An Exact Algorithm for a Dynamic, Time-Dependent and Stochastic Evacuation Problem

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1 Introduction

Determination of optimal evacuation plans for a large burning building or a building that has come under attack by enemy or natural catastrophe is addressed. In a network representation of the evacuation problem, the network represents the layout of the circulation systems of the building, where nodes correspond with locations inside the building (such as offices, meeting rooms, lobbies, lavatories, and building exits) and arcs correspond to the passageways that connect these locations (such as staircases, elevator shafts, doorways, corridors and ramps). The nodes at which the people are located when the evacuation begins are called source nodes and the exits are referred to as sink nodes. The arc weight is the time it takes to traverse the arc. When large numbers of people must be evacuated from the building simultaneously, issues concerning capacity of the network arcs arise. The capacity of an arc is the number of people that can pass through the associated passageway per unit of time. The arc capacities are dependent upon the size and type of passageway that the arcs represent. In this work, the SEscape (Safest Escape) algorithm is proposed. It seeks the pattern of flow that has the highest probability of successfully shipping all supply from a set of source nodes to a sink node. Specifically, a conceptual framework and algorithmic steps are provided for determining optimal evacuation paths in capacitated networks, where arc traversal times are time-dependent and arc capacities are discrete random variables with distribution functions that vary with time. Capacities are assumed to be recaptured over time, i.e. a dynamic (in addition to time-dependent) network flow problem is considered

Emergency evacuations are often characterized by dangers that strengthen and spread over time. Circumstances in an evacuation induce the possibility that successful egress may be inhibited by partial or complete failure of key escape paths. Moreover, one cannot know how the situation will progress with certainty even if the exact location and type of event that initiated the need for the evacuation is known. Thus, it is crucial to model the dynamic and uncertain nature of capacities inherent in such situations. Figure 1 depicts the hypothetical evolution of damage to one story of the Pentagon on September 11, 2001 and shows the necessity of considering such damage evolution and predictions of future remaining capacities in providing evacuation instructions. In the figure, corridors that were destroyed or completely disabled are removed from the network representation and capacities on remaining arcs likely deviated from normal values.

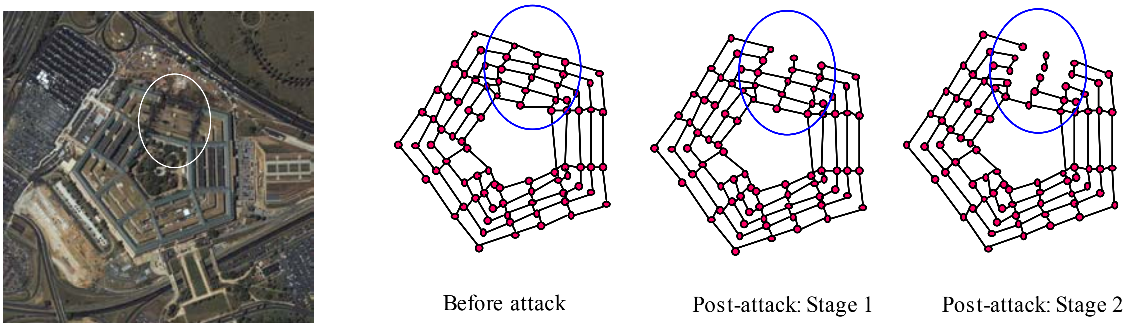

Figure 1. Post-attack hypothetical damage evolution of one story of the Pentagon.

In determining the optimal instructions, it is important to explicitly consider the probable ways that the damage might evolve, being careful not to route an evacuee to an arc that would have a high likelihood of failing by the time he or she arrived at that location. Instructions that do not consider the evolution of damage over time and threats of probable additional destruction and deterioration can result in suboptimal decisions that can lead to unnecessary imposed risk and unnecessary lost lives.

Several network flow problems have been modeled in static networks. Yamada (1996) employed a classical minimum cost flow algorithm to evaluate the current evacuation plan for Yokosuka City, Japan. Calvete (2003) studied the general equal flow problem, which is defined as the minimum cost flow problem with side constraints. The side constraints require each arc in a specified subset to carry the same amount of flow.

