Constraint Programming and Column Generation Methods to Solve the Dynamic Vehicle Routing Problem for Repair Services

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Extended Abstract

The objective of this research was motivated by a real problem, which is the dynamic dispatch of technicians of Xerox Chile to repair failures of their machines along the day. Xerox Corporation is an international company that offers color and black-and-white digital printers, digital presses, multifunction devices, and digital copiers. The proposed scheme is based upon the classic formulation of the Vehicle Routing Problem with Soft Time Windows (VRPSTW), and it is formulated and solved as a Dantzig-Wolfe decomposition by using set partitioning - column generation along with classical insertion heuristics, under a constraint programming approach.

The strategic objective of Xerox is based upon client satisfaction, and within this context, the technical service of their machines is one of the most important activities of the company in Chile, and that is why this study was motivated. Clients have different priorities, which define different goal response times for clients at different priority levels. The goal response time is defined as the maximum allowable time for a technician to reach the client measured from the time of the service request. Nowadays, technicians use two transportation modes: public transport and vans provided by the company. Choosing a mode depends on the area of the incident and the current location (status) of busy technicians. The system proposed here only considers the transport of technicians and equipment in vans, however it can easily be reformulated in order to include public transport modes.

Another interesting feature of this specific problem is that usually there are some technicians specialized in certain type of machines. The objective function for assigning technicians and

jobs is to minimize two components: the sum of the differences between goal response times of clients and the effective service time provided by Xerox and the sum of travel times. The way in which Xerox assigns their jobs is determined by the currently relative position of clients in a queue kept by the dispatcher sorted by priority.

Service times depend on the specific failure of the visited machine, as the final service (repair) time not always matches the description of the failure provided by the client at the time of the request. Once the repairman reaches a specific machine to be repaired, he can estimate the repair time with much more certainty. Travel times are estimated from historic data of the company.

Recent technological advances in communication systems and information management now allow the exploitation of real-time information for dynamic vehicle routing and scheduling. A very exhaustive review of the various approaches for solving problem of this type s found in Gendreau *et al.* (1998b).

Broadly speaking, we found two major approaches for solving the problem. One research line is based on queuing theory, focused on the study of the Dynamic Traveling Salesman Problem (DTSP)(Pasaraftis, 1988) and on the Dynamic Traveling Repairman Problem (DTRP) (Bertsimas and Van Ryzin, 1991). Then assignment policies depend on both the spatial and temporal distribution of the calls and on the observed work load. There are no real applications of this solution approach in the industry so far. A second algorithmic approach is based upon finding reasonable routes to be followed by each technician every time a new call enters the system, using insertion techniques and metaheuristics, such as Tabu Search (Gendreau *et al.*, 1999) or Ant System (Montemanni *et al.*, 2002).

Real application of the last approach for dynamic dispatch of vehicles can be found in Weintraub *et al.* (1999), developing a system based on insertion heuristics for an electrical distribution company, and also in Madsen *et al.* (1995) that develop heuristics for dispatching technicians of a gas distribution company.

The focus of this research follows an algorithmic approach, in which the decision maker updates some predefined routes for each technician every time a new request enters the system. The general scheme of the algorithm is shown in figure 1.

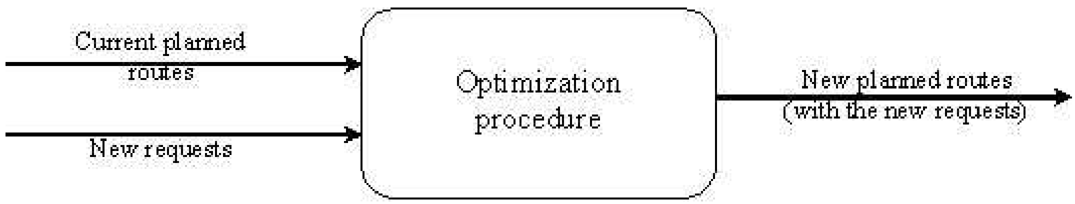

Figure 1: Generating initial routes

The modeling scheme was adapted from the classical formulation of the VRPSTW (Cordeau *et al.*, 2002), by considering the following adjustments:

- For this problem, only the upper bound of the soft time window (related to the goal response time of each client) is considered, since the objective is to serve each requirement as soon as possible.

