Traffic assignment by paired alternative segments
Extended abstract

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The user equilibrium (UE) traffic assignment is a corner stone in travel forecasting and traffic impact analysis. Many algorithms have been proposed over the years for solving the UE model. For many years the most popular algorithm in practical applications and in software packages was the Frank-Wolfe (FW) algorithm [LeBlanc et al., 1975], which relies on storing total link flows. A previous paper [Bar-Gera, 2002] presents an origin-based assignment (OBA) algorithm, and demonstrates its ability to achieve any desired level of convergence in reasonable computing time. Other algorithms that achieve high levels of convergence are route-based [e.g. Larsson and Patriksson, 1992].

Traditional applications of UE models require only estimates of total link flows, which are uniquely determined by the UE assumption. More and more, analyses of UE model results are based on route flows, which are not uniquely determined by the UE assumption. Bar-Gera and Luzon [2006] show that choosing arbitrarily a single solution from all UE route flow solutions may introduce significant and undesirable errors. Rossi et al. [1989] suggest that the Maximum Entropy User Equilibrium (MEUE) route flow solution is the most likely one. The MEUE solution satisfies desirable consistency properties as discussed hereon. In general, well converging algorithms (route-based or origin-based) typically suffer from fairly poor consistency, unless special attention is devoted to the issue. The challenge of this research is to develop a well converging algorithm that maintains reasonable consistency.

A fundamental insight regarding UE models is that the set of UE routes has a special structure that can be captured by a set of local Paired Alternative Segments (PAS). The main concept can be illustrated by two simple situations. The first situation is when travelers need to make a sequence of independent choices, where every choice is between two alternative segments. For example, considering the network in Figure 1, travelers from origin 1 to destination 8 must follow segment s_i; then they choose between segments s_1 and s_1'; they continue along segment

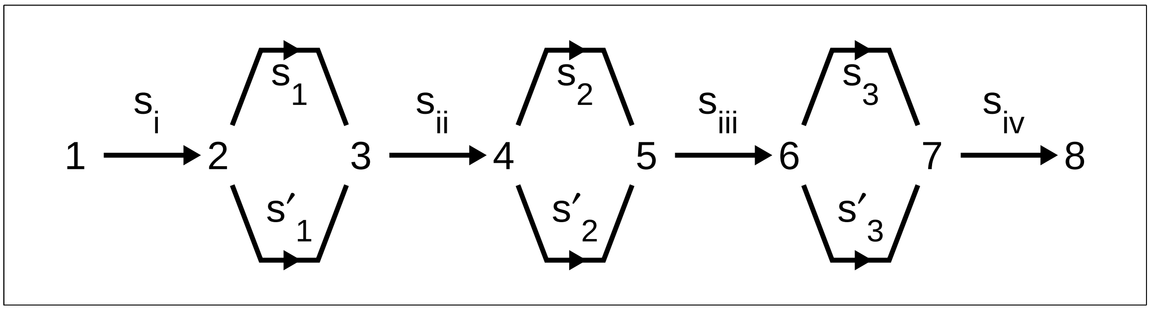


Figure 1: An simple example of routes and paired alternative segments (k=3)

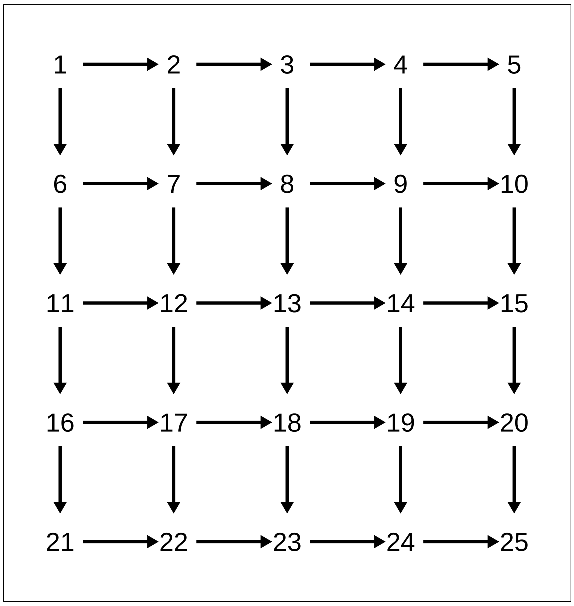


Figure 2: An example of an a-cyclic grid network (k=4)

