A POVERTY IMPACT ROAD PLANNING MODEL

E.R. Petersen & A.J. Taylor

School of Business,
Queen's University
Kingston, Ontario

May, 2007

The support of the Asian Development Bank and the Ministry of Communications, People's Republic of China, is gratefully acknowledged.

Introduction

The Area Wide Road Network Model (ARNM) is an integrated road network model that calculates the optimal set of road projects subject to specified budget constraints and rural road improvement objectives. This model provides road planners with a tool that can be used to prepare, in an integrated manner, a strategy for building expressways and rural roads that balances the objectives of an efficient transportation system with the needs for better links to poor communities. A network is used to model the roadway system for the entire region. The nodes of the network represent major centers or junctions, and the links represent the connecting roadways. Proposed roadway projects can either be upgrades to existing links or entirely new links. The set of proposed roadway projects comprise local road improvements, links connecting communities to expressways, links between major centers, and through expressways. Each proposal will have impacts on both local poverty alleviation and overall economic return that may be complementary or conflicting. Traffic is specified as both local link specific movements and through, or area-wide, origin-to-destination flows. Up to twelve different vehicle types are permitted, with the cost elasticity of demand specified for each.

For a given set of proposals, the ARNM calculates the optimal equilibrium traffic flow on each link, where through flows are routed through the network to minimize the user cost for each flow. Budget constraints are imposed on the solution. For example, a single budget constraint may be imposed on all projects. Alternatively, the budget may be subdivided with, say, 40 percent directed to rural roads and the remaining budget constraining the selection of other projects. Within these budget constraints, the projects that maximize the overall economic return are selected. Up to three different budget constraints are permitted.

The ARNM provides post-optimality analysis for any solution. Global economic and poverty impact measures are calculated for the total project. For each individual road project, the net economic benefits and economic internal rate of return (EIRR), and the benefits accruing to the poor and very poor are calculated based on the consumer surplus and demographics associated with each type of vehicle traffic. The Poverty Impact Ratio (PIR), or fraction of overall benefits flowing to the poor, is also computed.

2. **Roadway Capacity and Congestion**

The capacity of a roadway is defined by its speed-flow curves. The form of the speed-flow curves are those specified in the Rust PPK[2] report which assumes the speed of each vehicle type converge (exponentially) to a common speed at the converging congestion. As congestion increases, each vehicle type has this common speed until saturation. Denote the total traffic per day in passenger car units (pcu) as *v*, and the roadway capacity in pcu/day as *c*. The converging congestion level, or the value of *v/c* that corresponds to congestion, is denoted by *m*. For Expressway and Motorways Class I and II the equations have the form:

$$speed = \begin{cases} a\exp(b(v/c)^2), & v/c \le m = 0.80 \\ a_1 \exp(b_1 (v/c)^8), & v/c > m = 0.80. \end{cases}$$

Where a, a_1, b, and b_1 are parameters.

For Class II, III and IV roadways, the equations have the form:

$$speed = \begin{cases} a\exp(b(v/c)^2), & v/c \le m \\ a_1 \exp(b_1 (v/c)^2), & v/c > m. \end{cases}$$

(Note that often a linear function is used in the last equation when $v/c>m$, however, the above expression is very near linear and has the advantage that is always non-negative.)

The data for these curves for each vehicle type are defined by the following:

free-flow speed, *a*

v/c at the “knee” or convergence fraction *m*

speed at the “knee” or convergence fraction *m*,

speed at saturation, v/c=1.

These data are used to derive the parameters a, a_1, b, b_1 used in the above equations. The equations are specified for each class of roadway, for Flat terrain, Hilly terrain, and Mountainous terrain. The speed-flow curves for a typical Class IV road in Hilly terrain is shown in Figure 1.

