A Bilevel Network Flow Model for Planning HazMat Shipments♣

Lucio Bianco, *Massimiliano Caramia*, *Stefano Giordani*

Dipartimento di Ingegneria dell'Impresa
Università di Roma "Tor Vergata"
Via del Politecnico 1, I-00133 Roma, Italy
{bianco, caramia, giordani}@disp.uniroma2.it

Abstract

In this work we consider the problem of hazmat global route planning, where a set of hazmat shipments has to be routed over a transportation network in order to transport a given amount of hazardous materials from specific origin points to specific destination points. One of the main goal in hazmat global route planning is the minimization of the total risk imposed by hazmat transportations on the public and environment. As a matter of fact, risk equity has also to be addressed, since several hazmat shipments has to be carried out on the network. We provide a bilevel programming formulation for the hazmat global route planning that takes into account both total risk minimization and risk equity.

Keywords: Hazardous materials; Transportation planning; Bilevel optimization.

1. Introduction

The transportation of hazardous materials (hazmats), though may be classified among the most general freight transport issues, is an activity that presents extremely typical characteristics which make its planning, management and control a particularly complex task. What differentiates hazmat shipments from the transportation of other materials is the risk associated with an accidental release of hazardous materials during transportation. To reduce the occurrence of dangerous events it is necessary to provide appropriate answers to safety management associated with dangerous goods shipments.

Risk assessment and hazmat shipments planning are two of the main research fields in hazmat transportation. Risk is the primary ingredient that distinguishes hazmat transportation problems from other transportation problems. In the literature, a lot of work has already been done in risk assessment, by modeling risk probability distribution over given areas, for example, taking into account the risk related to the carried object and the transport modality (Abkovitz et al. 1984) and the environmental conditions (Patel and Horowits, 1994). There are several excellent review articles addressing the literature related to modeling of risk for hazmat transportation; however, there is no universally definition of risk. In this work we refer to the traditional definition of risk over a link, that is the societal risk defined as the product of the population along the link within the neighborhood and the probability of an accident (Erkut and Verter, 1998).

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The main issue of hazmat shipments planning is routing hazmat shipments, that involves a selection among the alternative paths between origin-destination pairs. From a carrier's perspective, shipment contracts can be considered independently and a routing decision needs to be made for each shipment, which we call the local route planning problem. At the macro level, hazmat routing is a "many to many" routing problem with multiple origins and an even greater number of destinations. In the sequel, we refer to this problem as global route planning.

The local routing problem is to select routes between a given origin-destination pair for a given hazmat, transport mode, and vehicle type. Thus, for each shipment order, this problem focuses on a single-commodity and a single origin-destination route plan. Since these plans are often made without taking into consideration the general context, certain links of the transport network tend to be overloaded with hazmat traffic. This could result in a considerable increase of accident probabilities on some road links as well as leading to inequity in the spatial distribution of risk.

Transport costs remain as the carriers' main focus. In contrast, the government has to consider the global problem by taking into account all shipments in its jurisdiction. This leads to a harder class of problems that involve multi-commodity and multiple origin-destination routing decisions. Moreover, besides the minimization of the total risk imposed on the public and environment, a government agency may need to consider promoting equity in the spatial distribution of risk. This becomes crucial in the case in which certain population zones are exposed to intolerable levels of risk as a result of the carriers' routing decisions.

Therefore, in the global route planning for hazmat shipments, the main problem is that of finding minimum risk routes, while limiting and equitably spreading the risk in any zone in which the transportation network is embedded. As a matter of fact, risk equity has to be taken into account also whenever it is necessary to carry out several hazmat shipments from a given origin to a given destination. In this situation, the planning effort has to be devoted to distribute risk uniformly among all the zones of the geographical crossed region. This concept is well defined in (Keeney, 1980), where a measure of the collective risk is determined with explicit reference to the equity.

Hazmat local route planning has attracted the attention of many OR researchers, while the global route planning problem has attained relatively little attention in the literature. The results in this latter area include the works of Gopalan et al. (1990), Lindner-Dutton et al. (1991) and Marianov and ReVelle (1998). The works of Akgün et al. (2000), Dell'Olmo et al. (2005) and Carotenuto et al. (2007) on the problem of finding a number of spatially dissimilar paths between an origin and a destination can also be considered in this area. For a complete survey on hazmat logistics the reader is referred to Erkut et al. (2007).

In this paper, we give a more general formulation than those present in the literature for the following hazmat shipment global route planning problem: a set of hazmat shipments has to be routed over a transportation network in order to transport a given amount of hazardous materials from specific origin points to specific destination points with the aim of minimizing the total risk of the shipments and spreading the risk equitably over the geographical region in which the transportation network is embedded. The main advances presented are:

- flow based formulation as opposed to path based formulations in the literature; this helps in solving the problem without iteratively computing paths and avoiding each route search to be biased by the previously found paths;
- proposal of a bi-level optimization model, with total risk and equity as objective functions.

