Analytical Modeling of Stochastic Rerouting Delays for Dynamic Multi-Vehicle Pick-up and Delivery Problems

Cristián E. Cortés* R. Jayakrishnan†

*Department of Civil Engineering
Universidad de Chile
Casilla 228-3, Santiago, Chile
ccortes@ing.uchile.cl

†Department of Civil Engineering
University of California, Irvine, CA. 92967, U.S.A
rjayakri@uci.edu

Extended Abstract

This paper develops a framework for incorporating stochasticity into the routing rules for taking decisions in the context of the dynamic passenger pickup and delivery problem. Suppose we have a set of vehicles *VF*. At any time we assume that each vehicle $j \in VF$ has been assigned a sequence of tasks that include pickups and deliveries, and can be represented by a sequence set $CS_j = \{1,2,..k,..N_j\}$, in which the k^{th} element of the sequence represent a specific stop along vehicle *j*'s route. Service requests enter the system dynamically and decisions have to be taken in real time based upon some insertion-type algorithm at the dispatching module. The final decision is to insert the new request into the best sequence CS_j (the one with minimum incremental insertion cost), subject to the feasibility constraints. This paper develops an adaptive-predictive scheme to handle such problems. The schemes developed are for application in a transit system context with large vehicle fleets operating with real-time routing, but may have applicability for many general purpose pickup and delivery contexts. In fact, the idea of viewing this problem from a real-time control perspective as our paper does, is itself not that common in the literature.

One assumption behind most of the scheduling-routing rules described in the literature is that travel times between stops are considered fixed in the cost expressions (for example Wilson and Weissberg, 1976; Wilson and Colvin, 1977; Jaw *et al.*, 1986, Bodin *et al*., 1983, Psaraftis, 1983a, 1983b, 1986). In this paper, we propose an analytical way to estimate the expected travel time, say of vehicle j, between two scheduled stops, assuming that all conditions and attributes of vehicle j are known by the dispatching module before it arrives at the upstream stop position of such a segment. Since the analysis is based on fulfilling the travel desires of unknown customers appearing in real-time, it is assumed that most of the predefined routes and schedules should dynamically change as a new call enters the system. Thus, one important issue to be highlighted is that, when computing an analytical expression for any kind of decision cost function (whether it affects the user or the operator), there is an additional source of stochasticity in the calculation of the expected travel time between two points on a transportation network, apart from the typical non-recurrent congestion of traffic affecting all vehicle travel times. This additional delay can be called the *stochastic rerouting delay for dynamic vehicle routing (SRDDVR)*.

In other words, for all the already-scheduled users, the expected travel (or waiting) time that they will incur during their trip will be strongly affected by any future reassignment of the vehicle assigned to pick them up at their origin spot, or drop them at their destination in case they are already on the vehicle. We introduce stochastic prediction into the routing rules deciding the assignment in order to incorporate a more realistic measure of travel (waiting) time experienced by the users as well as the operator into the decision cost formulation, which eventually could change some of the dispatching module decisions, resulting in better solutions closer to the desired dynamic social optimal equilibrium.

The paper introduces an approach for estimating the expected travel time of a transit vehicle to any of its scheduled stops under stochastic demand, generated dynamically over time without any previous knowledge of these new requests by the dispatching module at the time of the call. The methodology is based upon real-time updates of probabilities to predict each vehicle's expected travel time between two already-scheduled stops. The main idea is to use the dynamics of the system to help predict behavior so that this can be used to make better decisions. Since logical decisions in assignment involve discrete actions, conceptually this problem could be considered as a form of quasi-optimal hybrid adaptive predictive control (H-APC) system.

Predictive control, as the name implies, is a form of control, which incorporates the prediction of a system behavior into its formulation. The prediction serves to estimate the future values of a variable based on available system information. The more representative the information on the system is, the better the accuracy of the prediction. An estimate of the future system variables can then be used in the design of control laws to achieve good control performance, which is usually to drive or maintain the output to a desired set-point. This class of control methods, which incorporates information and assumptions pertaining to the future values of the system output, are generally referred to as predictive control. Basically, if a good model is available *a priori* to describe the dynamic relationship between the system input and outputs, it may be used as the predictive model (Clarke and Mohtadi, 1989; Morari and Lee, 1999).