Numerous works consider the dynamic nature of network attributes that is present in many real world applications. A survey of algorithms, applications and implementations of several dynamic network flow problems is provided by Aronson (1989). Choi et al. (1988) addressed the problem of finding building evacuation plans, where each arc's capacity is a function of the rate of flow on the incoming arcs, as a network flow problem with side constraints. Fleischer and Tardos (1998) extended algorithms for dealing with several discrete-time dynamic flow problems to solve their continuous-time counterparts. Chen and Chin (1990), Rosen et al. (1991), and Calvete (2004) studied a variant of the quickest flow problem: the quickest path problem. The quickest path problem seeks a single path for shipping the supply from a source to a sink with minimum total traversal time, where the traversal time of an arc depends on the rate of flow on that arc.

While many papers have addressed network flow problems in dynamic (but time-invariant) networks, few works have considered the time-dependent characteristics of network attributes. Miller-Hooks and Stock Patterson (2004) proposed the TDQFP algorithm for the quickest evacuation problem in time-varying capacitated networks with dynamic flow characteristics. Cai et al. (2001) developed solution algorithms to solve the time-varying minimum cost flow problem with three waiting policies at nodes.

Works discussed thus far consider only deterministic network flow problems. Other works address the connectivity and reliability of networks, where the nodes or arcs may randomly fail with known probability. Frank and Gaul (1982) provided bounds and approximations to the probability that the entire network is connected. Jentsch (1998), Lucet and Manouvrier (1999), and Lin (2001) proposed procedures for assessing system reliability in static networks with stochastic arc capacities. System reliability is defined as the probability that the network can accommodate a given level of flow. None of these works provide routing plans.

Other works have also addressed stochastic network flow problems, where routing plans are determined. For example, Glockner and Nemhauser (2000, 2001) considered a dynamic, time-invariant network flow problem with random arc capacities. They provided a multistage stochastic linear programming formulation of this stochastic minimum cost network flow problem. Karbowicz and Macgregor Smith (1984) proposed a simulation-based methodology for determining the evacuation paths along which to send evacuees. They consider both distance and evacuation time in finding the paths. While their approach can address the time-varying and stochastic nature of the evacuation problem, at least heuristically, it is expected to perform poorly in congested networks. To the best of our knowledge, no existing methodology is proposed for efficiently determining optimal evacuation paths, where the uncertain, dynamic and time-varying conditions inherent in circumstances warranting evacuation are explicitly considered.

2 Conceptual framework and solution approach

In this work, we address the problem of determining optimal evacuation plans in dynamic networks with time-varying arc traversal times and stochastic, time-varying arc capacities. In capacitated networks, a set of path flows that is optimal in one realization may not be feasible in another. And a single solution that is feasible in all realizations may not exist. Thus, a solution approach that considers, at least implicitly, the probability of successfully completing the shipments in all realizations is required.

We assert that to ship the supply through a network with stochastic arc capacities, particularly in emergency evacuation, a set of path flows that has the highest probability of ensuring successful arrival at the destination is preferred. The example given in Figure 2a is used to illustrate this. Two paths, paths I and II exist between node 1 and node 2. The corresponding capacities are given probabilistically, e.g. the possible capacities of path I are 0 with probability 0.7 and 4 with probability 0.3: P$\{c_I = 0\}$ = 0.7 and P$\{c_I = 4\}$ = 0.3, where P$\{c_i = n\}$ denotes the probability that the capacity of path *i* equals *n*.