The proposed model is experimentally evaluated on an Italian geographical region.

2. The Bilevel Optimization Model

In a bileveling mathematical programming (see e.g. Bialas and Karwan, 1984) one is concerned with two optimization problems where the feasible region of the first problem, called upper level (or leader) problem, is determined by the knowledge of the other optimization problem, called lower level (or follower) problem. Problems that naturally can be modelled by means of bilevel programming are those for which variables of the first problem are constrained to be the optimal solution of the lower level problem.

In general, bilevel optimization is issued to cope with problems with two decision makers in which the optimal decision of one of them (the leader) is constrained by the decision of the second decision maker (the follower). The second level decision maker optimizes his/her objective function under a feasible region that is defined by the first level decision maker. The latter, with this setting, is in charge to define all the possible reactions of the second level decision maker and selects those values for the variable controlled by the follower that produce the best outcome for his/her objective function. A general formulation of such problem is the following:

$$\min_{(x_1,x_2)\in S} f_1(x_1,x_2)$$
$$x_2 \in \arg\min_{x_2\in S(x_1)} f_2(x_1,x_2)$$

where x_1 is a vector of m_1 real valued components representing the variables controlled by the upper level program, and x_2 is a vector of m_2 real valued components controlled by the lower program. S is the common feasible region, and $S(x_1)=\{x_2\in R^{m_2}:(x_1,x_2)\in S\}$. When both the leader and the follower problems are linear then we have a bilevel linear/linear program that can be explicitly defined e.g. as follows:

$$\begin{aligned}
&\min_{x_1} f_1 = k_1x_1 + k_2x_2\\
&\quad A_1x_1 + A_2x_2 \geq b_1\\
&\quad x_1 \geq 0\\
&\text{where } x_2 \text{ solves}\\
&\min_{x_2} f_2 = k_3x_1 + k_4x_2\\
&\quad A_3x_1 + A_4x_2 \geq b_2\\
&\quad x_2 \geq 0
\end{aligned}$$

where $A_1\in R^{m_3\times m_1}$, $A_2\in R^{m_3\times m_2}$, $A_3\in R^{m_4\times m_1}$, $A_4\in R^{m_4\times m_2}$, $k_1,k_3\in R^{m_1}$, $k_2,k_4\in R^{m_2}$, $b_1\in R^{m_3}$, $b_2\in R^{m_4}$.

In the hazmat literature there is a previous work done with bilevel optimization, that is based on the idea of modelling the decision of the government and the carriers, respectively (Kara and Verter, 2004). In the paper of Kara and Verter (2004), the authors study the problem of how to design a road network available to dangerous goods carriers by an ad-hoc government agency such that the road segments selected are those that minimize the total risk. Indeed, on the (sub)-network that minimize the total risk each carrier should select a number of paths typically minimizing the total cost of such paths, and thus the best should be that such paths overlap with those selected by the government, and this is why the problem is a bilevel programming one.

As for the other papers in the hazmat literature, we note that also the model by Kara and Verter (2004) uses paths to determine the routes onto which the dangerous materials should be shipped.

In this paper, we propose a new model based on flows. In our attempt to design a model based on flows, we posed a first question on how capacities on the transportation network should have been defined, and, in particular, if capacities have sense in the studied problem. Indeed, in a classical flow problem, flow conservation laws and capacity constraints are to be obeyed, and in our setting the role played by capacities is not clear since the overall quantity of the commodities to be shipped does not undergo to a violation of the arc capacities. This is the main reason for which we decided to consider a bilevel program formulation. To be more specific, we assumed the existence of two decision makers, one willing to define a feasible flow on the network that produces the minimum total risk on the population, and the other that, interpreting the optimal flow of the previous (lower level) decision maker as an arc capacity vector, minimizes the maximum risk on the network, i.e., coping with an equity risk distribution objective function. Note that, differently from what happens in the paper of Kara and Verter (2004), the two actors belong to the same strategic decision area, i.e., in this paper we do not consider the carrier point of view.