Conceptually, an adaptive control system must receive continuous information about the present state of the plant so as to *identify* the process; it must compare present system performance to the desired or optimum performance and make a *decision* to adapt the system so as to tend it toward optimum performance; and finally it must initiate proper *modifications* so as to drive the system towards the optimum.

The approach of Adaptive Predictive Control (APC) that combines the above two schemes, has received some attention over the last decade. Basically, if a good model is available *a priori* to describe the dynamic relationship between the system input and outputs, it may be used as the predictive model (Soeterboek, 1992; Camacho and Bordons, 1995).

In addition, hybrid systems typically arise when continuous plants are coupled with controllers that involve discrete logic actions. Although hybrid systems are encountered in many practical situations, up to now most controllers for such systems have been designed using ad hoc and heuristic procedures. Due to the complex nature of hybrid systems, it is infeasible to come up with generally applicable control design methods (Bemporad and Morari, 1999). Combining Adaptive Predictive Control and Hybrid Systems is what we call Hybrid Adaptive Predictive Control (H-APC) in the context of this paper.

The premise is that, since calls are generated dynamically and the pick-up and delivery decisions are taken in real time based on system information at the decision time, decision rules should depend on the *expected number of future insertions* into pre-established vehicle routes. Note that passenger assignments could change over time because of changes in system conditions. Hence, the proposed *SRDDVR* scheme essentially considers stochastic travel times expected for future assignments. These travel times can be calibrated online, but could also use historical information on system performance under discrete logic actions embedded in the dispatch decisions, just as in other APC systems. This capability of fine-tuning the system is what essentially brings out the quasi-optimal nature of the solutions.

Assume an influence area *A*, with a transit service network of length *D* in distance units [DU]. A fleet of transit *VF* vehicles of size *NF* is currently in operation traveling within the area according to predefined routing rules. The demand for service is unknown and is generated dynamically in real-time (assume a rate μ_{CD} in calls per time units [call/TU]). Routing and scheduling decisions have to be taken in real time, to handle such demand with the available vehicles. The predictive controller in this case is represented by the dispatching module, which takes routing decisions in real time based on the information it has from the system (process) and the expected values for travel times and attributes of its vehicle fleet (model).

Unlike in the traditional predictive control schemes, time steps are not directly applicable in this case, since the predictive controller is taking the routing decisions only whenever a call enters the system. In other words, the control mechanism is activated whenever a new request comes in. Hence, instead of defining a fixed time step, in this scheme it is necessary to formulate the problem in terms of "epochs" $e(s)$, where $e(s)$ will represent the time interval between requests $s-1$ and s.

The state of the system at instant s (i.e, when request s enters the system) will be determined by the attributes of each vehicle, namely $ATTR_j(k,k+1)$ for vehicle j while on route sequence segment $(k,k+1)$. Vehicle attributes include all vehicle features when leaving the k^{th} stop of its sequence. In our formulation we have incorporated clock time of arrival $tCL_j(k)$, the vehicle load $L_j(k)$ (that could be replaced by the available space for accommodating new requests on the stretch), the cumulative travel-time experienced by all passengers on board after leaving stop k, $TR_j(k)$, and a measure of the available space for potential passengers in the proximity of segment $(k,k+1)$, $WSP_j(k,k+1)$.

The last variable introduced here represents the system's available capacity (measured in vehicle seats) within the "catchment area" of segment $(k,k+1)$ when vehicle j is expected to traverse such a route segment. The catchment area concept is a practical variable added here to limit the spatial influence of vehicle route segments.