s_{ii}; choose between segments s_2 and s'_2; continue along segment s_{iii}; choose between segments s_3 and s'_3; and finally follow segment s_{iv} to the destination. Travelers that need to make k independent decisions of this type, have 2^k routes to choose from. In addition, the local PAS $L_2 = (s_2, s'_2)$ is also a component of the choice set of travelers from origin 3 to destination 6, as well as travelers from origin 1 to destination 6 and travelers from origin 3 to destination 8.

The second simple situation is when the network consists of an a-cyclic grid of $k+1$ by $k+1$ nodes, like the case shown in Figure 2. All route choices in such a network, for all OD pairs, can be described by combinations of the k^2 "around the block" local PASs, like $([1,2,7],[1,6,7])$ or $([12,13,18],[12,17,18])$. On the other hand, the number of routes from corner to corner (1 to 25 in this example) is $\begin{pmatrix} 2k \\ k \end{pmatrix} \approx 2^{2k}$. (Recall that for $k=10$, $k^2 = 100$ but $2^{2k} \approx 1,000,000$.)

In general networks the structure may be much more complex, but there is always a set of *basic* PASs such that the difference between any two UE routes can be described as a combination of choices related to basic PASs, and the number of basic PASs is typically several orders of magnitudes smaller than the number of routes [Bar-Gera, 2006]. Realizing the importance of PASs in the UE model leads directly to the development of a Traffic Assignment by Paired Alternative Segments (TAPAS) algorithm.

The general principle of the algorithm is very simple. Suppose that we have a current solution, represented by link-flows disaggregated by origins, and suppose that we have identified a local PAS, $L = (s_1(L), s_2(L))$. Suppose without loss of generality that the cost of segment s_1, c_{s_1}, is greater than the cost of segment s_2, c_{s_2}. For every relevant origin $p \in P(L)$, let $g_1(L, p)$ be the minimum origin-based link flow for origin p among the links of segment $s_1(L)$. Note that $G_1(L) = \sum_{p \in P(L)} g_1(L, p)$ is the total amount of flow that can be shifted from s_1 to s_2. If after shifting a flow of $G_1(L)$ from s_1 to s_2 still $c_{s_1} > c_{s_2}$, then the total amount of flow should be shifted; otherwise, a simple line search can be used to determine the necessary shift in order to equalize c_{s_1} and c_{s_2} (and to minimize the convex optimization objective function). The overall algorithm is based on storing a set of PASs which are considered in an iterative order for flow shifts, and in addition on a procedure to update the set of PASs periodically.

In order to update the set of PASs, we find the tree of shortest routes from origin p, and identify all links used by flows from origin p which are not part of the minimum cost tree for origin p. For every such "problematic" link, a, it is desirable to have a PAS that will allow reducing the amount of flow from origin p that uses link a. If there is no such PAS in the current set of PASs, we may wish to detect a new PAS for link a. The merge node of such PAS should be the link head, $n_m = a_h$; and one of its segments should consist of links used by origin p, including in particular the "problematic" link, a. It is reasonable to prefer a PAS whose other segment is part of the minimum cost tree for origin p. It is also reasonable to prefer a "local" PAS with short segments, as it is more likely to be shared by other origins. Therefore, the search for a new PAS focuses mainly on the identification of the diverge node n_d. Initially n_d is set to the min-cost predecessor of the merge node n_m. If there is a segment of used links from n_d to the tail of the problematic link a_t, we are done. Otherwise, we substitute n_d with its min-cost predecessor, and repeat the same process again. Once a new PAS is found, all origins that use the high cost segment are added to the list of relevant origins for this PAS.