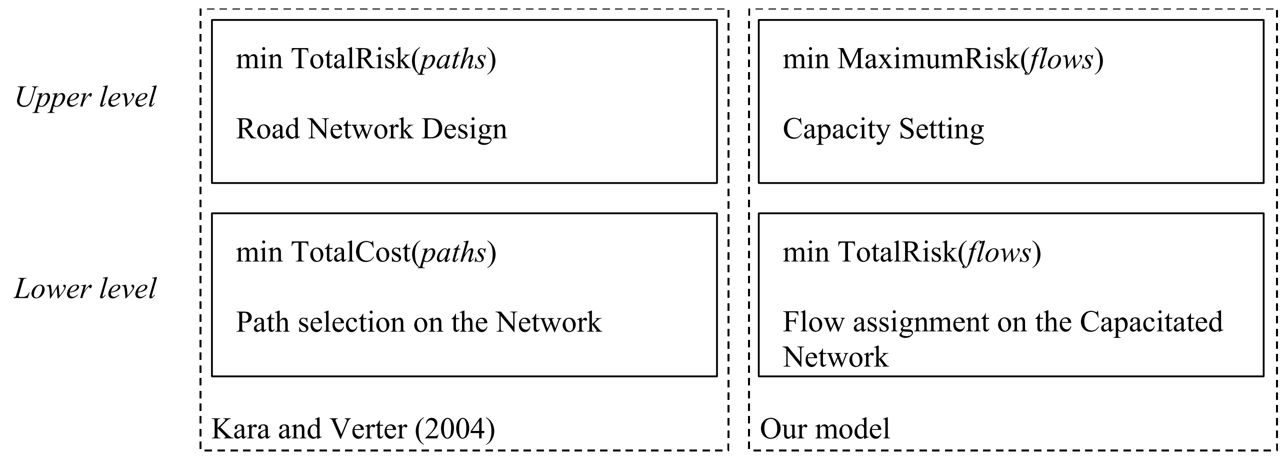

Figure 1: Comparison between Kara and Verter (2004) and the proposed model.

In Figure 1, we report the comparison between our model and that of Kara and Verter (2004). In particular, our problem is a network design one, where the goal is not that of determining a subgraph of the whole transportation road network, but it is to determine capacities leading to a balanced risk over the population as evenly as possible.

Let the transportation network be represented as a directed graph $G = (N, A)$, with N and A being the set of n nodes and the set of m links (arcs) of the network, respectively. Let C be the set of hazmat shipments, and, for each hazmat shipment $c \in C$, let s^c and t^c be respectively the source node (origin point) and the sink node (destination point), and let d^c be the amount of hazmat to be shipped from s^c to t^c.

We assume that the risk is computed on each link of the network and is proportional to the flow traversing such an arc. Let $s(x_{ij}^c)$ be a function that models the risk on arc $(i, j) \in A$ due to the flow of hazmat shipment $c \in C$ on that arc: we assume that this function is linear in flow x_{ij}^c assigned to arc (i, j) related to commodity (hazmat shipment) c, that is, $s(x_{ij}^c) = \rho_{ij}^c x_{ij}^c$, where ρ_{ij}^c is the risk per flow unit.

Roughly speaking, we model the hazmat problem imposing that the feasible region S represents all the feasible flows in the transportation network where arc capacities are defined by the optimal solutions of the upper level program. Note that once capacities are fixed by the leader decision maker, the follower's problem becomes a minimum cost flow problem, where the arc cost models the risk of traversing an arc through the linear relation between flows and risks. To be more specific, the lower level problem is a multi-commodity network flow problem, where a specific hazmat shipment c (commodity c) is associated with a couple $(s^c; t^c)$ of source-sink nodes.

Assume then that the variables of the leader are capacities, i.e., he/she wants the maximum risk associated with arc network to be minimized, given a feasible flow that is controlled by the followers' formulation, i.e., the decision maker that solves a minimum cost flow problem, given the capacity of the upper level decision maker. It is easy to understand that the lower level formulation might give different feasible flows based on the capacities given by the first decision maker and that similarly the capacities of the upper program are affected by the flow of the second decision maker. Thus, let x_1 be the vector of flows, and x_2 be the vector of capacities. Given that the objective function of the follower is $f_2(x_1, x_2)$ we can write the following program that solves the lower level program

$$\min_{x_1}\{f_2(x_1,x_2) : g_2(x_1,x_2) \le 0, h_2(x_1,x_2) = 0\}$$

where

$$f_2 : R^{m_1} \times R^{m_2} \to R$$

$$g_2 : R^{m_1} \times R^{m_2} \to R^p$$

$$h_2 : R^{m_1} \times R^{m_2} \to R^q$$

$$g_2(x_1,x_2) = g^1(x_1,x_2),\ldots, g^p(x_1,x_2)$$

$$h_2(x_1,x_2) = h^1(x_1,x_2),\ldots, h^q(x_1,x_2)$$

Let now $\Psi(x_2)$ be the set of optimal solutions of the previous problem, then the bilevel program can be formulated as a unique problem, that is the problem of the leader decision maker, as follows:

$$\min_{x_2}\{f_1(x_1(x_2),x_2): g_1(x_1(x_2),x_2)\le 0, h_1(x_1(x_2),x_2)=0, x_1(x_2)\in \Psi(x_2)\}$$

where

$$f_1 : R^{m_1}\times R^{m_2}\to R$$

$$g_1 : R^{m_1}\times R^{m_2}\to R^k$$

$$h_1 : R^{m_1}\times R^{m_2}\to R^\ell$$

Note that, as it will appear clearer next, g_2, h_2 are the classical capacity constraints and flow conservation laws, respectively.