The control $u(s)$ can be visualized as the dispatching module routing decisions, represented by the set of sequences assigned to every vehicle before $e(s)$. Analytically,

$$x(s) = \left\{ ATTR_j(k, k+1) \right\}\Big|_{e(s)} \qquad \forall\ j : 1, \ldots, NF \tag{1}$$

$$u(s) = \left\{ CS_j \right\}\Big|_{e(s)} \qquad \forall\ j : 1, \ldots, NF \tag{2}$$

Notice that the sequences CS_j remain fixed during the whole interval $e(s)$, since during that time the controller does not realize any action. The vector of process measurements y_m will be computed measuring the observed segment travel times $tS_j(k, k+1)$, occurring during the time interval $e(s)$ for all vehicles and for all pair of stops $(k, k+1)$. These pairs of stops are adjacent stops in the current sequence CS_j. Notice that, in order to measure the expected travel time, the vehicle is required to arrive at stop k +1 sometime within time interval $e(s)$. That is,

$$y_m(s) = \left\{ tS_j(k, k+1) \right\}\Big|_{\substack{tCL_j(k+1) \in e(s) \\ (k,k+1) \in CS_j|_{e(r)<e(s)}}} \tag{3}$$

where $tCL_j(k+1)$ represents the clock time at which vehicle *j* leaves stop $k+1 \in CS_j$, and $CS_j\big|_{e(r)<e(s)}$ denotes a vehicle *j* sequence being defined for an epoch $e(r)$ previous to $e(s)$. Figure 1 shows an example of a certain control strategy applied by the dispatching module at the time request *s* enters the system, for a hypothetical case with *NF = 5*.

In what follows, $\hat{x}(s), \hat{y}(s)$ will represent the system predictions based on modeling parameters calibrated before epoch $e(s)$. In equations,

$$\hat{x}(s) = \left\{ E[ATTR_j(k, k+1)] \right\}\Big|_{e(s)}$$

$$\hat{x}(s) = \left\{ E[tCL_j(k)], E[L_j(k)], TR_j(k), WSP_j(k, k+1) \right\}\Big|_{e(s)} \tag{4}$$

$$\forall\ j, \forall\ k, k+1 \in CS_j$$

$$\hat{y}(s) = \left\{ E[tS_j(k, k+1]\right\}\Big|_{e(s)} \qquad \forall\, j, \forall\, k, k+1 \in CS_j \tag{5}$$

Expressions (4) and (5) show an estimation of the system state, comprising vehicle features and expected travel times over predefined segments, the latter as a function of the expected number of insertions. The modeling procedure embedded into the computation of the expected vehicle travel time, will remain invariant for a period predefined by the modeler. In case of applying just a predictive approach, the parameters of the model would be considered always fixed independent of the observed measurements y_m. Note also that the prediction horizon H_p is not fixed in this formulation, and it can be computed as the latest expected stop time among the sequences of all the vehicles. That is,

$$H_p(s) = \underset{j:1,\ldots,NF}{Max} \left\{ E[tCL_j(N_j)\right\}\Big|_{e(s)} \tag{6}$$

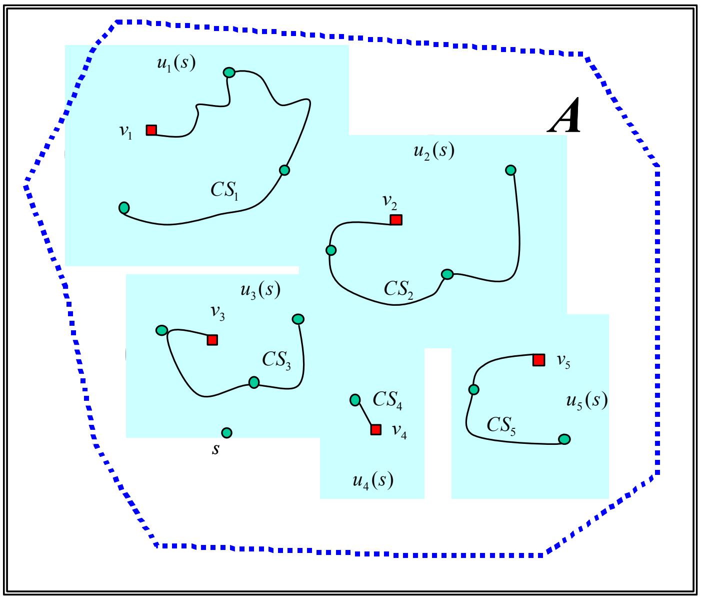

Figure 1: Graphical representation of controller actions u(s)