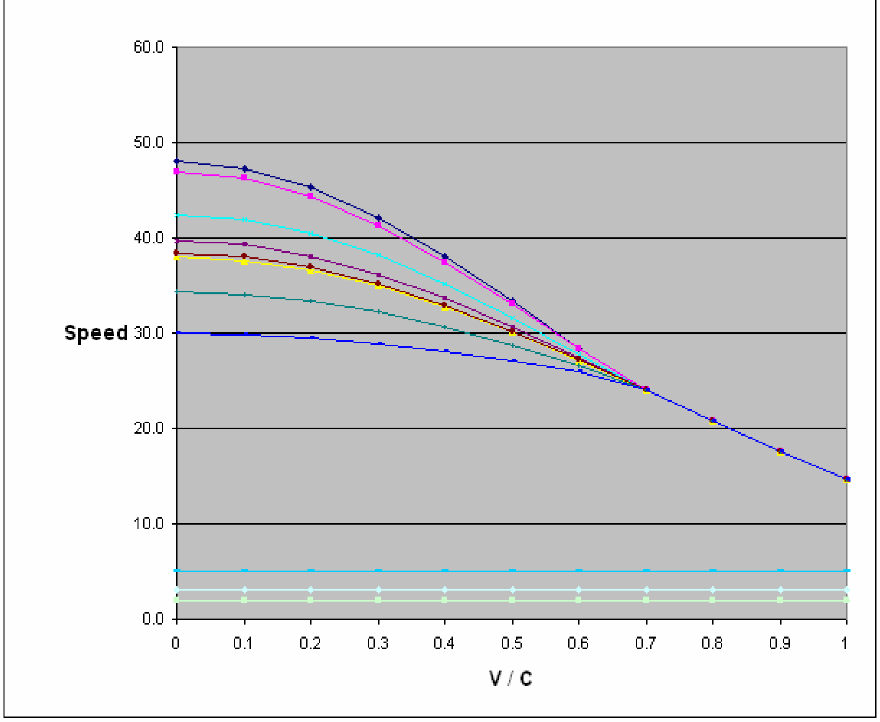

Figure 1: Speed-Flow Class IV Road, Hilly Terrain

Seasonal roads are modelled using a separate sub-model for each of the dry and wet seasons. A composite model combines the calculations from the two sub-models.

Through traffic will be assigned to routes based on user cost. When toll roads are present, not all traffic is assigned to the toll road since the existence of tolls will cause some vehicles to divert to the parallel non-toll road. Logit toll diversion curves are used to predict this behavior. These models have the form:

$$P = \frac{1}{1 + \exp(a(C_{toll} - C_{free}) + bT}$$

where

P = probability of utilizing the toll route
C_{toll} = economic cost using the toll road
C_{free} = economic cost via the non - toll route
T = total toll on the toll route
a, b are parameters.

Different diversion curves may be specified for each vehicle type.

3. Traffic

A forecast of traffic for the planning period and for future time periods is exogenously generated and input. Both local and through traffic by vehicle type is defined using a traffic demand function with respect to base-case user cost. Given the traffic levels, both local and through, the traffic over each link is then calculated. The cost of each vehicle trip is calculated, and based on these costs new traffic levels are calculated. This iterative calculation is repeated until equilibrium is obtained for each set of selected project options.

Up to twelve different vehicle types can be modeled. Table 1 is an example of possible vehicle types, together with their pcu equivalent value.

Table1:Vehicle Types

Vehicle Type No.	Vehicle Type	name	Equivalent pcu	description
1	Passenger Car	CAR	1	
2	Mini Bus	MB	1	seats ≤ 19 seats
3	Large Bus	BUS	1.5	bus with seats >19
4	Light Truck	LT	1	load capacity ≤ 2t
5	Medium Truck	MT	1.5	load capacity > 2t~≤7t
6	Heavy Truck	HT	2	load capacity > 7t~≤14t
7	Extra Heavy Truck & Truck Trlr	EHT&TT	3	load capacity > 14t,
8	Motor Cycle	MCycle	0.5	
9	Motorized Slow Moving	MSM	2	
10	Slow Moving - Man Power	SM-MP	1	
11	Slow Moving - Livestock Power	SM-LP	4	
12	Bicycle	bike	0.2	

4. Costs and Benefits

Project alternatives are evaluated based on the accrued economic costs and benefits. Economic costs measure the resources being used and therefore exclude taxes and other transfer payments. Economic costs are specified as follows:

1. Road-provider costs

Maintenance costs: both routine and periodic. Roadway maintenance costs are specified for each link on a per km and/or traffic usage.

Capital cost for each project: land, construction, drainage, bridges and pavements.

2. Road-users cost, specified for each current and projected link, as distance and time costs per unit for each vehicle type. This cost is aggregated base on the following component costs:

Vehicle operating cost (VOC): fuel, tires, crew wages and depreciation.

Payload time cost: passengers travel time and cost of goods in transit. Vehicle operating cost depends on the terrain and the road roughness.

Tolls.

Accident cost: personal (loss of life or injury), loss and damage to goods in transit, and property damage.

Given the traffic levels, both local and through, the traffic over each link is calculated and the cost of each vehicle trip is calculated. Based on these costs, new through traffic levels are calculated. This iterative calculation is repeated until equilibrium is obtained as shown in Figure 2.

