An Exact Approach for the Symmetric Capacitated Vehicle Routing Problem with Two Dimensional Loading Constraints

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1 Introduction

We consider a special case of the classical symmetric capacitated vehicle routing problem (*SCVRP*), in which we impose additional two - dimensional constraints on the loading of the vehicles. In this problem a fleet of K identical vehicles must serve n clients, each with a given demand, consisting in a lot of total weight d_i, and of m_i rectangular and two-dimensional items, of dimensions $h_\ell \times w_\ell$ ($\ell = 1, \dots, m_i$). Thus, a feasible two-dimensional packing ($2CL$) must be found for each of the containers to be used in a solution. We consider that each client must be served by just one vehicle (cluster constraint) and that, in order to facilitate operations, when unloading items of one lot, no items of successive lots must be moved inside the vehicle (loading constraint).

The problem derives from a company operating in the furniture market, and has several applications in the distribution of goods that cannot be placed one over the other. The problem is NP-Hard in the strong sense, since it is the combination of $SCVRP$ and of $2CL$, strictly deriving from two dimensional Bin Packing Problem, both NP-Hard in the strong sense. In the following, we refer to it as *2CL-SCVRP*.

We propose an exact approach, in which the routing part is solved through a Branch-and-Cut algorithm, that uses CPLEX 8.1 as LP solver. When a heuristic solution is found, the sets of items, corresponding to the lots that are to be loaded in each vehicle, are passed to a Branch-and-Bound procedure, to check the feasibility of the loadings. If the output from the checking is unfeasible for the items associated to a set of clients then a new constraint is added to the original problem, imposing that some clients must not be included in the same tour.

New lower bounds are proposed for the Branch-and-Bound procedure, and different heuristics, derived from the combination of the literatures of routing and packing, are included in order to improve the performance of the algorithm.

2 Problem Description

Given a central depot, the classical VRP calls for the determination of the optimal set of tours to be performed by a fleet of vehicles, in order to satisfy the demands of a given set of clients. In the capacitated symmetrical VRP ($SCVRP$), the vehicles have a maximum transport capacity, and all arcs can be travelled along both directions, producing the same cost.

The VRP problem was first introduced by Dantzig and Ramser in 1959, and since then it has been widely studied. A first effective heuristic was proposed in 1964 by Clarke and Wright. For a recent survey see Toth and Vigo (2001). Additional formulations can be found in Laporte and Norbert (1987), while Descrochers, Lenstra and Savelsbergh (1990) propose a classification scheme. Other surveys on exact and heuristic algorithms can be found in Christofides, Mingozzi and Toth (1979), Magnanti (1981), Bodin, Golden, Assad and Ball (1983), Christofides (1985), Golden and Assad (1985), Laporte (1997), Laporte and Osman (1995), Fisher (1995), Toth and Vigo (1998), Golden, Wasil, Kelly and Chao (1998). VRP is a very difficult optimization problem and indeed there are several benchmark instances in literature with 75 customers that are still unsolved by exact algorithms. Therefore, the problem addressed in this paper is expected to be even harder because it also considers 2-dimensional packing constraints.

We consider the case in which all vehicles are identical, and the demands of the clients are defined by a lot of given weights associated to the 2-dimensional rectangular items. We define V as the set of vertices corresponding to the depot and clients and E as the set of edges. We use the following notation:

- K = number of vehicles,
- n = number of clients,
- c_e = cost for travelling along edge $e \in E$,
- $\delta(S)$ = set of edges leaving $S \subseteq V$,
- $E(S)$ = set of edges with both vertices in $S \subseteq V$,
- D, H, W = weight, height and width capacities of the vehicles,
- d_i = total weight of the lot demanded by client i (for $i =, \ldots, n$),
- m_i = number of items in the lot demanded by client i (for $i = 1, \ldots, n$),
- h_ℓ, w_ℓ = height and width of the item ℓ in the lot of client i (for $i = 1, \ldots, n; \ell = 1, \ldots, m_i$).

When trying to load a set of lots into a vehicle, we have to check both that the maximum weight capacity is respected, and that the two-dimensional packing is feasible. Since both the container and the items are rectangular, we consider orthogonal packing (items are packed with their edges parallel to the sides of the container). Moreover, we consider an heterogeneous packing (heights and widths of the items can be different), and we assume (for simplicity) that no rotation is allowed.

Additional constraints derive from the aim of facilitating the operations of transport and delivery of the goods. Thus, we first impose that one client must be visited by only one vehicle. As a direct consequence, and without considering the stability of the vehicle, we do not take into account the weights of the single items, but only the one of the complete lot (in the latter we refer to these constraints as to the *cluster constraints*). Moreover, we impose that, when unloading items of one lot, no item of lots that are successive unload in the tour must be moved. Robot-loading is assumed (when loading and unloading, forks only make vertical movements).