In the following, we present the bilevel formulation where the *upper level formulation* looks for risk equity, while the *lower level formulation* looks for total risk minimization.

$$\textit{Upper level formulation}\begin{cases} \lambda^* = \min_y \lambda \\ \sum_{c\in C}\rho_{ij}^c x_{ij}^c \le \lambda, \quad \forall (i,j)\in A & (1) \\ y_{ij}\ge 0, \quad \forall (i,j)\in A & (2) \\ \text{where } x_{ij}^c \text{ solves the lower level problem} \end{cases}$$

$$\textit{Lower level formulation}\begin{cases} R_{tot}^* = \min_x \sum_{c\in C}\sum_{(i,j)\in A}\rho_{ij}^c x_{ij}^c \\ \sum_{(i,j)\in FS(i)} x_{ij}^c - \sum_{(j,i)\in BS(i)} x_{ji}^c = \begin{cases} d^c, & i=s^c, \forall c\in C \\ 0, & \forall i\in N\setminus\{s^c,t^c\}, \forall c\in C \\ -d^c, & i=t^c, \forall c\in C \end{cases} & (3) \\ \sum_{c\in C} x_{ij}^c \le y_{ij}, \quad \forall (i,j)\in A & (4) \\ x_{ij}^c\ge 0, \quad \forall (i,j)\in A, \forall c\in C & (5) \end{cases}$$

The *upper level formulation* models the problem of assuring an equitable distribution of the risk over the network given a multi-commodity hazmat flow by the lower level decision maker. The model minimizes the maximum risk λ allowed on each arc of the network, finding appropriate are capacities to the network. Constraints (1) say that the risk induced over the population of each arc cannot be greater than λ, while (2) are nonnegative constraints on the arc capacities.

The *lower level formulation* models the problem of minimizing the total risk over the network induced by a multi-commodity flow, given the capacities imposed by the leader decision maker. Being *FS*(*i*) and *BS*(*i*) respectively the forward and backward stars of each node $i \in N$, constraints (3) impose the conservation of flow at nodes for each commodity. Constraints (4) say that the total flow on arc $(i, j) \in A$ should not exceed the arc capacity value y_{ij}. Note that flows are variables of the lower level model and capacities of the upper level, thus the minimization of R_{tot} is assumed over the x_{ij}^c variables, and λ over the y_{ij} variables.

Therefore, our formulation can be rewritten as

$$
P1\left\{
\begin{array}{l}
\min_{y} \lambda \\
\sum_{c\in C} \rho_{ij}^c x_{ij}^c \le \lambda, \quad \forall (i,j)\in A \\
y_{ij} \ge 0, \quad \forall (i,j)\in A \\
P2\left\{
\begin{array}{ll}
\min_{x} \sum_{c\in C} \sum_{(i,j)\in A} \rho_{ij}^c x_{ij}^c & \\
\sum_{(i,j)\in FS(i)} x_{ij}^c - \sum_{(j,i)\in BS(i)} x_{ji}^c = \begin{cases} d^c, & i = s^c, \forall c\in C \\ 0, & \forall i\in N\setminus\{s^c,t^c\}, \forall c\in C \\ -d^c, & i = t^c, \forall c\in C \end{cases} & (3) \\
\sum_{c\in C} x_{ij}^c \le y_{ij}, \quad \forall (i,j)\in A & (4) \\
x_{ij}^c \ge 0, \quad \forall (i,j)\in A, \forall c\in C &
\end{array}
\right.
\end{array}
\right.
$$

Fortuny-Amat and McCarl (1981) proposed a method to solve the linear/linear bilevel formulation, by replacing the lower level model with the Kuhn-Tucker optimality conditions into the upper level problem, thus obtaining a unique optimization problem. Therefore, in the following, we will transform our bilevel program into a new problem with a single objective function, assuming that *P*2 is the primal formulation from which we want to define its optimality conditions, i.e., complementary slackness and primal and dual feasibility. To this aim let us define:

γ_i^c = dual variables associated with primal constraints (3), where $i \in N$, $c \in C$;
η_{ij} = dual variables associated with primal constraints (4) where $(i, j) \in A$;
w_{ij} = slack variables of the primal constraints (4), where $(i, j) \in A$;
z_{ij}^c = slack variables of the dual constraints, where $(i, j) \in A$, $c \in C$.