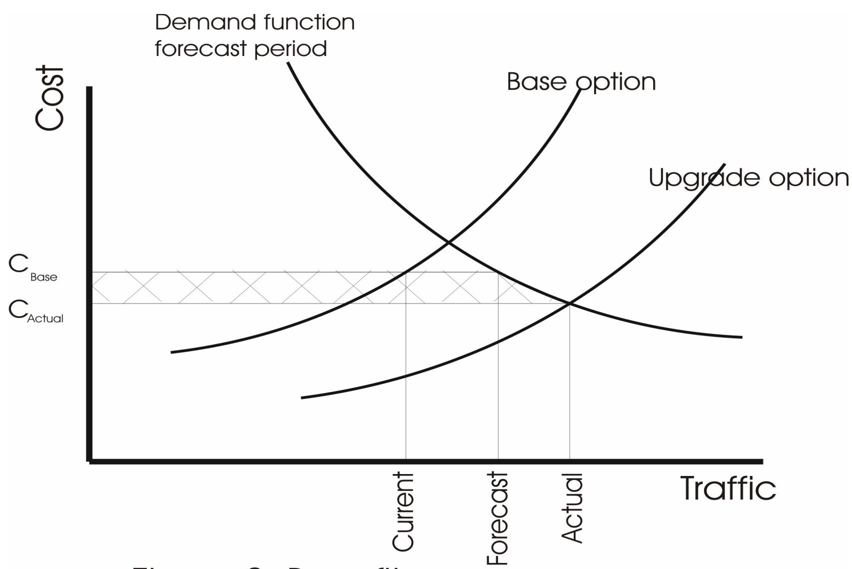

Figure 2: Benefits

If a roadway is improved, costs are lowered and the users realize the benefits. If traffic is fixed, with inelastic demand, then the benefit realized by each type of vehicle is the change in cost times the number of vehicles. With variable demand traffic, the benefit for each type of vehicle is the consumer's surplus, which is shown as the shaded area in Figure 2. Similarly, if the roadway is not improved, costs increase, resulting in negative benefits for each type of vehicle travelling, and reduced traffic over the link. The net total benefit is the sum of local traffic benefits over all existing links plus the sum of through traffic benefits over all OD flows.

7. **The Investment Planning Model**

The planning model selects the set of road projects that maximize total economic benefits subject to the budget constraints. If a link is upgraded, then costs on that link decrease, inducing more traffic to move. Similarly, if a link is not upgraded, costs increase with traffic, resulting in negative benefits and reduced traffic growth. The through or origin-to-destination traffic will select the minimum cost path from origin to destination. This path and trip cost will change as different projects are selected. A mathematical formulation of the area network model, and the investment planning model follows:

<u>Notation</u>

$\mathrm{N} =$ set of nodes, $\mathrm{i}, \mathrm{j} \in \mathrm{N}$.

$\mathrm{A} =$ set of arcs, $a \in \mathrm{A}$,

$i(a) =$ starting node of arc a

$j(a) =$ ending node of arc a.

$\mathrm{P} \subset \mathrm{A} =$ set of potential road projects, $p \in \mathrm{P}$

$z_p = 0,1; 1$ if project p selected, 0 otherwise,

$\mathrm{P}_1 \subset \mathrm{P}$ the projects subject to constraint 1,

$\mathrm{P}_2 \subset \mathrm{P}$ the projects subject to constraint 2,

$\mathrm{P}_3 \subset \mathrm{P}$ the projects subject to constraint 3.

$\mathrm{P}_1 \cap \mathrm{P}_2 = \phi,\ \mathrm{P}_2 \cap \mathrm{P}_3 = \phi,\ \mathrm{P}_1 \cap \mathrm{P}_3 = \phi$

$\mathrm{P}_1 \cup \mathrm{P}_2 \cup \mathrm{P}_3 = \mathrm{P}$,

$C_p =$ capital costs of project p,

$\alpha_p = \frac{i(1+i)^N}{(1+i)^N - 1}$, annual capital carrying cost,

where i is the cost of capital and N is the life of the project,

$M_p =$ increase in annual maintenance with the project.

$\mathrm{V} =$ set of vehicles, $v \in \mathrm{V}$.

T = set of time periods, $t \in \mathrm{T}$

$t = 0$, status quo,

$= 1$, planning period

$= 2$, future period.

Through traffic:

D = set of O - D traffic flows, $m \in \mathrm{D}$

$o(m) =$ origin node,

$d(m) =$ destination node,

$v_m =$ vehicle type for flow m,

$\hat{q}_m^t$ = forecast traffic flow in vehicles per day, period t,

q_m^t = actual traffic flow in vehicles per day, period t,

with demand function $q = q_m(c_m^t)$,

$c_m^t =$ cost along the optimal path for vehicle type v_m,

y_{am}^t = vehicle flow on arc a, time t, $m \in \mathrm{D}$, of type v_m.