In an APC scheme, the major difference would be to add an adaptation mechanism via the calibration of certain parameters associated to the model procedure, whenever needed. In other words, after a period of evaluation of the performance of the model, it could be necessary to recalibrate some procedures and update certain observed expected values, in order to better reproduce the observed values measured in real time from the operation of the system. This test can be carried out every H_c epoch, with $H_c < H_p$ (receding horizon concept). See Figure 2 for a representation of the whole H-APC process for Stochastic Rerouting Delay.

Analytically, the expected travel time will depend upon the network travel time $tSN(a_j(k), a_j(k+1))$, the expected number of non-scheduled insertions $E[I_j(k,k+1)]$, and a parameter $\overline{\Psi}$ defined below. In addition, $E[I_j(k,k+1)]$ depends on the conditions of the system, the demand rate, and a set of parameters $\hat{\mathrm{B}}(s)$ describing the probability of vehicle-call assignment given the corresponding state conditions.

These parameters have been calibrated before the epoch $e(s)$ and are assumed to be representative of the system behavior during $e(s)$.

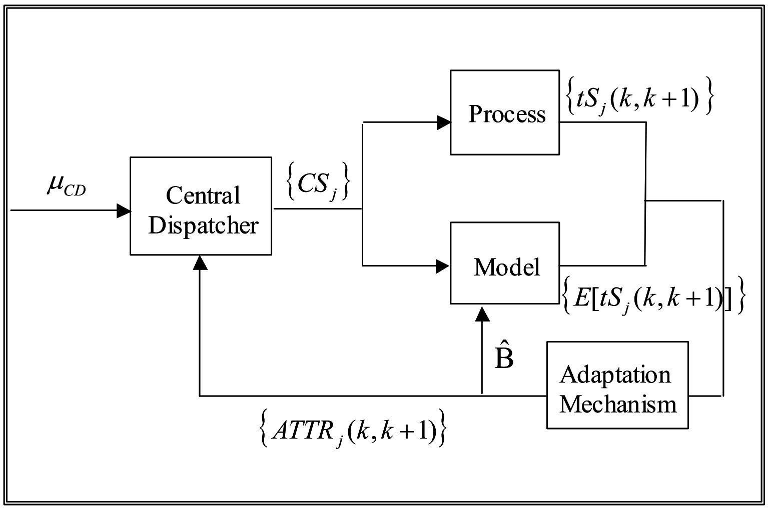

Figure 2: Overall block diagram of an H-APC approach for computing Stochastic Rerouting Delay

Analytically,

$$E[tS_j(k,k+1)]\Big|_{e(s)} = F\Big(tSN(a_j(k), a_j(k+1)), \overline{\Psi}, E[I_j(k,k+1)\Big)\Big|_{e(s)} \tag{7}$$

where

$$E[I_j(k,k+1)]\Big|_{e(s)} = G\Big(E[ATTR_j(k)],\mu_{CD},\hat{\mathrm{B}}(s)\Big)\Big|_{e(s)} \qquad (8)$$

hence,

$$E[tS_j(k,k+1)]\Big|_{e(s)} = F\Big(tSN(a_j(k),a_j(k+1)),\overline{\Psi},G\Big(E[ATTR_j(k,k+1)],\mu_{CD},\hat{\mathrm{B}}(s)\Big)\Big)\Big|_{e(s)} \qquad (9)$$

$\overline{\Psi}$ is a measured average of the expected additional travel time incurred by a vehicle due to an extra insertion into its original route, and it can be updated from system information every update time H_c as well as the set of parameters $\hat{B}(s)$. The functions *F* and *G* may not have closed forms, as our experience with the transit system context shows. This is why special care is needed in applying the conventional APC model to our problem, as described in this paper.

