A Quadratic-Programming Approach for the Signal Control Problem in Large-scale Congested Urban Road Networks

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***Abstract*— The problem of designing real-time traffic signal control strategies for large-scale congested urban road networks is considered. A generic, simple, network-wide problem formulation is presented in the format of a discrete-time optimal control problem whose numerical solution is achieved by use of quadratic-programming algorithms. Some procedures enabling the application of the proposed approach in real-time are outlined. Finally, a simulation-based investigation of the signal control problem for a realistic example is aimed at demonstrating the feasibility and real-time efficiency of the proposed approach when compared with a linear multivariable feedback regulator and a nonlinear optimal control approach that is based on a fairly accurate traffic flow model.**

I. Introduction

In view of the imminent traffic congestion and lack of possibilities for infrastructure expansion in urban road networks, the importance of efficient signal control strategies, particularly under saturated traffic conditions, can hardly be overemphasized. It is generally believed that real-time (traffic-responsive) systems responding automatically to the prevailing traffic conditions, are potentially more efficient than clock-based fixed-time control settings, possibly extended via a simple traffic-actuated (micro-regulation) logic.

On the other hand, the development of network-wide real-time signal control strategies using elaborated network models is deemed infeasible due to the combinatorial nature of the related optimization problem [1]; as a consequence, some developed or implemented signal control strategies include many simplifications or heuristics which may render the strategies less efficient, particularly under saturated traffic conditions, unless a high effort is put in the fine-tuning of many parameters included in the signal control strategy.

The purpose of this paper is to investigate the efficiency of a new signal control methodology [2], which offers a computationally feasible technique for real-time network-wide control of the junction green times. This methodology combines traffic flow modeling based on the so-called store-and-forward modeling (SFM) paradigm, mathematical optimization and optimal control. More specifically, a generic mathematical model for the traffic flow process in large-scale urban networks is developed first, upon which an optimal control approach is applied for the design of traffic signal control strategies that aim at minimizing and balancing the link queues so as to reduce the risk of queue spill-back. The derived optimization problem is of the quadratic-programming (QP) type, i.e. it involves a quadratic objective function with linear constraints.

In order to evaluate the efficiency of the proposed quadratic-programming control (QPC) approach, we compare its open-loop behaviour with the closed-loop behaviour of a linear multivariable regulator (LQ) that is employed in the signal control strategy TUC [15], and the open-loop behaviour of a nonlinear optimal control (NOC) approach [3] that is based on a more accurate traffic flow model.

II. Background

A variety of traffic signal control strategies for urban networks have been developed during the past few decades. Without attempting a survey of this vast research area we will address a few selected strategies (for an up-to-date account we refer the reader to [1]), some of which have been implemented in real-life conditions while others are still in the research and development stage.

Fixed-time strategies for isolated junction control (stage-based approaches SIGSET [4], SIGCAP [5] based on the well-known Webster's delay formula) or network-wide coordinated control have been widely used due to their simplicity of implementation in networks with undersaturated traffic conditions. Arterial progression schemes that maximize the bandwidth of progression (MAXBAND [6], MULTI-BAND [7]), and more general network optimization schemes that minimize delay, stops or other measures of disutility (TRANSYT-7F [8]) are also in use. The main drawback of these strategies is that their settings are based on historical rather than real-time data.

SCOOT [9] and SCATS [10] are two well-known and widely used coordinated traffic-responsive strategies. These well-designed strategies function effectively when the traffic conditions in the network are below saturation but their performance deteriorates when severe congestion persists during the rush period. Other elaborated model-based traffic-responsive strategies such as PRODYN [11] and RHODES [12] employ dynamic programming while OPAC [13] employs exhaustive enumeration. Due to the exponential complexity of these solution algorithms, the basic optimization kernel is not real-time feasible for more than one junction.

Store-and-forward modeling of traffic networks was first suggested by Gazis and Potts [14] and has since been used in various works notably for road traffic control. This modeling philosophy offers a major advantage: it allows highly efficient optimization and control methods to be used for large-scale congested urban networks. A recently developed strategy of this type is the signal control strategy TUC [15].