Now, we report the new optimization problem *P*3 with a single objective function, where we considered the optimality conditions of the lower level decision maker.

$$
P3\begin{cases}
\min\limits_{(x,y)} \lambda \\
\sum\limits_{c\in C} \rho_{ij}^c x_{ij}^c \le \lambda, \quad \forall (i,j)\in A \\
\sum\limits_{(i,j)\in FS(i)} x_{ij}^c - \sum\limits_{(j,i)\in BS(i)} x_{ji}^c = \begin{cases} d^c, \quad i = s^c, \forall c\in C \\ 0, \quad \forall i\in N \setminus \{s^c, t^c\}, \forall c\in C \\ -d^c, i = t^c, \forall c\in C \end{cases} \\
\sum\limits_{c\in C} x_{ij}^c + w_{ij} = y_{ij}, \quad \forall (i,j)\in A \\
\rho_{ij}^c - \gamma_i^c + \gamma_j^c - \eta_{ij} - z_{ij}^c = 0, \quad \forall (i,j)\in A, c\in C \qquad (5) \\
\eta_{ij} w_{ij} = 0, \forall (i,j)\in A \qquad (6) \\
x_{ij}^c z_{ij}^c = 0, \forall (i,j)\in A, c\in C \qquad (7) \\
z_{ij}^c, x_{ij}^c \ge 0, \quad \forall (i,j)\in A, \forall c\in C \\
w_{ij}, y_{ij}, \eta_{ij} \ge 0, \quad \forall (i,j)\in A \\
\gamma_i^c \text{ free}, \quad \forall i\in N, \forall c\in C \qquad (8)
\end{cases}
$$

It is easy to see that complementary slackness conditions (6) and (7) are quadratic constraints. Moreover (see constraints (8)), dual variables associated with primal equality constraints are free in sign.

Constraints (6) and (7) can be linearized by introducing four binary variables, namely $\delta_1, \delta_2, \delta_3$ and δ_4 and a large number *M*, as reported in the next binary linear program *P*4.

$$
P4\begin{cases}
\min\limits_{(x,y)} \lambda \\
\sum\limits_{c\in C} \rho_{ij}^c x_{ij}^c \le \lambda, \quad \forall (i,j)\in A \\
\sum\limits_{(i,j)\in FS(i)} x_{ij}^c - \sum\limits_{(j,i)\in BS(i)} x_{ji}^c = \begin{cases} d^c, \quad i = s^c, \forall c\in C \\ 0, \quad \forall i\in N\setminus\{s^c,t^c\}, \forall c\in C \\ -d^c, i = t^c, \forall c\in C \end{cases} \\
\sum\limits_{c\in C} x_{ij}^c + w_{ij} = y_{ij}, \quad \forall (i,j)\in A \\
\rho_{ij}^c - \gamma_i^c + \gamma_j^c - \eta_{ij} - z_{ij}^c = 0, \quad \forall (i,j)\in A, c\in C \\
\eta_{ij} \le M\delta_1, \quad \forall (i,j)\in A & (9) \\
w_{ij} \le M\delta_2, \quad \forall (i,j)\in A & (10) \\
x_{ij}^c \le M\delta_3, \quad \forall (i,j)\in A, c\in C & (11) \\
z_{ij}^c \le M\delta_4, \quad \forall (i,j)\in A, c\in C & (12) \\
\delta_1 + \delta_2 = 1, & (13) \\
\delta_3 + \delta_4 = 1, & (14) \\
z_{ij}^c, x_{ij}^c \ge 0, \quad \forall (i,j)\in A, \forall c\in C \\
w_{ij}, y_{ij}, \eta_{ij} \ge 0, \quad \forall (i,j)\in A \\
\gamma_{ij}^c \text{ free}, \quad \forall (i,j)\in A, \forall c\in C \\
\delta_1, \delta_2, \delta_3, \delta_4 \in \{0,1\},
\end{cases}
$$

Note that, due to constraints (13) and (14), pairs of constraints (9)-(10) and (11)-(12) are such that complementary slackness relations in $P3$ are obeyed.

The PL01 program $P4$ can be solved in practice by considering separately the four binary patterns for the δ variables, i.e., (δ_1, δ_2, δ_3, δ_4) $\in$ {(1, 0, 1, 0); (1, 0, 0, 1); (0, 1, 1, 0); (0,1, 0, 1)}, and solving the related linear program at the optimum and then choosing the solution with the smallest value of λ. This represents another major difference with the model of Kara and Verter (2004) where also variables related to the two decision makers are integers and thus this kind of reduction cannot be afforded.

3. Experimental results

We study a real-world case study to prove the effectiveness of the proposed model, using the commercial solver CPLEX 8.0.1 (www.ilog.com). We considered the road network of the Lazio region (located in the middle of Italy), and, in particular, its main transport roads for an overall size of 331 macro-nodes and 879 arcs (see Figure 2). Risks values have been provided by a local agency and range from 50 to 250 per ton. of hazmat transported. We considered up to 10 origin-destination pairs each associated with 200 tons to be routed.

Results are reported in Figures 3 and 4. Figure 3 shows the total risk on the network for an increasing number of origin-destination pairs (values range from 295788 to 6290000). Figure 4 depicts the trends of the maximum risk λ for the same shipment range (values ranges from 17160 to 34320). Figure 5 reports the number of simplex iterations executed by CPLEX to solve the model $P4$ in this setting.