Following the adaptation, the estimation quality is checked every update time H_c. Thus

$$\hat{\mathrm{B}}(s+1) = \begin{cases} \hat{\mathrm{B}}(s) & \text{if } tS_j(k,k+1)\big|_{e(s)} \approx E[tS_j(k,k+1)]\big|_{e(s)} \\ \hat{\mathrm{B}}(x(s-r),y_m(s-r)) \ \text{ for } r:0,\ldots,H_e & \text{otherwise} \end{cases}$$

The estimation process $\hat{\mathrm{B}}(\cdot)$ requires data obtained from the performance of the system for a time period H_e measured backward since the occurrence of event *s*. This parameter H_e is defined arbitrarily by the modeler in order to perform a robust and representative estimation of the stochastic model embedded in the definition of the expected number of pick-up insertions as will be discussed in the next section.

In summary, the H-APC general structure is applied to the specific problem of computing the delays from rerouting of transit vehicles in real time, under stochastic and unknown demand generated dynamically. Due to the unknown internal relations between components of the system, and considering that the general expressions found in this formulation do not seem to have a convenient closed forms, the conventional adaptive predictive modeling techniques need to be applied with care for the dynamic pick up and delivery problem. The event-driven nature of controller decisions with the continuous nature of the system response is addressed to stress that the method here is in fact a new hybrid form of the conventional techniques. In the paper, we develop a detailed modeling approach to solve the problem. In addition, unknown parameters have been calibrated for estimating expected vehicle travel times in real time, assuming random demand generated dynamically, and a set of optimized embedded decision rules applied by the dispatching module. The framework is developed for application to a real-time large scale transit fleet routing problem, but has applicability in other contexts as well.

Acknowledgements

This research was partially financed by the California PATH Program, by the University of California Transportation Center, by Fondecyt, grant 1030700, Chile, and the Millennium Nucleus "Complex Engineering Systems".

References

A. Bemporad and M. Morari, “Control of systems integrating logic, dynamics, and constraints”, *Automatica* 35(3), 407-427 (1999).

L. Bodin, B. Golden, A. Assad and M. Ball, “Routing and scheduling of vehicles and crews. The state of the art”, *Computers and Operations Research* 10, 69-211 (1983).

E.F. Camacho and C.Bordons, *Model Predictive Control in Process Industry.* Springer - Verlag. London (1995).

D.W. Clarke and C. Mohtadi, “Properties of Generalized Predictive Control”, *Automatica* 25(6), 859-882 (1989).

J. Jaw, A. Odoni, H. Psaraftis and N. Wilson, “A heuristic algorithm for the multi-vehicle advance request dial-a-ride problem with time windows”, *Transportation Research* 20B(3), 243-257 (1986).

M. Morari and J.H. Lee, “Model Predictive Control: Past, Present and Future”, *Comp. Chem. Engng.* 23, 667-682 (1999).

H.N. Psaraftis, “An exact algorithm for the single vehicle many-to-many dial-a-ride problem with time windows”, *Transportation Science* 17(3), 351-357 (1983a).

H.N. Psaraftis, "Analysis of an $O(n^2)$ heuristic for the single vehicle many-to-many Euclidean dial-a-ride problem", *Transportation Research* 17B(2), 133-145 (1983b).

H.N. Psaraftis, "Scheduling large-scale advance-request dial-a-ride systems", *American Journal of Math. And Mgmt. Sciences* 6, 327-367 (1986).

H.N. Pasaraftis, "Dynamic Vehicle Routing Problems," in *Vehicle Routing: Methods and Studies*, B.L. Golden and A.A. Assad (eds), 223-248, North-Holland, Amsterdam, 1988.

R. Soeterboek, *Predictive Control: Unified Approach*. Prentice Hall (1992).

N.H.M. Wilson and H. Weissberg, "Advanced dial-a-ride algorithms research project: Final report", Report R76-20, Department of Civil Engineering, M.I.T., Cambridge, MA, 1976.

N.H.M. Wilson and E. Miller, "Advanced dial-a-ride algorithms research project phase II: Interim report", Report R77-31, Department of Civil Engineering, M.I.T., Cambridge, MA, 1977.