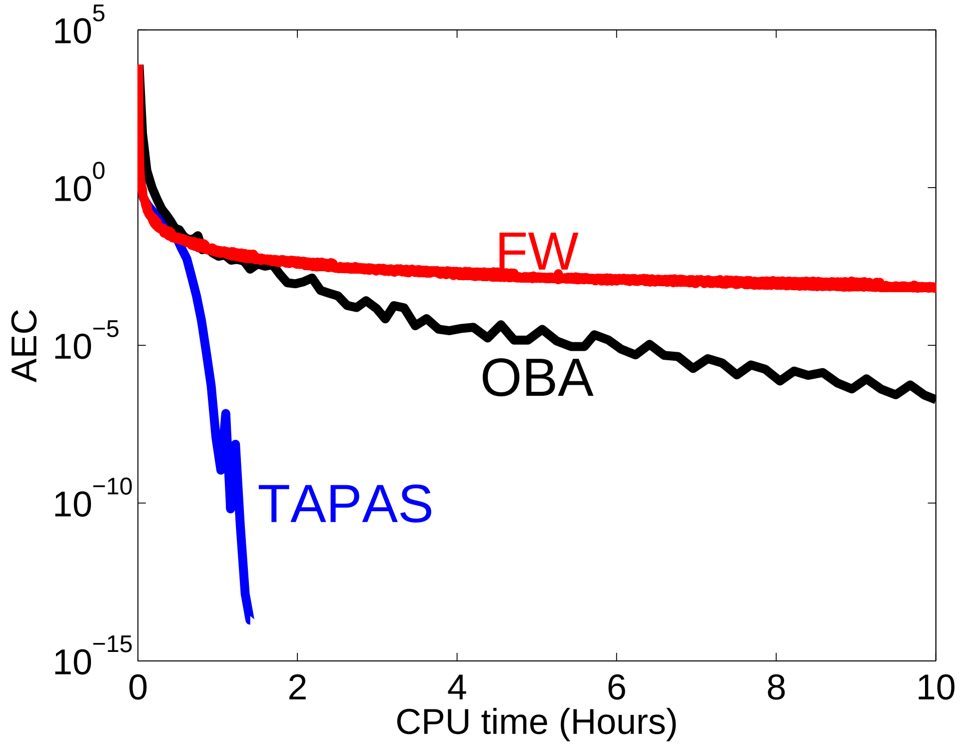
The main challenge in developing a TAPAS algorithm is to determine how to divide the computational effort between the different tasks, and particularly when to search for new PASs and for relevant origins for the PASs. On one hand, timely identification of new PASs and relevant origins are clearly crucial to allow effective progress of the algorithm. On the other hand, in many cases the search does not lead to any changes in the set of PASs or in the lists of relevant origins, and thus may be a waste of time. As part of this research several experiments regarding this issue are being conducted and will be reported.

Figure 3 shows a comparison of convergence efficiency between the current TAPAS implementation, OBA [Bar-Gera, 2002] and FW [LeBlanc et al. 1975]. The level of convergence here is measured by the Average Excess Cost (AEC). The results are for a detailed model of the Chicago Region with 1,790 zones, 12,982 nodes and 39,018 links. All codes are in C. The computations were performed on Windows PC machines with 2GB RAM running at 600-700 MFLOPS. The results clearly show the advantage of TAPAS in comparison to the other algorithms.