More recently, a number of strategies have been proposed employing various computationally expensive numerical solution algorithms, including genetic algorithms [16], [17], multi-extended linear complementary programming [18], and mixed-integer linear programming [19], [20]. In [19], [17], and [20] the traffic flow conditions are modeled using the cell transmission model [21], a convergent numerical approximation to the first-order hydrodynamic model of traffic flow for network links. However, these approaches are in a relatively premature stage and their implementation and feasibility in real-life and real-time conditions are still questionable.

III. Problem Formulation

The urban road network is represented as a directed graph with links $z \in Z$ and junctions $j \in J$. For each signalized junction j, we define the sets of incoming I_j and outgoing O_j links. It is assumed that the offset, the cycle time C_j, and the lost time L_j of junction j are fixed. In addition, to enable network offset coordination, we assume that $C_j = C$ for all junctions $j \in J$. Furthermore, the signal control plan of junction j is based on a fixed number of stages that belong to the set F_j, while v_z denotes the set of stages where link z has right of way (r.o.w.). Finally, the saturation flow S_z of link $z \in Z$, and the turning movement rates $t_{w,z}$, where $w \in I_j$ and $z \in O_j$, are assumed to be known and fixed.

By definition, the constraint

$$\sum_{i \in F_j} g_{j,i} + L_j = (\text{or } \leq) \, C \tag{1}$$

holds at junction j, where $g_{j,i}$, is the green time of stage i at junction j. In addition, the constraint

$$g_{j,i} \geq g_{j,i,\min}, \quad i \in F_j \tag{2}$$

where $g_{j,i,\min}$ is the minimum permissible green time for stage i at junction $j \in J$, is introduced to guarantee allocation of sufficient green time to pedestrian phases.

Consider a link z connecting two junctions M and N such that $z \in O_M$ and $z \in I_N$ (Fig. 1). The dynamics of link z are given by the continuity equation

$$x_z(k+1) = x_z(k) + T\left[q_z(k) - s_z(k) + d_z(k) - u_z(k)\right] \tag{3}$$

where $x_z(k)$ is the number of vehicles within link z at time kT, $q_z(k)$ and $u_z(k)$ are the inflow and outflow, respectively, of link z in the sample period $[kT, (k+1)T]$; with T the discrete-time step and $k = 0, 1, \ldots$ the discrete-time index. In addition, d_z and s_z, are the demand and the exit flow within the link, respectively. For the exit flow we set $s_z(k) = t_{z,0} q_z(k)$, where the exit rates $t_{z,0}$ are assumed to be known.

Queues are subject to the constraints

$$0 \leq x_z(k) \leq x_{z,\max}, \quad \forall \, z \in Z \tag{4}$$

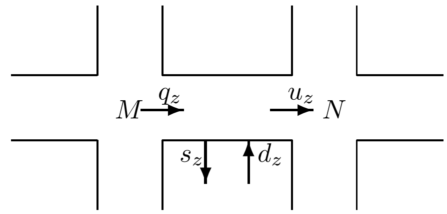

Fig. 1. An urban road link.

where $x_{z,\max}$ is the maximum admissible queue length. This constraint may automatically lead to a suitable upstream gating in order to protect downstream areas from oversaturation during periods of high demand.

The inflow to the link z is given by $q_z(k) = \sum_{w \in I_M} t_{w,z} u_w(k)$, where $t_{w,z}$ with $w \in I_M$ are the turning movement rates towards link z from the links that enter junction M.

We now introduce a critical simplification for the outflow u_z that characterizes the suggested modeling approach. Assuming that space is available in the downstream links and that x_z is sufficiently high, the outflow (real flow) u_z of link z is equal to the saturation flow S_z if the link has r.o.w., and equal to zero otherwise. However, if the discrete time step T is equal to C, an average value for each period (modeled flow) is obtained (Fig. 2) by

$$u_z(k) = G_z(k) S_z / C \tag{5}$$

where G_z, is the green time of link z, calculated as $G_z(k) = \sum_{i \in v_z} g_{j,i}(k)$.