Local traffic:

$\hat{l}_{av}^t$ = forecast local flow in vehicles per day on arc a, vehicle type v, with no improvement

l_{av}^t = actual local flow in vehicles per day on arc a, vehicle type v, time t,

with demand function $l = l_{av}(c_{av}^t)$,

The total flow on arc $a \in A$, vehicle v in pcu is

$x_a^t = \sum_{v \in V} l_{av}^t u_v + \sum_{m \in D, v_m = v} y_{am}^t u_v$ where u_v is the conversion factor for vehicle type v into pcu.

Costs and Benefits:

Define $c_{av}^t(x_a^t, z)$ = the long-run variable cost for vehicle type v on arc a in time period t,

which depends on total traffic flow x_a^t and the set of selected projects z, and

c_{av}^0 = current (time 0) long-run variable cost for vehicle type v on arc a.

Then the benefit to local traffic for vehicle type v on arc a is

$$b_v^a = \left[\mathrm{c}_{\mathrm{av}}^0 - c_{av}^1\left(x_a^1, z\right)\right]\left(\hat{l}_{av}^1 + l_{av}^1\right)/2.$$

The benefit to through traffic for vehicle type v_m on OD m is

$$b_v^m = [c_m^0 - c_m^1(z)](\hat{q}_m^1 + q_m^1)/2.$$

With this notation, the investment planning problem can be formulated as the following non-linear integer programming problem:

$$\max Z(z) = \sum_{a \varepsilon A} \left(\sum_{v \in V} (c^0_{av} - c^1_{av}(x^t_a, z))(\hat{l}^1_{av} + l^1_{av})/2 \ _p \right) + \sum_{m \in M} [c^0_m - c^1_m(z)](\hat{q}^1_m + q^1_m)/2 \ - \sum_{p \varepsilon \mathrm{P}} (\alpha_p C_p + M_p) z_p$$

subject to the through traffic flow constraints:

$$\sum_{i(a)=i} y^t_{am} - \sum_{j(a)=i} y^t_{am} = \begin{cases} q^t_m, i = o(m) \\ -q^t_m, i = d(m) \\ 0, \text{otherwise} \end{cases} \qquad \text{for all } i \in \mathrm{N}, m \in \mathrm{D}$$

the budget constraints

$$\sum_{p \in P_1} z_p C_p \le budget1$$

$$\sum_{p \in P_2} z_p C_p \le budget2$$

$$\sum_{p \in P_3} z_p C_p \le budget3,$$

the total flow in pcu on each arc

$$x^t_a = \sum_{v \in V} l^t_{av} u_v + \sum_{m \in D, v_m = v} y^t_{am} u_v \ , \ a \in \mathrm{A}, t \in \mathrm{T},$$

and the non - negativity and integer constraints

$$l^t_{av} \ge 0 \ , \ y^t_{av} \ge 0, \ a \in \mathrm{A}, v \in \mathrm{V}, t \in \mathrm{T}$$

$$z_p = 0,1, \ p \in \mathrm{P}.$$

Several solution methods are possible. The following approach based on implementation in a spreadsheet will be used:

For each set of feasible projects **z**:

1. A shortest path iterative DP algorithm is used to route the through traffic;
2. Costs are calculated for each local and through traffic flow;
3. A new traffic flows are calculated;
4. Steps 2 & 3 repeated until a supply-demand equilibrium is found;
5. Steps 1 to 4 repeated until a general equilibrium is found;
6. The objective function Z(**z**) is calculated;
7. The EXCEL solver is used to solve the following program:

$$\max Z(z)$$
$$\textit{subject to}$$
$$\sum_{p\in P_1} z_p C_p \le budget1$$
$$\sum_{p\in P_2} z_p C_p \le budget2$$
$$\sum_{p\in P_3} z_p C_p \le budget3,$$
$$z_p = 0,1,\ p\in \text{P}.$$

A dynamic programming algorithm is used to calculate the shortest (i.e. minimum cost) path for each OD pair. Let f_i be the minimum cost from node i to the origin node *o(m).* Also define $A(i)$ to be the set of non-directed arcs that extend from node i. Then the shortest cost path satisfies the following dynamic programming functional equations.

$$f_i = \min_{a\in A(i)} \left\{ c_a + f_{j(a)} \right\}$$
$$f_{o(m)} = 0$$
$$c_m^t = f_{d(m)}.$$

This functional equation must be solved recursively to identify the minimum cost path from *o(m)* to *d(m).* The flow assignment is equivalent to minimizing operating costs subject to the linear conservation of flow constraints.