- Service times are quantitatively longer and less accurate than travel times, therefore the former will play a more relevant role than the latter into the proposed decision rules.

Let $I = \{m_1, \ldots, m_k, m_{k+1}, \ldots, m_I, m_{I+1}\}$ denote the set of machines, and let $K = \{t_1, \ldots, t_k\}$ be the set of technicians. The set $\{m_1, \ldots, m_k\}$ includes those machines being currently repaired by a technician, while $\{m_{k+1}, \ldots, m_I\}$ represents the set of machines waiting in queue. m_{I+1} is simply a fictitious machine representing the final stop of each technician's route. Additionally, let b_i be the upper bound of the time window associated with machine i and s_i, the expected repair time for machine i. The technician specialization is modeled through the binary parameter c_{ki} which is 1 if technician k is able to repair machine i, 0 otherwise. Let t_{ij} be the travel time from machine i to machine j. The end of the working day is L. There are two types of variables in this model: flow variables x_{ijk}, equal to 1 if technician k attends client i and client j sequentially, 0 otherwise; and two temporal variables w_{ij} which is the time when technician k visits client i; and δ_{ik} that represents the violation of the soft time window of client i by machine k.

Thus, the formulation is as follows:

$$\min_{x,\delta} \left[\sum_{k \in K} \sum_{i \in I} \delta_{ik} + \beta \sum_{k \in K} \sum_{i,j \in I} t_{ij} x_{ijk} \right] \qquad (1)$$

subject to

$$\sum_{k \in K} \sum_{j \in I} x_{ijk} = 1 \qquad \forall i \in \{I \backslash m_{I+1}\} \qquad (2)$$

$$\sum_{\mathrm{i}} x_{ijk} - \sum_{\mathrm{i}} x_{jik} = 0 \qquad \forall j \in \{m_{K+1}, \ldots, m_I\}, \forall k \in K \qquad (3)$$

$$\sum_{j \in I} x_{ijk} = 1 \qquad \{i \in \{m_1, \ldots, m_K\}, k \in K \backslash \; i = m_p, k = t_o \Leftrightarrow p = o\} \qquad (4)$$

$$w_{ik} + s_i + t_{ij} - w_{jk} \leq (1 - x_{ijk}) * M \qquad \forall i, j \in I, \forall k \in K \qquad (5)$$

$$x_{ijk} \leq c_{jk} \qquad \forall i, j \in I, \forall k \in K \qquad (6)$$

$$w_{ik} \leq L \sum_{j \in I} x_{jik} \qquad \forall i \in I, \forall k \in K \qquad (7)$$

$$\delta_{ik} \geq [w_{ik} - b_i] \qquad \forall i \in I, \forall k \in K \qquad (8)$$

$$x_{ijk} \in \{0, 1\}$$

$$w_{i,k}, \delta_{i,k} \geq 0 \qquad \forall i, j \in I, \forall k \in K \qquad (9)$$

The objective function (1) account for the total cost, that is the sum of violation of the soft time windows and travel time. Constraints (2) restrict the assignment of each machine to exactly one vehicle link. Next, constraints (3) and (4) characterize the flow on the path to be followed by technician k, that is (3) forces each client to be served by only one technician and (4) characterize the initial client of each route. Additionally, constraints (5) guarantee schedule feasibility with respect to time considerations (Cordeau *et al.*, 2002) and constraint (6) insure that only qualified technicians can be considered for each repair job. Note that for a given k, constraints (7) force $w_{ik} = 0$ whenever machine i is not visited by vehicle k. Constraints (8) characterize the violation of the soft time window. Finally, (9) impose binary conditions on the flow variables, and the positive nature of time variables.