In contrast to other SFM-based approaches (see for instance [22]), we will now introduce the green times G_z of each link z as additional independent variables. The reason behind this modification is that we want to preserve model validity also under nonsaturated traffic conditions [2]. The introduced link green times G_z are constrained as follows:

$$0 \leq G_z(k) \leq \sum_{i \in v_z} g_{j,i}(k), \quad \forall \, j \in J. \tag{6}$$

The main reason for introducing independent G_z in the problem formulation lies in the following observation: if the queue x_z is not sufficiently long or even zero; or if the downstream link queue is too long to accommodate a high inflow; then the constraints (4) will become active and will reduce the corresponding stage greens accordingly. As an illustrative example, assume that at a certain cycle there are two links z and w having r.o.w. simultaneously during a stage (M, i), and that $x_z \approx 0$ while $x_w \gg 0$ (Fig. 3). If G_z and G_w are not independently introduced, we have by definition $G_z = G_w = g_{M,i}$. Then, the stage green $g_{M,i}$ will be strictly limited by the constraint $x_z \geq 0$ although link w may need a longer green phase for dissolving x_w. In contrast, by introducing G_z and G_w independently, the algorithm can guarantee $x_z \geq 0$ by choosing G_z accordingly short without constraining G_w and the stage green. Similarly, if the link r downstream of link z is close to spillback (see Fig. 3),

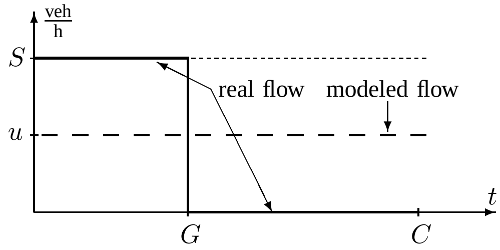

Fig. 2. Simplified modeling of link outflow.

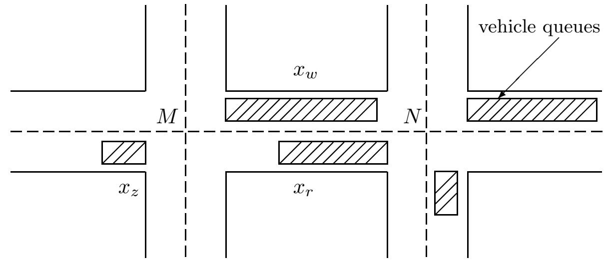

Fig. 3. A two-way link connecting two junctions M and N.

the constraint $x_r \leq x_{r,\max}$ can be guaranteed by choosing G_z accordingly short without constraining the green time of other links that are having r.o.w. during the same stage.

Replacing (5) in (3) leads to a linear state-space model for road networks of arbitrary size, topology, and characteristics

$$\mathbf{x}(k+1) = \mathbf{x}(k) + \mathbf{B}(k)\mathbf{G}(k) + T\mathbf{d}(k) \tag{7}$$

where $\mathbf{x}(k)$ is the state vector (with elements the number of vehicles x_z of each link z); $\mathbf{G}(k)$ is the link control vector with elements the green times G_z of each link z; $\mathbf{d}(k)$ is the disturbance vector with elements the demand flows d_z of each link z; $\mathbf{B}$ is a matrix of appropriate dimensions containing the network characteristics, and may be time-variant, if the involved saturation flows and turning rates are time-variant.

On the basis of the presented SFM and constraints, a (dynamic) optimal control problem may be formulated over a time-horizon K, starting with the known initial state $\mathbf{x}(0)$ in the state equation (7). In order to minimize the risk of oversaturation and spillback of link queues, one may attempt to minimize and balance the links' relative occupancies $x_z/x_{z,\max}$. A quadratic criterion that addresses this control objective has the form

$$\mathcal{J} = \frac{1}{2}\sum_{k=0}^{K}\sum_{z\in Z}\frac{x_z^2(k)}{x_{z,\max}}. \tag{8}$$

This criterion is physically reasonable as well as convenient from the numerical solution point of view. Alternatively, one may minimize the total time spent (which leads to a linear objective function) but this may increase the risk of link queue spillback. The resulting QP problem reads: minimization of the cost criterion (8) subject to (1), (2), (4), (6), (7). In summary, the optimization problem has three types of time-dependent decision variables, namely the stage green times $g_{j,i}(k)$, the state variables $x_z(k)$, and the link green times $G_z(k)$. This QP problem (with very sparse matrices) may be readily solved by use of broadly available codes or commercial software within few CPU-seconds even for large-scale networks and long time-horizons.