8. Poverty Impact

For each local road improvement project, ARNM calculates the consumer surplus for each vehicle type. Based on the demographics of the traffic for each type of vehicle, the benefit to the poor and the very poor is calculated. The poor's share of the projected benefits (the poverty impact ratio, or PIR) can then be compared to the poor's share of national or regional GDP to measure the impact of the proposal on poverty alleviation, Gajewski [1].

To make this calculation explicit, let $r = \{0, 1\}$, with $r = 0$ describing the very poor and $r = 1$ the poor. Let

$\beta_{av}^r, \beta_{mv}^r =$ the proportion of vehicle traffic v at poverty level r,

$\gamma_{av}^r =$ the proportion of local traffic vehicles owner by poverty level r,

$\delta_{av}, \delta_{mv} =$ the proportion of non - owner savings passed on,

for local traffic on link a, and through traffic m.

We assume that none of the vehicles that move the through or OD traffic are owned by the poor or very poor. Let the benefits be split into the time and vehicle operating components, with

$bT_v^a, bT_v^m =$ the time benefits, and

$bO_v^a, bO_v^m =$ the vehcle operating benefits,

Then the economic benefit to poverty level *r* is:

$$e_a^r = \sum_v \{ bT_v^a \, \beta_{av}^r + bO_v^a \left[\beta_{av}^r (\gamma_{av}^r + (1-\gamma_{av}^r)\,\delta_{av}) + (1-\beta_{av}^r)\,\gamma_{av}^r \,(1-\delta_{av}) \right] \} \text{ on link } a,$$

and

$$e_m^r = \sum_v \{ \; bT_v^m \, \beta_{mv}^r + bO_v^m \, \beta_{mv}^r \, \delta_{mv} \; \} \text{ for OD flow } m.$$

The total economic benefit to poverty group *r* from the selected projects is

$$e_r = \sum_{a \in A} e_a^r + \sum_{m \in M} e_m^r \,.$$

Total economic benefits to all users is

$$e = \sum_{v \in V} \left[\sum_{a \in A} b_v^a + \sum_{m \in M} b_v^m \right],$$

and the poverty impact ratios are;

$$PIR_0 = e_0 / e, \quad \text{for the very poor,}$$

and

$$PIR_1 = (e_0 + e_1)/e, \quad \text{for the poor and very poor.}$$

9. Multi-Criteria Decision Analysis

An alternate formulation for this decision problem is the following:

$$\max \; D(z) = \alpha(e_0 + e_1) + (1-\alpha)Z(z)$$
$$subject\ to$$
$$\sum_{p \in P} z_p C_p \leq total\ budget$$
$$z_p = 0,1,\; p \in \mathrm{P},\; 0 \leq \alpha \leq 1.$$

When α =0 the most economically efficient projects are selected. When α =1 the projects that contribute to the maximum poverty alleviation benefits are selected. As α varies from 0 to 1, the efficient frontier is determined, giving the decision maker a clearer understanding of the trade-off between economic efficiency and poverty alleviation.

10. Example of a Network

Figure 3 shows a map of a portion of Guangxi province in the People's Republic of China. In the figure, proposed road upgrades are shown in yellow, and numbered 2 through 9 in circles. A proposed bypass route from Baise to Longlin is also shown also in yellow, indicated by the circled number 1 at three points on the bypass route. National roads are shown in red, provincial roads in blue, and secondary roads in pink.