As shown in the past, the VRPTW problem is NP-Hard (Savelsbergh, 1985), which implies that under real conditions it is hard to find the optimal solution, particularly in dynamic systems when a real time response is needed. In order to deal with such a problem, we propose a Dantzig-Wolf decomposition, where the master problem is given by (1)-(2), i.e, the objective function (1), and the set covering of machines (2), in which each machine $i \in I$ is covered exactly once. The sub-problem is a model that generates feasible routes for each technician. Analytically:

Master problem: Choosing routes for each technician among a pool R.

$$\min \sum_{r \in R} c_r x_r \tag{10}$$

$$\text{s.t.}$$

$$\sum_{r \in R} \partial_{ir} x_r = 1 \qquad i \in I \tag{11}$$

$$x_r \in [0,1] \quad \forall r \in R \tag{12}$$

Where x_r is a binary variable indicating if route $r \in R$ is chosen. Each route starts at the initial position of the corresponding technician. ∂_{ir} is a binary parameter that indicates if route r contains machine i and c_r is the cost of route r. This problem is an integer problem solved using CPLEX 7.5 for the problem instances in reasonable time.

Sub Problem: Generate routes with minimum marginal cost. Solving the master problem over the current set of columns by using the simplex method gives the dual variables associated with constraints (11) necessary for the solutions of the subproblem. The subproblem generates the optimal route, by minimizing both time window violation and travel time minus the sum of dual variables of the clients in such a route, subject to (3)-(8). After this process, a route with minimum marginal cost is generated. If the computed marginal cost were negative then there would exist the possibility of improving the objective function of the master problem by adding this new column.

The model is formulated under a Constraint Programming framework (CP). This problem is NP-hard, and for instances of great size the computation time for setting the optimal solutions could be very high. However, in this case the size of each column generation problem is in the range of three to seven machines, which allows us to obtain the optimal solutions in a very short time (less than one second). This problem is solved using ILOG Solver 5.2.

The Constraint Programming constraints are based on the logical relations of the problem. In particular, the time windows constraint aid significantly in reducing feasible solutions. This implies that in the search tree a large percentage of nodes are eliminated.

Then, the subproblem generates an optimal route of length measured in number of clients visited L , as described below, which minimizes time window violation along with the travel time of serving that route minus the sum of dual variables of the clients in the route.

The General Process: Next, a summary of the general scheme of solution is described.

1. Generate a pool of initial routes:

   Let C is the average number of machines to be served by each technician, and A indicates a range of reasonable positions from this average.

```
For all machine m_i{
    For all technician t_k{
        For all L in [C-A,C+A]]{
            Create with CP a new route of length L,
            which starts at the present position of
            a technician t_k and includes machine m_i.
            //check if route has not been created before,
              otherwise generate new route.
        }
    }
}
```

   By running this routine, the feasibility of the solution is assured, in fact, there exist at least one route containing each machine .The above described procedure is shown in figure 2.

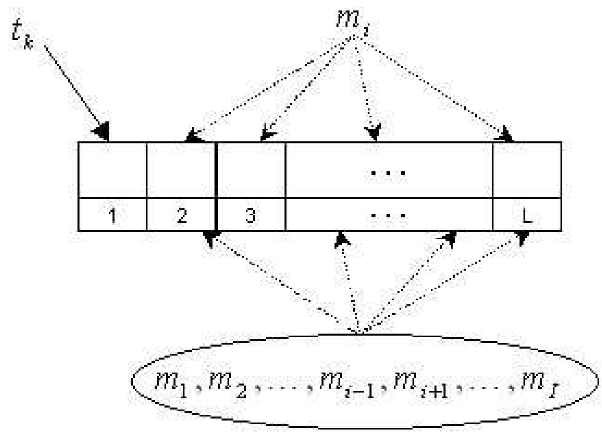

Figure 2: Generating initial routes

   For this route m_i must be in some position between 2 and L.

2. Solve a linear relaxation of the master problem with the generated pool of routes.

3. From the dual variables associated to the constraints of each machine in the master problem, generate new routes with the minimum reduced cost, adapted from the subproblem. If there exists a route with negative reduced cost cr^* , go to 4, otherwise go to 5.