IV. Discussion

In this section we present some remarks pertaining to the consequences of the simplification (5) and to the application of the proposed open-loop QPC methodology in real time.

Let us first discuss the consequences of simplification (5). First, the updating of the control decisions cannot be effectuated more frequently than at every cycle which, however, is deemed sufficient for fast network-wide real-time control reactions; on the other hand, this feature limits the real-time communication requirements between junction controllers and the central computer to one message exchange per cycle, in contrast to the second-by-second communication requirements of other signal control systems such as SCOOT [9]. Second, the model is not aware of short-term queue oscillations due to green-red switchings within a cycle, because it models a continuous (uninterrupted) average outflow from each network link (as long as there is sufficient demand). Finally, offset and cycle time have no impact within the SFM and must be either fixed or updated in real time independently [23]. These consequences of simplification (5) is the price to pay for avoiding the explicit modeling of red-green switchings which would render the resulting optimization problem discrete (combinatorial) and lead to exponential increase of computational complexity as in [9-11, 14-18].

For the application of the open-loop QPC methodology in real time, the corresponding algorithm may be embedded in a rolling horizon (model-predictive) scheme. More precisely, the optimal control problem may be solved on-line once per cycle using the current state (current estimates of the number of vehicles in each link) of the traffic system as the initial state and predicted demand flows; the optimization yields an optimal control sequence for K cycles whereby only the first control (signal control plan) in this sequence is actually applied to the signalized junctions of the traffic network. Note that the saturation flows S_z and the turning-movement rates $t_{w,z}$, may be assumed to be time-variant and may be estimated or predicted in real time by well-known recursive estimation schemes [24]; in addition, the predicted demand flows $\mathbf{d}(k)$ may be calculated by use of historical information or suitable extrapolation methods (e.g., time series or neural networks). This rolling-horizon procedure avoids myopic control actions while embedding a dynamic optimization problem in a traffic-responsive environment.

Finally, it should be stressed that, in contrast to the LQ approach [15], in QPC methodology the control decisions are based on the explicit minimization of the cost criterion subject to all control and state constraints. Therefore, the aforementioned methodology could be also utilized as off-line network optimization tool for calculating optimum signal control plans, since the traffic flow model (7) and related constraints incorporate all necessary network characteristics.

V. Application Results

To demonstrate the real-time feasibility and efficiency of the proposed approach to the problem of urban signal control, the urban network of the city centre of Chania, Greece, is considered. For this network, we compare the closed-loop behaviour of the LQ approach [15] with the open-loop behaviour of the proposed QPC approach and with the open-loop behaviour of the NOC approach described in [3]. To ensure fair and comparable results all methodologies are evaluated by use of the same simulation model that is outlined in the next section.

A. The Simulation Model

We now describe the simulation model that will set the stage for our subsequent investigations. The basic idea here is to construct a traffic flow model that is more accurate than the linear SFM (7) and that will be used for simulating signal control strategies. For this reason, we define a nonlinear outflow function that models the real traffic flow process more accurately than (5). More precisely, assuming that the model's time step is $T \ll C$, the outflow $u_z(k)$ is given by

$$u_z(k) = \begin{cases} 0 & \text{if } x_{\mathrm{d},z}(k) \geq c x_{\mathrm{d,max}}(k) \\ \min\left\{\frac{x_z(k)}{T}, \frac{G_z(k) S_z}{C}\right\} & \text{else} \end{cases} \tag{9}$$

where $x_{\mathrm{d},z}(k)$ is a downstream link of link z with $t_{z,\mathrm{d}} \neq 0$, and parameter $c \in (0,1]$. By introducing (9), the state variables are allowed to change their value more frequently than the control variables. More precisely, typical discrete-time model steps T for the traffic flow model (3) using (9) may be in the order of 5 s while the control variables change their value in discrete-time control steps T_{c}, e.g. at each cycle. Note that the basic simplification of SFM, i.e. a continuous link outflow (rather than zero flow during red and free flow during green), is still maintained in this model.