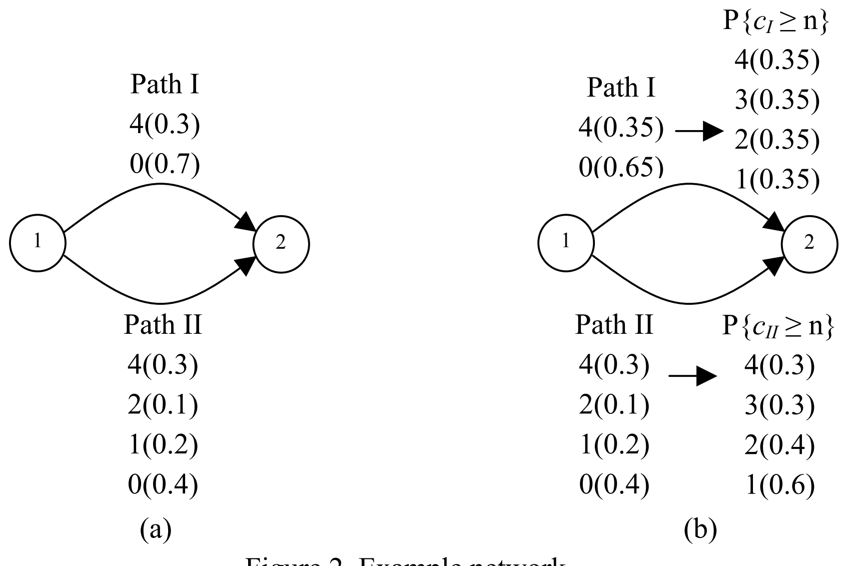

Figure 2. Example network.

Because the probability that four units can successfully traverse path I is equal to that of path II (P$\{c_I \geq 4\}$ = P$\{c_{II} \geq 4\}$ = 0.3), one may deem both paths equally good for shipping four units from node 1 to node 2. However, by doing so, one would consider only the case where all four units are successfully shipped to node 2. That is, all four units would be considered as one package and the possibility that one and two units may get through path II with probability 0.2 and 0.1, respectively, would be ignored. By considering these probabilities, one can see that path II is preferable to path I. The solution approach proposed here considers this likelihood that each single unit will successfully traverse each arc on its way to the sink.

Often, stochastic problems are addressed by considering the expected values of the random characteristics. An example (see Figure 2b) is given to illustrate that employing the expected capacity may not be particularly useful in this application domain. The example network shown in Figure 2b differs only slightly from that given in Figure 2a, i.e. $P\{c_I = 4\}$ is changed from 0.3 to 0.35 and $P\{c_I = 0\}$ is set to 0.65. With this adjustment, path II is no longer exclusively preferred to path I for shipping four units of flow. Rather, it is preferable to split the flow across both Paths I and II. Suppose that each arc capacity is replaced by its expected value. In the example, the expected capacities of paths I and II are 1.4 and 1.6, respectively. With this information, one would choose to ship all of the flow along path II and would not split the flow among the two paths.

Another possible measure for evaluating the paths is the value of flow that is expected to successfully traverse the network, i.e. the expected flow. While the expected capacity of an arc is constant, the expected flow along an arc depends on the arc's possible values of capacity, corresponding probabilities of occurrence, and the number of units that one attempts to ship. For example, the expected flow for shipping two units on path II can be computed as follows.

$$0 \cdot P\{c_{II} = 0\} + 1 \cdot P\{c_{II} = 1\} + 2 \cdot P\{c_{II} \geq 2\} = 0 \cdot 0.4 + 1 \cdot 0.2 + 2 \cdot (0.1+0.3) = 1.0.$$

Employing similar computations, the expected flow of path I when shipping four, three, two and one units along this path are 1.4, 1.05, 0.7 and 0.35, respectively, and the corresponding expected flow of path II are 1.6, 1.3, 1.0 and 0.6, respectively. Again, by using the expected value, one would choose to ship all of the flow along path II instead of splitting the flow among the two paths.

To develop a methodology to solve for the optimal pattern of flow (i.e. that would choose to split the flow across Paths I and II in the second example), we define the term network utility as the probability that all individual units of flow can successfully traverse the paths to which they are assigned. We refer to the problem of determining the path flows with maximum network utility as the Safest Escape problem and the corresponding algorithm as the SEscape algorithm. The algorithm can be decomposed into two main components: 1) generation of a set of paths connecting the source nodes and sink with nonzero probability of having adequate capacity to accommodate all supply units and 2) determination of the number of units to be sent along each path to complete the shipments. At each iteration of the algorithm, a path is selected such that the probability that the shipment of a single unit will be successfully completed is maximized.