4. Using the sub-problem generate all possible columns with reduced cost cr less than $cr^*\gamma$, and length L , where γ is a control parameter that satisfies $0 < \gamma < 1$, thus $cr \in [cr^*, cr^*\gamma] < 0$. This helps control the number of columns generated in each iteration. If cr^* is a large negative number, it may be convenient to generate many columns in a range $[cr^*, cr^*\gamma]$, where will be large negative numbers. Go to 2.
5. Solve IP associated with the master problem including all the columns generated in the previous steps, obtaining the final routes to be followed by each technician.

The running time of this procedure lies in the range between 30 and 120 seconds. Considering that decisions have to be taken dynamically, we decided to run the code only if the number of new request was high enough (10 new machines for example). Otherwise, the new service requests were inserted into the current routes using a procedure based on GENIUS-CP (Pesant *et al.*, 1997; Gendreau *et al.*, 1992, 1998a). Potential segments of insertion were chosen considering all pieces of route located close enough (in both senses space and time) from the location (time) of the new request. After checking all potential segments in time and space, the one with the minimum insertion cost is assigned to serve the new request, and the specific route is adapted accordingly.

The model was coded in ILOG Opl and solved using ILOG Cplex 7.5 and Solver 5.2. In our results, each technician was assigned not more than five machines per day. For higher number of total requests it was necessary to reschedule some of the machines for the next day, as not all could be served.

The consistency of the Column Generation approach was empirically checked by observing how the method converges to the optimal solution in case of small size problems. In fact, for small instances the optimal solution was obtained directly by solving the original Mixed Integer Problem (equations (1) to (9)). The next figure shows the rate of convergence of the CG method as a function of the number of iterations for a problem of twelve clients and three technicians, which is contrasted against the solution obtained from the exact model ER.

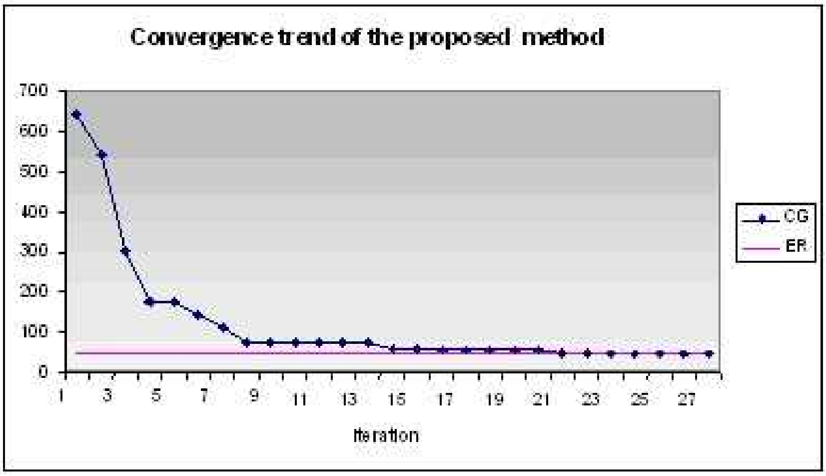

Figure 3: Convergence trend of the proposal method

Next, an example of the application of this scheme is shown, considering a first complete run at the beginning of the day (time 0), where the length of the day is 600 minutes, repair times fluctuate between 10 and 120 minutes, with goal response times between 0 and 600 minutes depending on different client priorities. The solution shows no time window violations and a very reasonable route pattern as well as a balanced distribution of assignments (see figure 4). The process time of this example was 25 seconds in a Pentium IV 2.2 MHz, 256 KB in RAM.