Replacing (9) in (3) we obtain a nonlinear state-space model for road networks of arbitrary size, topology, and characteristics [3]

$$\mathbf{x}(k+1) = \mathbf{f}\big[\mathbf{x}(k), \mathbf{g}(\kappa), \mathbf{d}(k)\big], \quad \kappa = [k/\tau] \tag{10}$$

where $\mathbf{f}$ is a nonlinear vector function; $\mathbf{g}(\kappa)$ is the control vector (with elements all the green times $g_{j,i}$ of stage i at junction j); κ is a discrete-time index, and $T_{\mathrm{c}} = \tau T$. In the sequel the nonlinear traffic flow model (10) is used as simulation model.

B. Network and Scenario Description

The urban network of the city centre of Chania consists of 16 signalized junctions and 71 links (Fig. 4). According to the notation of Section III, the following sets are defined: $J = \{1, \ldots, 16\}$, $Z = \{1, \ldots, 71\}$. The cycle time in the network is $C = 90$ s, and $T_{\mathrm{c}} = C$ is taken as a control interval for all strategies. For the simulation model we consider $T = 5$ s and $c = 0.85$.

Several tests were conducted in order to investigate the behaviour of the three alternative methodologies for different scenarios. The scenarios were created by assuming more or less high initial queues $x_z(0)$ in the origin links of the networks while the demand flows d_z were kept equal to zero. The optimization horizon for each scenario is 450 s (5 cycles).

TABLE I

Comparison of Assessment Criteria

Strategy / Scenario	LQ TTS	LQ RQB	QPC TTS	QPC RQB	NOC TTS	NOC RQB
1	31.1	532	30.4	445	29.9	461
2	15.2	223	13.8	183	13.5	184
3	9.3	79	8.9	63	8.8	65
Average	18.6	278	17.7	230	17.4	237
Improvement	—	—	4.5%	17.2%	6.1%	14.9%

C. Comparison of Objective Functions

For each of three distinct scenarios of initial states $\mathbf{x}(0)$ and for each control approach, two evaluation criteria were calculated for comparison. The total time spent

$$\mathrm{TTS} = T_{\mathrm{c}} \sum_{k=0}^{K} \sum_{z \in Z} x_z(k) \qquad (\text{in veh} \cdot \text{h}) \tag{11}$$

and the relative queue balance

$$\mathrm{RQB} = \sum_{k=0}^{K} \sum_{z \in Z} \frac{x_z^2(k)}{x_{z,\max}} \qquad (\text{in veh}). \tag{12}$$

Note that, as mentioned earlier, the control results of each strategy are applied to the nonlinear model (10). Eventually $x_z(k)$ over a whole cycle was calculated first as the average of the corresponding 5-s values resulting from (10), before applying the above criteria on the basis of $T_{\mathrm{c}} = C = 90$ s.

Table I displays the obtained results. As can be seen QPC and NOC lead to a reduction of both evaluation criteria compared to LQ. More specifically, when QPC is applied, the TTS and RQB are improved by 4.5% and 17.2%, respectively; when NOC is applied, the TTS and RQB are improved by 6.1% and 14.9%, respectively, compared to LQ.

NOC is seen to be superior to all other strategies in terms of the TTS. This is because the nonlinear traffic flow model used by NOC is more accurate than the linear model used by LQ or QPC (and is therefore used as a common simulator for the comparison).

Regarding the RQB, it can be seen that QPC is superior to all other strategies. On close examination, this is quite comprehensible as the RQB is the exact cost criterion considered by QPC, while, in the cost criteria considered by LQ and NOC there are partially competitive subgoals.

The average computational time per scenario for QPC and NOC is 10 s and 8 min, respectively.

D. Detailed Results

In the sequel we report on some more detailed illustrative results focussing on the particular junctions 12 and 13. These two junctions carry heavy loads, since they represent a major entrance to and exit from the city centre (see Fig. 4).