The load feasibility problem considered belongs to the category of *Container Loading Problems*, which have been widely studied in literature. Heuristic approaches have been proposed by Pisinger (1998, 2002) and Bortfeld (2002) (who also proposed genetic approaches as in Gehring (2001)). An analytical model for the 3-dimensional container problem has been presented by Chen, Lee and Shen (1995). General algorithms have been presented by Scheithauer (1992), who also proposed an LP-based bound and an approach for the related Multi-Pallet Loading Problem. Many of these studies derive or are strictly linked with studies on Bin-Packing and on Knapsack problems. Regarding the Bin-Packing Problem, we refer to Martello, Pisinger and Vigo (1998), for the Knapsack Problem to Martello and Toth (1980), and to Fekete and Schephers (1997).

We propose an integer mathematical programming model for the 2CL-SCVRP using a 0-1 variable z_e for each $e \in E$ and large families of linear inequalities. Due to the large number of constraints, a branch-and-cut algorithm will be described to solve the model. Indeed, this section presents separation procedures to find violated inequalities of these families and a primal heuristic to speed up the solution of the algorithm. The incomplete model on the z_e variables is named *master problem* while the separation procedures are named *subproblems.*

More precisely, the mathematical model will be solved through a branch-and-cut approach where some inequalities will be dynamically generated through an iterative procedure. At each iteration of the procedure a relaxed model (the master problem) will be solved and the optimal solution will be latter used to check if there are missing inequalities by solving a subproblem. If a violated inequality is detected, then a new iteration is applied with this new constraint as in standard *cutting-plane approaches.* The integrability of the solution is achieved by a branching scheme as in classical *branch-and-bound procedures.* The overall procedure is combined in a so-called *branch-and-cut algorithm.* See, e.g., Caprara and Fischetti (1998) for details on these approaches.

To model the 2CL-SCVRP we use a 0-1 variable z_e for each $e \in E$ representing that a vehicle must route edge e if and only if $x_e = 1$. Then the problem is:

$$\min \sum_{e \in E} c_e z_e \tag{1}$$

$$\sum_{e \in \delta(i)} z_e = 2 \qquad (i \in V \setminus \{0\}) \tag{2}$$

$$\sum_{e\in\delta(0)} z_e = 2K \tag{3}$$

$$z_e \in \{0,1\} \qquad (e \in E) \tag{4}$$

Some constraints are missing in the above model because, e.g., we need connectivity between the depot and the customers. This requirement can be considered by also taking into account the capacity of the vehicle as follows:

$$\sum_{e\in\delta(S)} x_e \geq r(S) \quad (S \subseteq V\backslash\{0\}, S \neq \emptyset) \tag{5}$$

where $r(S)$ is defined by:

$$r(S) = \left\lceil \sum_{i\in S} d_i/D \right\rceil \quad (S \subseteq V\backslash\{0\}, S \neq \emptyset) \tag{6}$$

Still, the model (1)–(5) in not valid since it considers the weight of the lots of items but not the height and width of the items (neither the robot-loading requirement). An improvement of the model is obtained by considering the areas of the items and containers. More precisely, let $A = H \times W$, and $a_i = \sum_{\ell=1}^{m_i} h_\ell \times w_\ell$ for each customer i; we can improve $r(S)$ with:

$$r'(S) = \max\left\{ r(S), \left\lceil \sum_{i\in S} a_i/A \right\rceil \right\} \quad (S \subseteq V\backslash\{0\}, S \neq \emptyset) \tag{7}$$

and consequently strengthening the capacity cuts constraints (5). The set of constraints are in a large number and therefore it is not immediate to consider all of them explicitly in a mathematical model. We will address the problem of generating dynamically some of them when required. This problem is named *separation problem* associated to constraints (5).

Nevertheless, the model (1)–(5) even using $r'(S)$ is not valid for the 2CL-SCVRP because the solutions of the above model could required unfeasible load for a container. To avoid this infeasible solutions we have included the following family of *subtour elimination constraints*:

$$\sum_{e\in E(S)} x_e \leq |S| - 1 \tag{8}$$

for all subset S such that $0 \in S$ and all the customers in S cannot be served by only one vehicle.

Now, model (1)–(8) is a valid model since each solution contains K cycles passing through the depot, each customer is in exactly one route, and all the items associated to the customers assigned to each route can be load in the vehicle with the required load constraints. Of course, the big disadvantage of this model is due to constraints (8). Indeed, unlike in the well-known Travelling Salesman Problem, this family of constraints is hard to be managed by a cutting-plane approach since the constraints are not defined for all subset S but only for some very special subsets. Nevertheless, the good result is that it is possible to develop an effective algorithm for the problem of finding a violated constraints (if any exists) when necessary. The analysis of this problem is addressed in the paper presented during the TRISTAN conference, together with a mathematical model and computational results.

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