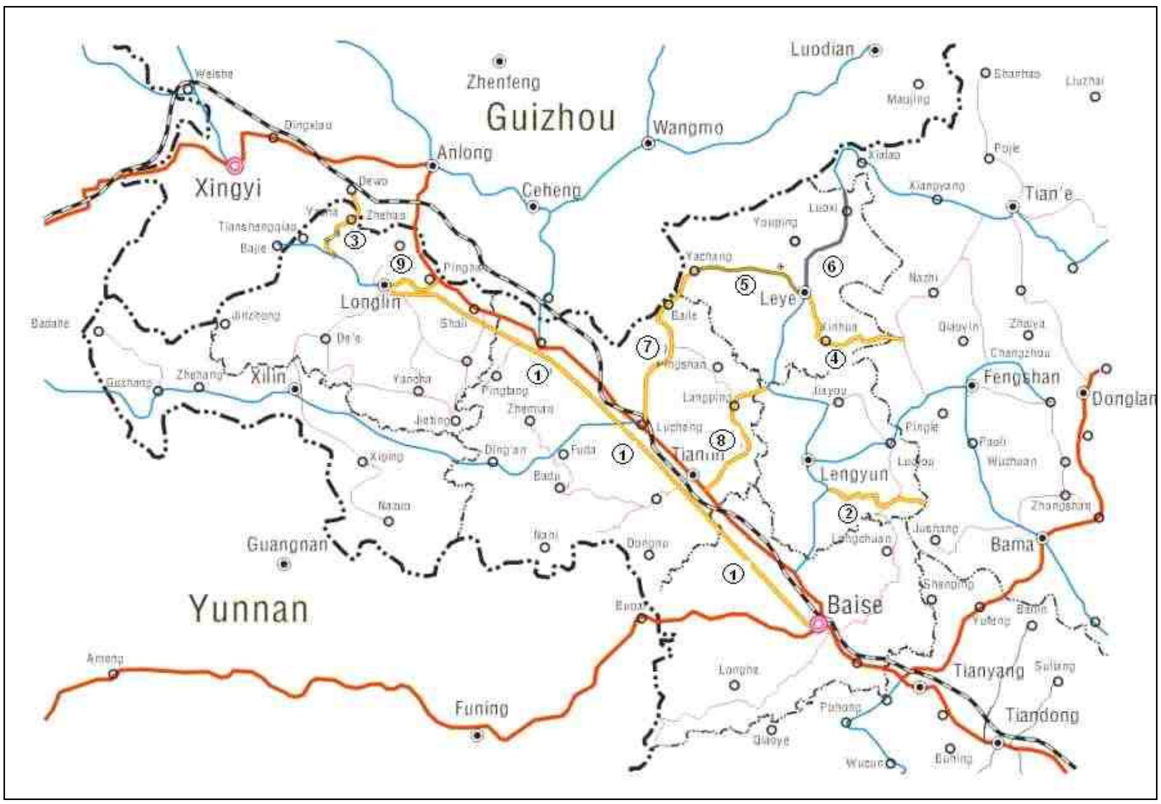

Figure 3. Road Map of Study Area

Careful examination of the natural routes and principal roadways within this region give rise to the ARNM network model shown in Figure 4. Each node has been designated with a number, starting at 1, and each existing link has been assigned a number starting at 101.

Roads with proposed upgrades are indicated with a rectangular box around the road number. Bypasses are assigned numbers beginning at the largest existing link number + 1, and are shown in the figure with dashed lines. Nodes are named, for reference, with the name of the nearest population center, though the node may correspond to an intersection of roads outside the center itself.