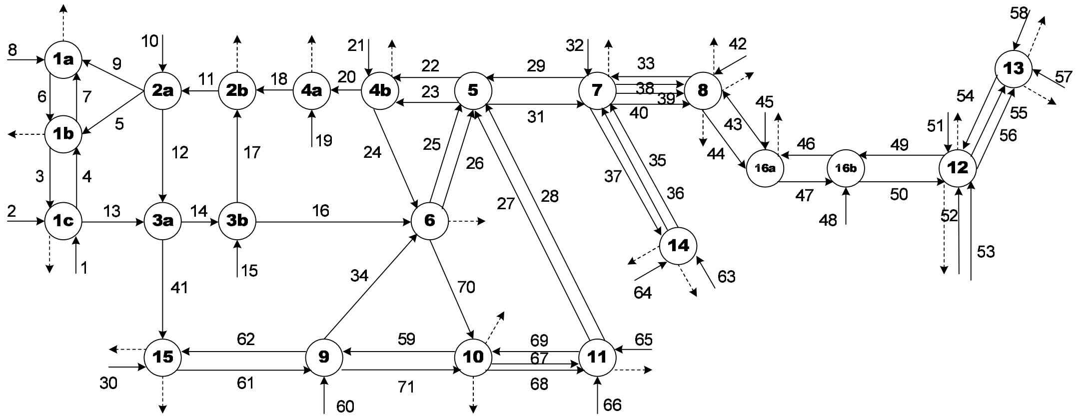

Fig. 4. The Chania urban road network.

For the aforementioned scenarios, the calculated optimal state and control trajectories demonstrate the efficiency of the three alternative methods to solve the urban signal control problem. Figures 5 and 6 depict the optimal trajectories for a particular scenario for the three methods. The main observations are summarized in the following remarks:

- Both QPC and NOC manage to dissolve the queues in a quite balanced way (see Figs. 5(b) and 5(c)) and thus, the desired control objective of queue balancing is achieved. Note that, these two strategies with different utilized traffic flow models accomplish the desired goal in a very similar way.
- The outflows of the origin links 57 and 58 enter the internal link 54 (solid line in Figs. 5(a)–5(c)) according to the green times of the corresponding junctions. It may be seen that QPC and NOC exhibit similar behaviour while managing particularly the queue of link 54 (see Figs. 6(b) and 6(c)).
- In contrast, the LQ strategy first allows the high initial queues to flow into the internal link 54 and then, in order to manage the developed long queue therein, it gradually increases the green time of stage 1 (see Fig. 6(a)) where link 54 has r.o.w. This somewhat slower behaviour is due to the reactive nature of the linear feedback regulator.

Both NOC and QPC deliver satisfactory results with similarly efficient control behaviour for different scenarios. Thus, taking into account that QPC needs substantially less computational effort than NOC [3], QPC may be considered as a quite satisfactory method for the solution of the urban signal control problem and a strong competitor of LQ in terms of efficiency and real-time feasibility.

VI. Conclusions and Future Work

The paper presented a generic quadratic-programming approach to the signal control problem in large-scale congested urban road networks. A simulation-based investigation of the signal control problem for a realistic example aimed at demonstrating the efficiency and feasibility in real-time conditions of the proposed approach when compared with the LQ approach taken by the signal control strategy TUC and a NOC approach that is based on a fairly accurate traffic flow model.

Future work will deal with: (a) the comparison of the proposed approach, embedded in a rolling horizon scheme, with other strategies (e.g. TUC) in more elaborated simulation involving external and internal demands and saturated traffic conditions as well as in real-life conditions; and (b) improvements of the NOC strategy to cope more efficiently with hard constraints on controls.