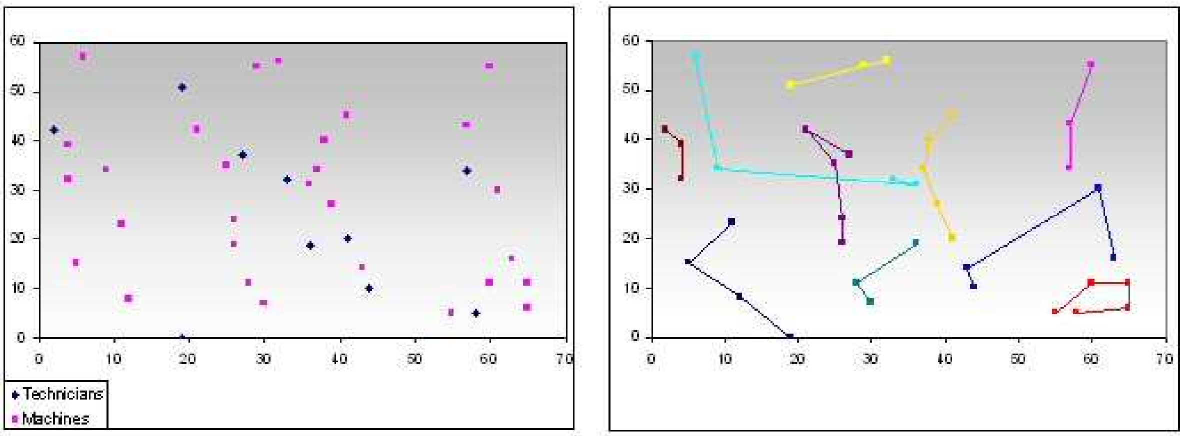

Figure 4: Graphical representation of an example. (Left. Clients) (Right. Generated Routes)

This paper is part of an ongoing project. Currently, we are incorporating the stochastic nature of repair times as well as the uncertainty of future and unknown demand into the problem by using techniques in the line of robust optimization.

Acknowledgements

This research was partially financed by Fondecyt, Chile, grants 1030700 and 1020784, and the Millennium Nucleus "Complex Engineering Systems".

References

D.J. Bertsimas and G. Van Ryzin, "A Stochastic and Dynamic Vehicle Routing Problem in the Euclidean Plane", *Operations Research* 39, 601-615 (1991).

J.-F. Cordeau, G. Desaulniers, J. Desrosiers, M. M. Solomon and F. Soumis, "The VRP with Time Windows", in *The Vehicle Routing Problem*, Paolo Toth and Daniele Vigo (eds), 157-193, 2002.

M. Gendreau, A. Hertz and G. Laporte, "A New Insertion and Postoptimization Procedures for the Traveling Salesman Problem", *Operations Research* 40, 1086-1093 (1992).

M. Gendreau, A. Hertz, G. Laporte and M. Stan, "A Generalized Insertion Heuristic for the Traveling Salesman Problem With Time Windows", *Operations Research* 43, 330-335 (1998a).

M. Gendreau, F. Guertin, J.-Y. Potvin, and E. Taillard, "Parallel tabu search for real-time vehicle routing and dispatching". *Transportation Science* 33(4), 381-390 (1999).

M. Gendreau and J.-Y. Potvin, "Dynamic vehicle routing and dispatching" in *Fleet Management and Logistic.* T.G. Crainic and G. Laporte (eds), 115-226, 1998b.

O.B.G. Madsen, K. Tosti and J. Voelds, "A Heuristic Method for Dispatching Repair Men" in *Annals of Operations Research* 61, 213-226, (1995).

R. Montemanni, L.M. Gambardella, A.E. Rizzoli and A.V. Donati. "A new algorithm for a Dynamic vehicle Routing Problem based on Ant Colony System". IDSIA, (2002).

G. Pesant, M. Gendreau and J.-M. Rousseau. "GENIUS-CP: A generic single-vehicle routing algorithm". In Proceedings of CP '97, 420-433. Springer-Verlag, 1997.

H.N. Pasaraftis, "Dynamic Vehicle Routing Problems," in *Vehicle Routing: Methods and Studies*, B.L. Golden and A.A. Assad (eds), 223-248, North-Holland, Amsterdam, 1988.

M.W.P Savelsbergh. "Local Search in routing problems with time windows". in *Annals of Operations Research*, 4, 285-305, (1985).

A. Weintraub, J. Aboud, C. Fernndez, G. Laporte and E. Ramrez, "An emergency vehicle dispatching system for an electric utility in Chile" *Journal of the Operational Research Society*, 50-7, 690-696 (1999).