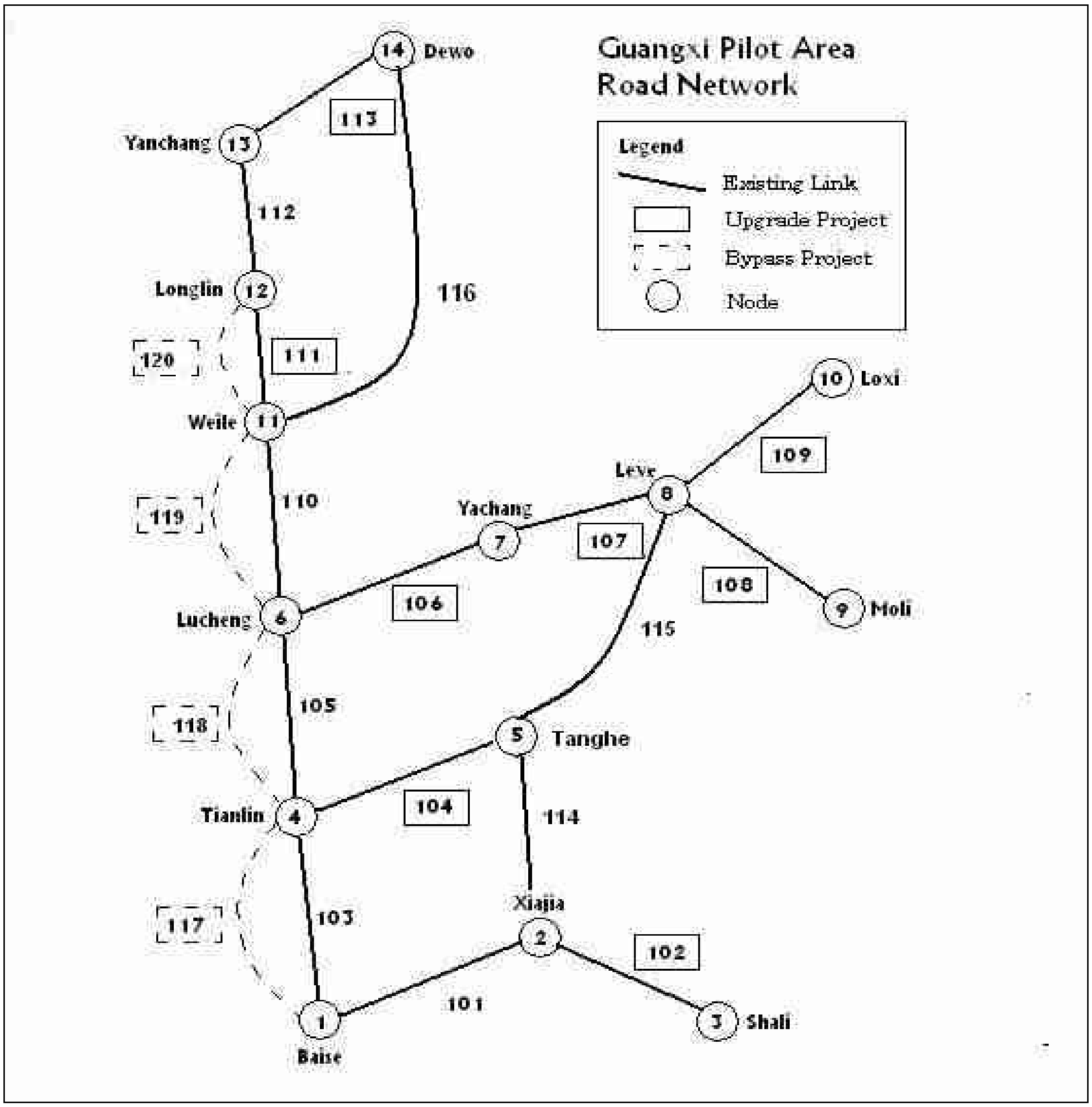

Figure 4. ARNM network model of study area

The road links 114 and 115 were included in this model since there is a substantial traffic from Baise to Loxi and beyond that passes over these roads, and this traffic could be affected if the bypass links 117 and 118, Baise to Lucheng, were completed and roads 106 and/or 107 from Lucheng to Leye were to be upgraded. However, roads beyond Loxi, Moli, and Shali were not included since road improvements in the study area will not significantly alter the traffic flow on these roads. The same is true for other roads leading to Baise from the south, from Dewo to the north, and leading west from the Baise-Longlin corridor.

For a given weighting parameter α, the model produces a description of the optimal road improvements and the associated economic information. Figure 5 shows a typical summary output for the Guangxi project. Detailed information on each investment is contained in auxiliary model worksheets. For this run of the model, maximum weight (α=1) is placed on the benefit to the poor and very poor.

	A	B	C	D	E	F	G	H	I	J	K
1	**ARNM Model Summary**										
2											Total User benefits
3	Time Period			t = 1	Planning						minus maintenance & opp. Cost of capital
4											239,850
5	Total User Benefits			573,943		Weight on Poor (α)			0.95		
6	Increase in Maint. Cost			22,175		Benefits to Poor			23,372		
7	Capital Carrying Cost			311,918		PIR			4.1%		
8											
9	EIRR			23.7%		Total Capital Invest.			2,329,857		
10						Budget			2,500,000		
11											
12	Existing Link Upgrades					New Alignment (bypass)					
13	Link #	Upgrade?	Between	And		Link #	Construct?	Between	And		
14	101	No	Baise	Xiajia		117	Yes	Baise	Tianlin		
15	102	No	Xiajia	Shali		118	No	Tianlin	Lucheng		
16	103	No	Baise	Tianlin		119	Yes	Lucheng	Weile		
17	104	Yes	Tianlin	Tanghe		120	No	Weile	Longlin		
18	105	No	Tianlin	Lucheng							
19	106	No	Lucheng	Yachang							
20	107	Yes	Yachang	Leye							
21	108	No	Leye	Moli							
22	109	Yes	Leye	Loxi							
23	110	No	Lucheng	Weile							
24	111	Yes	Weile	Longlin							
25	112	No	Longlin	Yanchang							
26	113	Yes	Yanchang	Dewo							
27	114	No	Xiajia	Tanghe							
28	115	No	Tanghe	Leye							
29	116	No	Weile	Dewo							

Figure 5 Typical Model Output

Figure 6 shows the trade-off as alpha changes from 0 to 1. Note that the efficient frontier is discrete and monotonically decreasing.