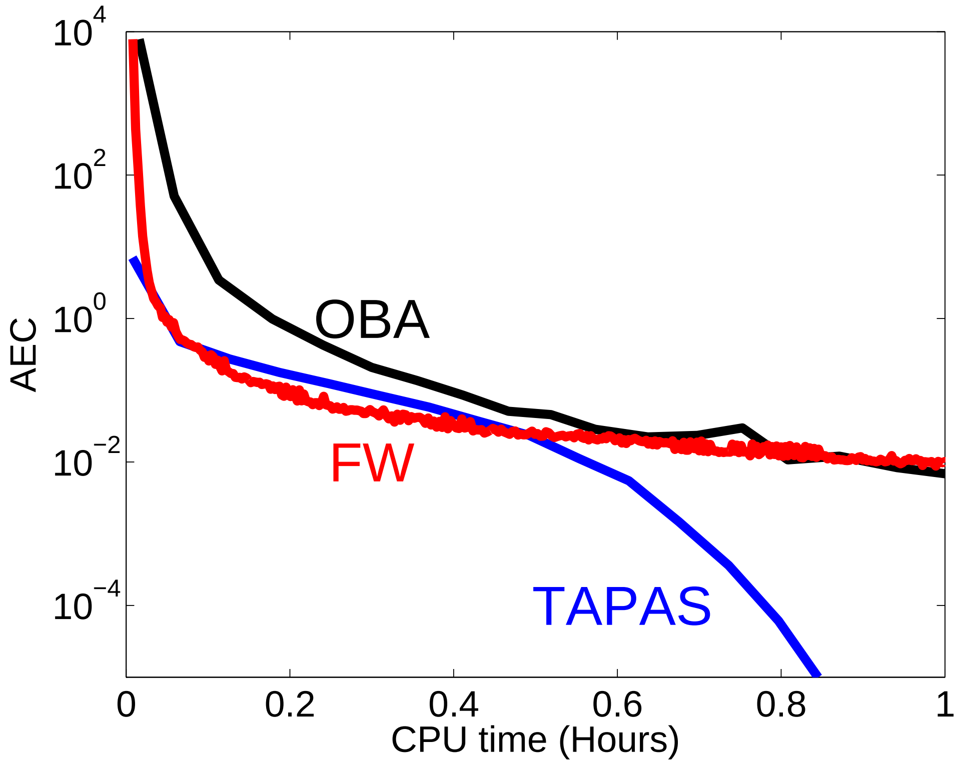
In addition to excellent convergence results, the identification of traveler choices by local PASs is also essential for consistency. Again the main idea of *consistency* can be illustrated by the simple network in Figure 1, which has three local PASs namely $L_1 = (s_1, s_1')$; $L_2 = (s_2, s_2')$; $L_3 = (s_3, s_3')$. If there are travelers that use the route that goes through segments s_1, s_2' and s_3, and other travelers that use the route that goes through s_1', s_2 and s_3', then it seems unreasonable (or inconsistent) to assume that there would be no travelers using the route that goes through segments s_1, s_2 and s_3.

More generally, PAS $L = (s_1(L), s_2(L))$ is considered *active* in a set of routes $\mathbf{R}' \subseteq \mathbf{R}$ if each of the segments of L is part of a route in $\mathbf{R}'$, that is $s_1(L) \subseteq r_1$; $s_2(L) \subseteq r_2$; and $r_1, r_2 \in \mathbf{R}'$. PAS L is said to be *partly used* by $\mathbf{R}'$ if one of the segments, say $s_1(L)$, is part of a route in $\mathbf{R}'$, say r_1, but the alternative route, r_2, obtained from r_1 by replacing $s_1(L)$ with $s_2(L)$, is not included in $\mathbf{R}'$. If there is an active PAS in $\mathbf{R}'$ that is partly used, then the set of routes $\mathbf{R}'$ is not consistent. In other words, a set of routes $\mathbf{R}'$ is *consistent* if there are no partly used active PASs. (A general condition for *perfect consistency* is discussed in [Bar-Gera, 2006].)

Assuming that choices at different PASs are independent, route flows are considered to



(a) Ten hours



(b) One hour

Figure 3: Convergence efficiency comparison for the Chicago Regional model

be consistent if they are proportional. That is if for every PAS $L = (s_1(L), s_2(L))$ the same global proportions $\rho_1(L), \rho_2(L) = 1 - \rho_1(L)$ are maintained by the route flows h_{r_1}, h_{r_2} of any pair of routes r_1, r_2 that differ by L, meaning that $h_{r_1} = \rho_1(L) \cdot (h_{r_1} + h_{r_2})$ and $h_{r_2} = \rho_2(L) \cdot (h_{r_1} + h_{r_2})$. This proportionality condition is in fact an optimality condition of the MEUE problem. Therefore, a consistent set of routes is essential for consistent/proportional/MEUE route flows. By considering all relevant origins for every PAS, as described above, the proposed algorithm is likely to yield reasonably consistent solutions.

To summarize, in this research we explore a new type of traffic assignment algorithm. The algorithm is based on identification of local paired alternative segments. Flow adjustments in the algorithm are applied to these paired segments, in a way that corresponds to consistency considerations and entropy maximization. Above all the algorithm exhibits high efficiency in achieving any level of convergence, and particularly when highly converged solutions are needed.

Acknowledgments

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