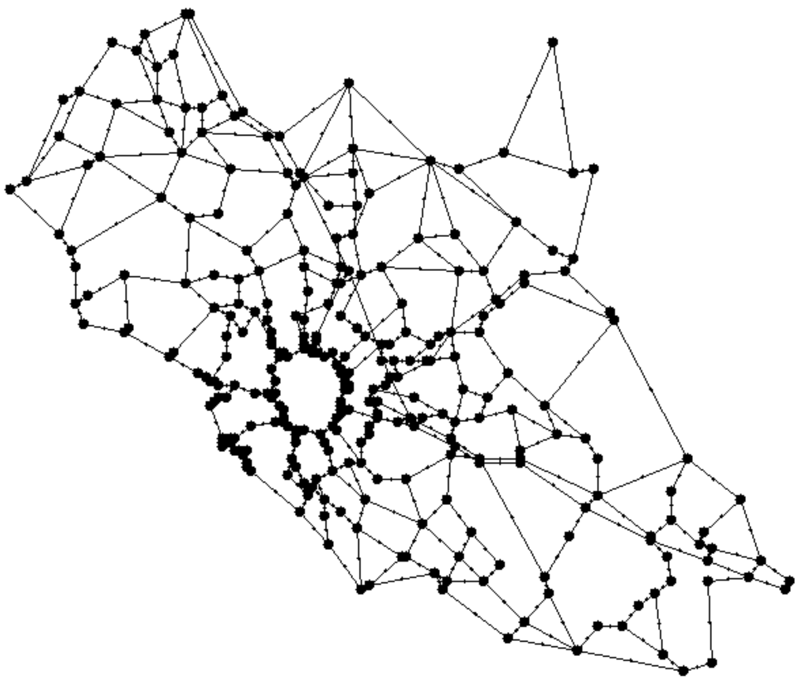

Figure 2: The transportation network of Lazio.

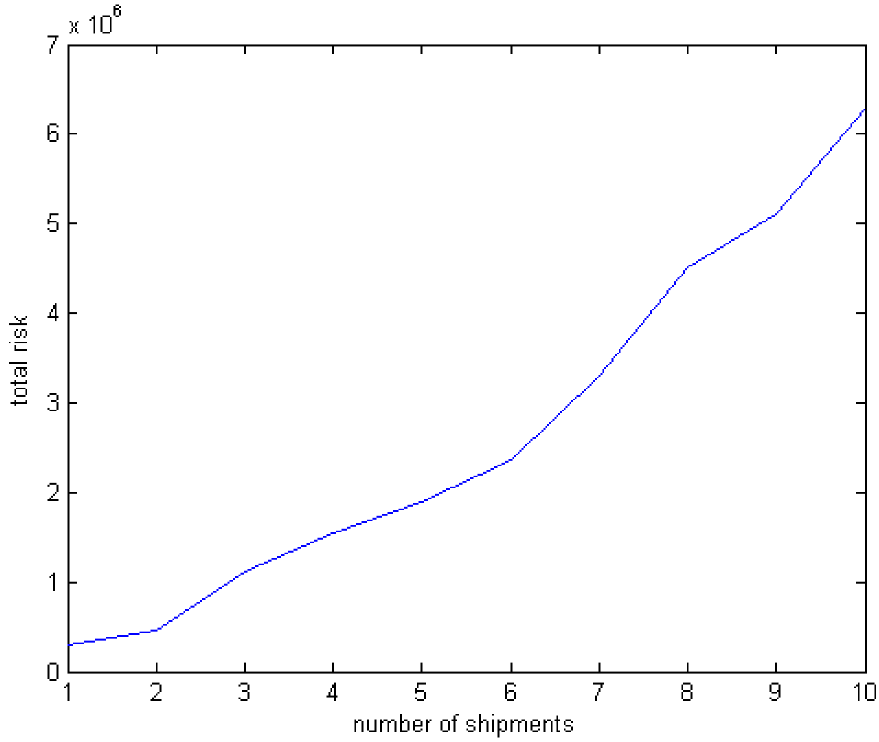

Figure 3: total risk over the number of shipments.