VII. Acknowledgment

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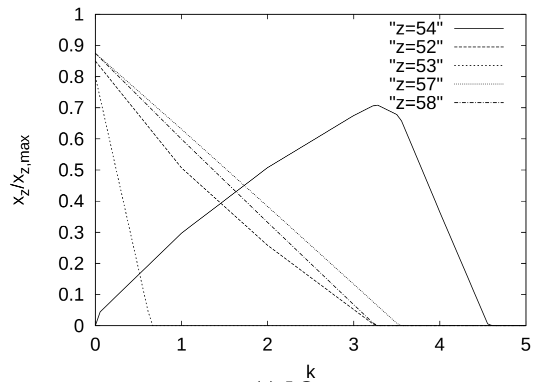
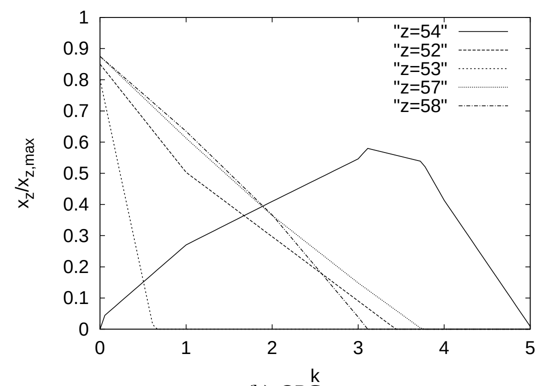
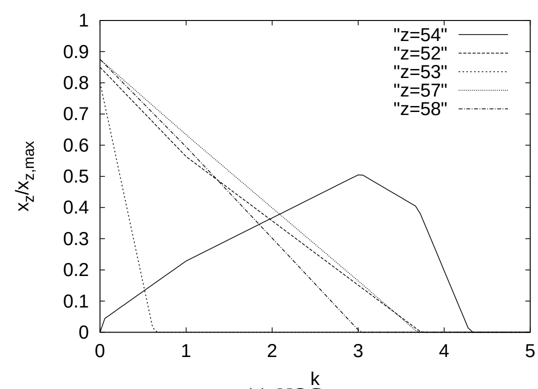


(a) LQ

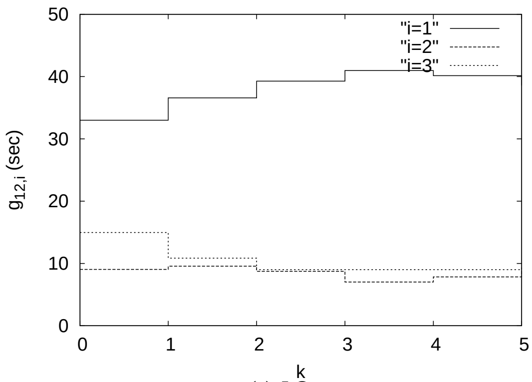


(b) QPC

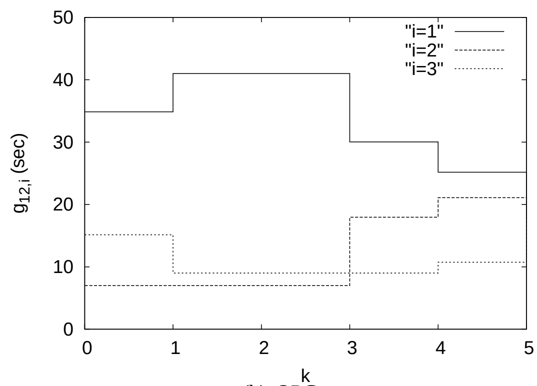


(c) NOC

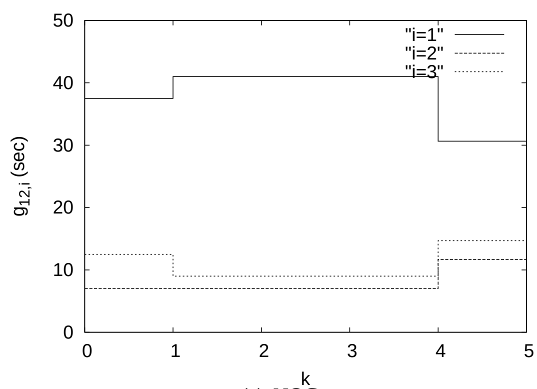
Fig. 5. Relative occupancies within the links at junctions 12, 13.



(a) LQ



(b) QPC



(c) NOC

Fig. 6. Optimal control trajectories of junction 12.

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