalCosts$;
update B^{t+1} by a local search move;
} // *close Iterationloop 2*
} // *close Iterationloop 1*
Step 4: compute the average value of $minTotalCosts$ over the samples
chosen in *Iterationloop 1* for the given B^t;
$overallCosts :=$ costs of step 1, plus average costs of step 4;
Step 5: update B^t by a local search move;
$c_{t+1} := \rho \cdot c_t + (1-\rho) \cdot \frac{c\left(H_0^t \cup B^t \cup \left(H_1^{t-1} \setminus B^{t-1}\right)\right) - c\left(H_0^t \cup B^t\right)}{\left|H_1^{t-1} \setminus B^{t-1}\right|}$;
$\bar{c}_{t+1} := \rho \cdot \bar{c}_t + (1-\rho) \cdot \frac{c\left(H_0^t \cup B^t \cup \left(H_1^{t-1} \setminus B^{t-1}\right)\right) - c\left(H_0^t \cup B^t\right)}{\varphi\left(H_1^{t-1} \setminus B^{t-1}\right)}$;
increment t;
} // *close Iterationloop 0*;

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