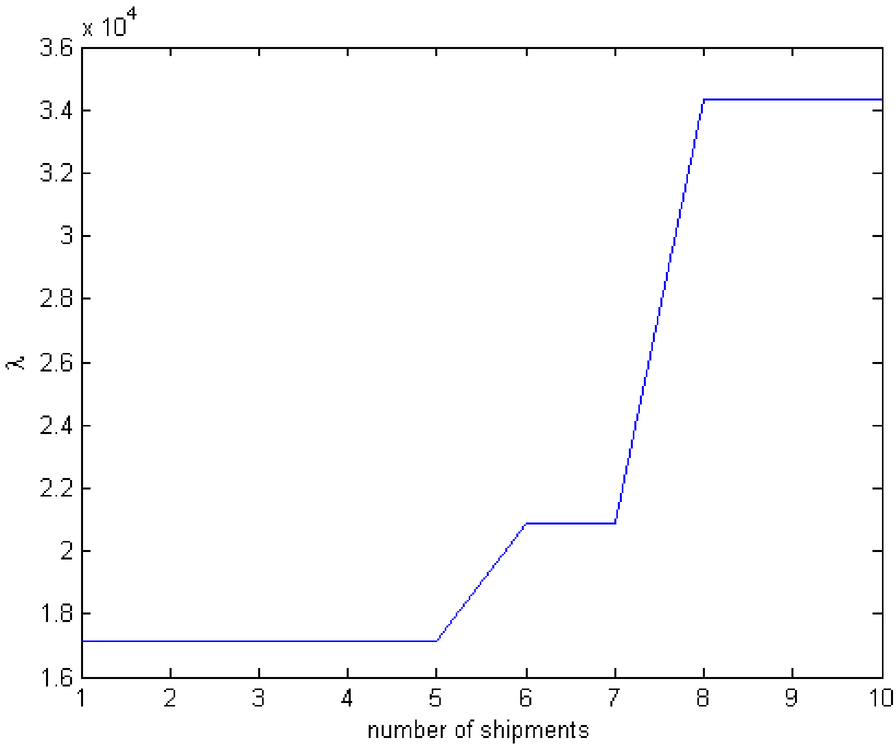

Figure 4: maximum risk over the number of shipments.

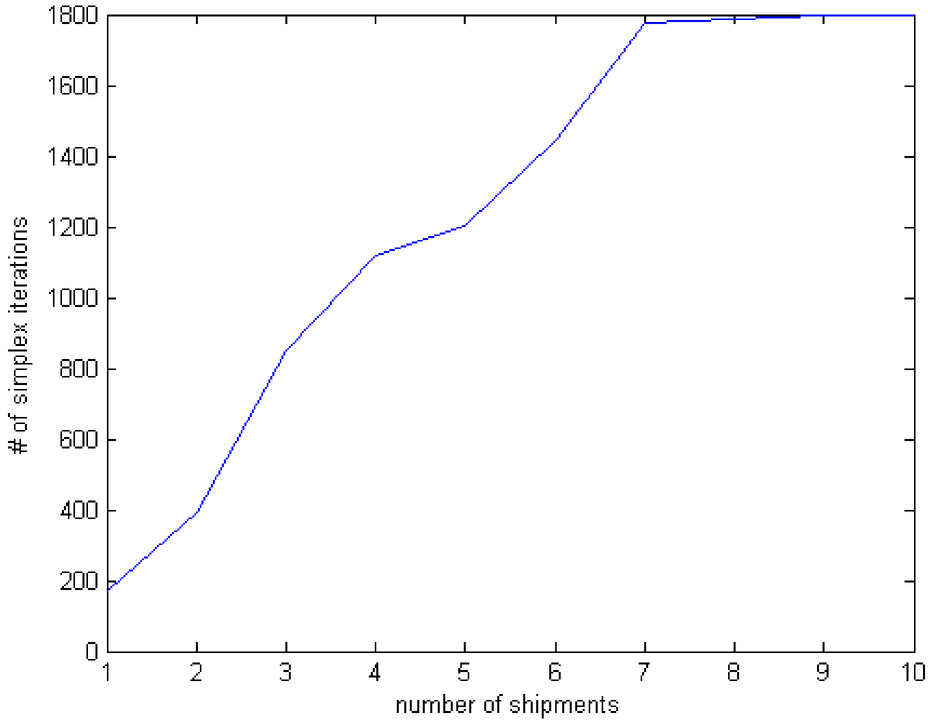

Figure 5: the number of simplex iterations performed by the solver.

Moreover, we experimented with a variable amount of hazmat to be routed for each shipment, keeping fixed the number of origin-destination pairs to 10. Figures 6 and 7 show the trends of the total risk and the maximum risk, respectively, over increasing hazmat tons per shipment ranging to 20 to 640 tons. Note that in the latter two figures the behaviour of the two objective functions is almost linear since the origin-destination pairs are fixed and the quantity of hazmat to be shipped increases of the same amount for each of them.

4. Conclusions

In this work, we have proposed a bilevel network flow model for hazmat global route planning. The proposed model aims to minimize total risk and risk equity. The performance of the model has been evaluated on a real test case over an Italian region. Several refinements may be introduced to the basic model presented in this paper. For example, one may change the objective of the lower level formulation considering the total cost of the shipments; another possible modification is to consider also the population density of the people living in the neighbour of each arc, and in the upper level formulation minimize the maximum exposure of the population of each arc.