To illustrate this concept of network utility, we revisit the example given in Figure 2b. Suppose there are four supply units at node 1. The first unit should be assigned to path II since the probability that it will arrive at node 2 by way of this path is greater than that through path I (i.e. $P\{c_{II} \geq 1\} = 0.6 > P\{c_I \geq 1\} = 0.35$). Bearing in mind that we cannot know *a priori* whether or not the first shipment will successfully arrive at its destination, path II is the preferred path for shipping the second unit, because the probability that path II can accommodate two units simultaneously, $P\{c_{II} \geq 2\}$, is greater than the probability that path I can accommodate one unit, $P\{c_I \geq 1\}$. However, for the third unit, path I is preferred, because the probability that path II can accommodate three units simultaneously, $P\{c_{II} \geq 3\}$, is less than the probability that path I can accommodate one unit, $P\{c_I \geq 1\}$. This is also the case for shipping the last unit, where $P\{c_I \geq 2\} = 0.4 > P\{c_{II} \geq 3\} = 0.3$. Hence, the best routing plan for these four supply units is to ship two units along each path. The resulting network utility is $P\{c_{II} \geq 1\} \cdot P\{c_{II} \geq 2\}\ P\{c_I \geq 1\}\ P\{c_I \geq 2\} = 0.6 \cdot 0.4 \cdot 0.35 \cdot 0.35 = 0.0294$. Network utility differs from the probability of the realizations for which the flow can be accommodated with certainty, i.e. $P\{c_I \geq 2\} \cdot P\{c_{II} \geq 2\} = 0.35 \cdot 0.4 = 0.14$.

For clarity, the examples described herein have been static. To extend these concepts for networks with time-varying arc attributes, specifically arc traversal times and capacities, the arc travel times, capacities and flow must be expressed with respect to the arrival time at the nodes and the path selection process is performed over the time dimension.

The proposed SEscape algorithm can be viewed as an extension of the Time-Dependent Quickest Flow Problem (TDQFP) algorithm of Miller-Hooks and Stock Patterson (2004) for addressing quickest flow problems in deterministic, time-varying, dynamic networks. Unlike the TDQFP algorithm, the SEscape algorithm employs a probabilistic, time-dependent (PTD) residual network, with arc weight values that are related to the probability that the capacity of arc (i,j) at time t is no less than n, $P\{c_{ij}(t) \geq n\}$, as shown in Figure 2b. Through the special structure of the PTD residual network, the probability that each unit will succeed in traversing each arc can be readily obtained. In the TDQFP algorithm, at each iteration, the path with the earliest arrival time in a time-dependent residual network is chosen. In the SEscape algorithm, rather than seek this earliest arrival time path at each iteration, the algorithm chooses the path with the maximum probability of successfully shipping one unit of flow from a source node to the sink in the PTD residual network. An overview of the SEscape algorithm is presented next.

1. Generating Maximum Probability Paths: Given the PTD residual network, the path with maximum probability for sending one unit of flow from a source to the sink is obtained by the maximum probability path (MPP) algorithm (developed as part of this work). One unit of flow is shipped through this path.

2. Updating the PTD Residual Network: In the PTD residual network, the values related to the $\mathrm{P}\{c_{ij}(t) \geq n\}$ are maintained along each arc (*i,j*) at departure time *t*. These reduced arc capacities are updated at each iteration and backward arcs, where $\mathrm{P}\{c_{ji}(t') \geq n\}$ $t' = t + \tau_{ij}(t) \leq T$, for T the last time in the time period of interest, are added to the graph as needed. Backward arcs allow the return of capacity to an arc to cancel decisions made earlier (ensuring optimality in the larger problem).
3. Termination: The algorithm terminates when all supply has been assigned to paths.

Upon completion, the algorithm provides a set of paths from the source nodes to the sink and the corresponding number of units to be sent along each path such that the maximum network utility is achieved. The actual amount of flow that will be able to pass through each arc at a given point in time is not known until the solution is implemented and values of the arc capacities are revealed. The algorithm is pseudopolynomial. Numerical experiments have been conducted to assess the computational performance of this algorithm. The proposed conceptual framework and specific algorithmic steps can be used in evacuation planning, enabling safer evacuation of a building in the event of military attack, fire, natural disaster, or other circumstances warranting quick escape.

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