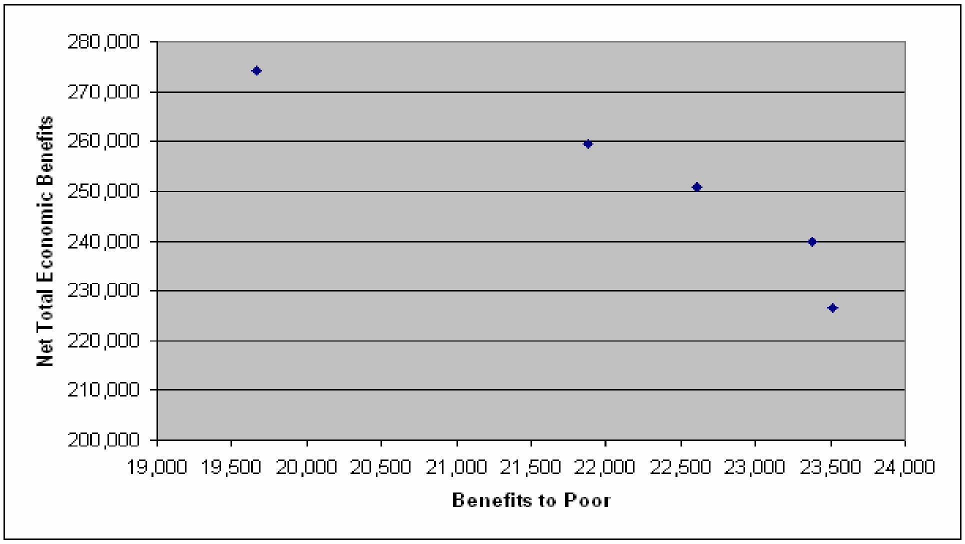

Figure 6. Contrasting Net Economic Benefits and Benefits to the Poor

Traffic engineers often use Economic Internal Rate of Return (EIRR) as a way to assess projects, and poverty specialists use a similar measure called the Poverty Impact Ratio (PIR) to identify the value of the project to the poor. The ARNM computes these values automatically. Although both measures have shortcomings, Figure 7 illustrates these measures at each point on the efficient frontier.

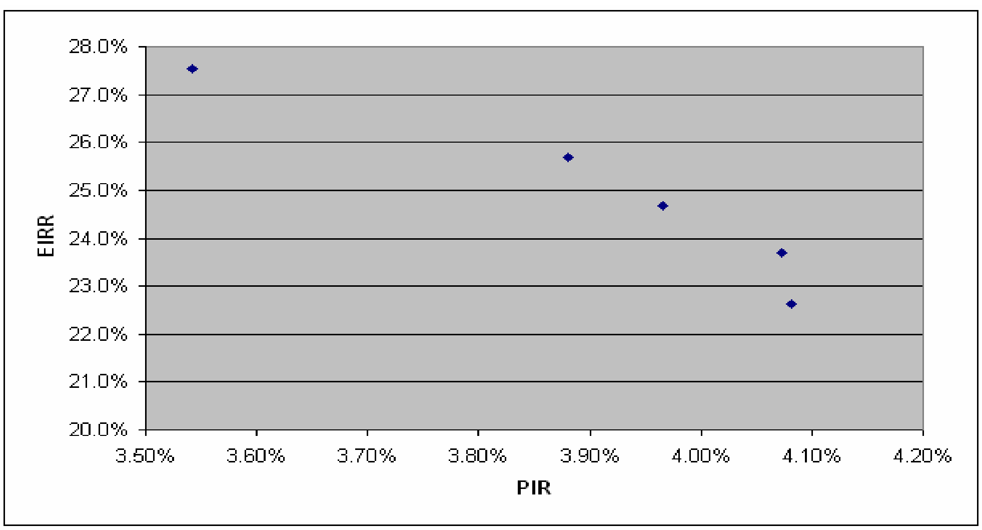

Figure 7. Contrasting Economic Returns to the Poverty Impact Ratio

11. Summary

This model provides a tool for governmental and transportation planners to evaluate alternative road improvements in a network. In particular, the model allows users to:

- Identify the optimal set of road investment projects in a network given constraints.
- Assess tradeoffs of alternatives with respect to total economic benefit and benefit to the poor.
- Facilitate measurement of the impacts on the economy and on the poor of alternative policy choices.
- Integrate engineering, cost, and budget issues

References:

1. G. Gajewski, and Mark Luppino, "Methods in Distribution and Poverty Impact Analysis of Projects", Louis Berger Corp., Western Economic Association International, 79th Annual Conference, June 2004.

2. World Bank, "Study of Prioritization of Highway Investments", Rust PPK Pty. Ltd, Feasibility Study Methodology, Washington DC, 1994.