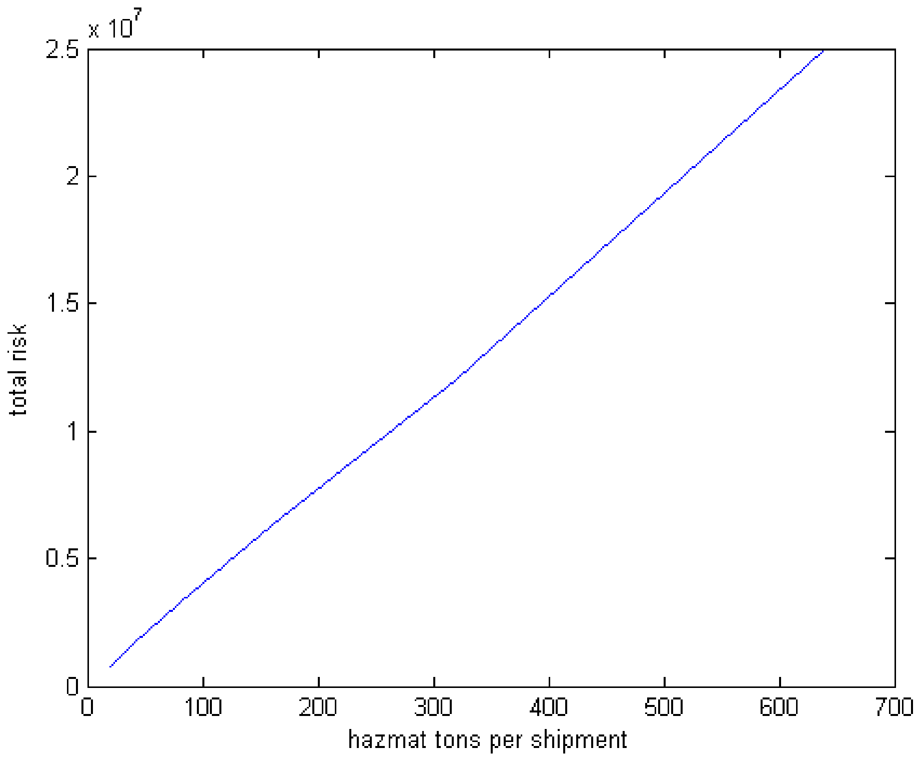

Figure 6: total risk on the network over increasing hazmat tons per shipment.

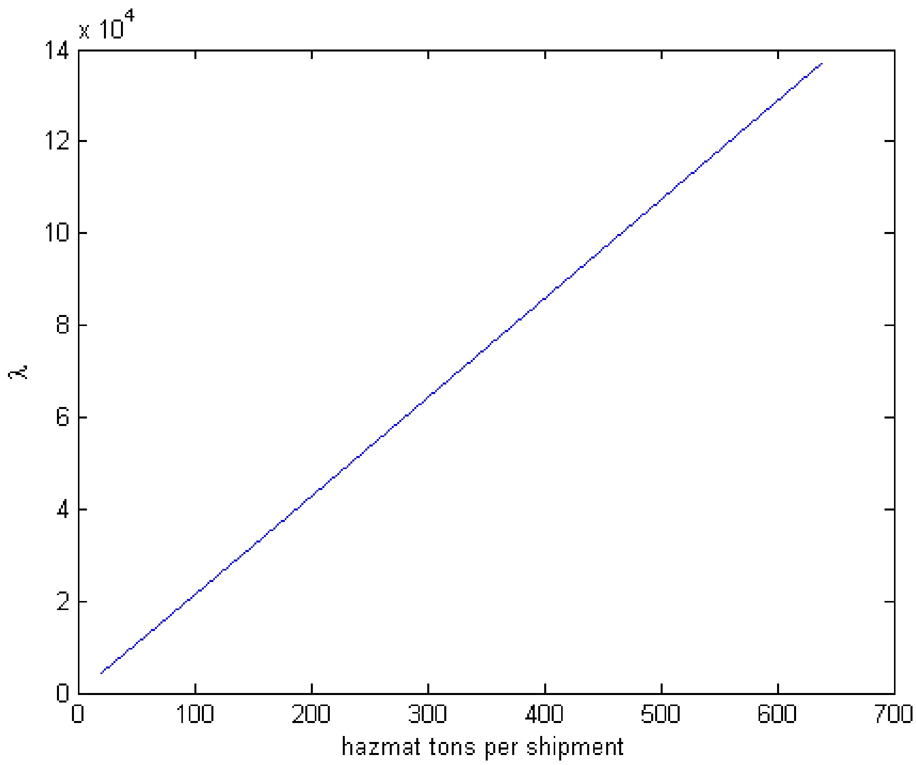

Figure 7: maximum risk on the network over increasing hazmat tons per shipment.

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