

# Dynamic Queuing and Spillback in an Analytical Multiclass Dynamic Network Loading Model

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## 1 Introduction

In order to analyze transportation networks for planning purposes, traffic assignment models have shown to be a useful tool. For this reason these models have been applied for many years now. Although static models are still widely used, the theory and practice of dynamic models have evolved significantly over the last 10 years. This resulted in a shift of focus from static traffic assignment to dynamic traffic assignment (DTA) in both research and (commercial) applications.

DTA models typically describe route choice behavior of travelers on a transportation network and the way in which traffic dynamically propagates through the network. A nice overview of DTA approaches is given in [1]. Two main approaches can be distinguished, namely (i) a pure analytical approach, and (ii) a simulation-based approach. In the pure analytical approach, the DTA problem (typically formulated as a variational inequality problem) is directly solved by using well-known optimization techniques. Examples are models proposed by [2], [3], and [4]. These models are usually limited to small hypothesized networks, as they use solution procedures that do not take advantage of the specific characteristics of the transportation problems. On the other hand, simulation-based models are specifically designed for transportation problems and can handle larger and more realistic networks. These simulation-based DTA models are nowadays widely available and can define the problem either on a microscopic level (e.g., PARAMICS, AIMSUN2 with a micro-simulator propagating the network flows), a mesoscopic level (e.g., DYNASMART, INTEGRATION), or on a macroscopic level (e.g., INDY, MARPLE with a dynamic network loading procedure performing the flow propagation, see [5]).

In this paper we will propose an analytical dynamic network loading (DNL) procedure as part of a macroscopic DTA model in which queuing and spillback are taken into account in a multiclass setting with different vehicle types. Attempts of others have some strong restrictions, as will be pointed out in Section 2, while the model proposed in this paper does not rely on these (unrealistic) restrictions. This will lead to a completely time-responsive dynamic queuing model without any assumptions on stationary inflows, queue lengths, and/or outflow capacities. The key to the dynamic queuing model is that we do not rely on travel time functions that look forward in time, but only use queuing and exit functions that look backward in time. The formulation correctly deals with time-varying link attributes, such as inflow and outflow capacities and maximum speeds, such that a wide range of dynamic traffic management (DTM) measures can be incorporated.

## 2 Approaches of queuing in dynamic network loading models

Many macroscopic simulation-based DTA models use DNL models that propagate the traffic flow on the links using link travel time functions (see e.g. [6]–[9]), which relate the travel time to the number of vehicles on the link at the time of link entrance. Although these models will generally be able to give good estimates for average travel times and flows, dealing with specific phenomena such as spillback and DTM measures in which changes in capacities play an important role constitute weak points. This is mainly due to the fact that the (forecasted) travel times are assumed fixed while traversing the link, while in reality the travel time is not known before a vehicle exits the link, as there may be dynamically changing queues and dynamically changing outflow restrictions.

In the literature a few possible approaches for dealing with queues and spillback have been proposed. Often, these models consider a horizontal queue (as opposed to the unrealistic vertical queues) on a link that is artificially split up into a free-moving part followed by a queuing part. There are typically two difficulties: (a) the queue length may change while traversing the link, and (b) the outflow capacity may change while traversing the link. By means of trajectories illustrated in Figure 1 we will explain the differences in the approaches and how they deal with the above mentioned difficulties.

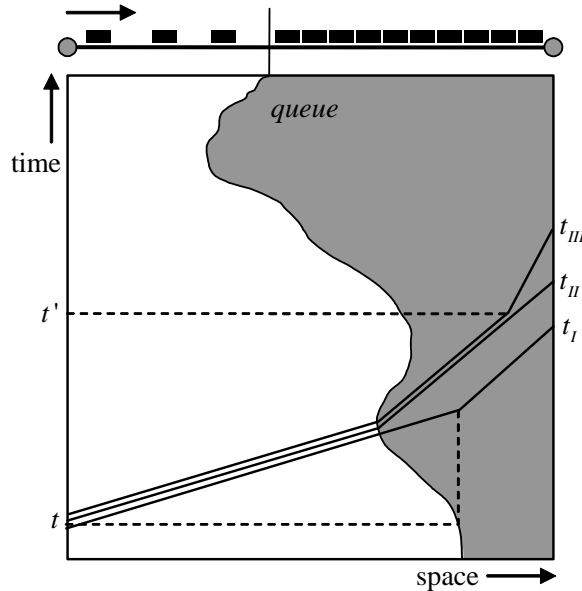


Figure 1: Different queuing approaches

Consider a link with a free-moving and a queuing part. In the figure is depicted how the queue grows and shrinks over time. Furthermore, consider a vehicle entering the link at time instant  $t$ . We are interested to know what time instant this vehicle will exit the link.

In [2] a model is proposed in which the queue length at link entrance (time  $t$ ) is considered, and then the link exit time is calculated by computing the time in the free-moving part and in the queuing part, assuming that neither the queue length, nor the outflow capacity will change (which we will call an *instantaneous queue* approach). This leads to exit time  $t_I$ . However, the queue length may change (due to vehicles entering the queue from the free-moving part and vehicles leaving the queue flowing out of the link). This problem was solved by [10] by realizing that all vehicles that are in the free-moving part at time  $t$  will have entered the queue when our considered vehicle enters the queue. By assuming a fixed outflow capacity, it is possible to determine the exact queue length, which leads to an exit time of  $t_{II}$  (so-called *variable queue* approach). Note that their approach is only valid if no overtaking finds place in the free-moving part, hence it will not hold in case of multiple vehicle types where first-in-first-out (FIFO) does not hold between all vehicles. In [11] FIFO is not required in the free-moving part, as they look backwards from the tail of the queue. However, they also compute the travel time assuming that the outflow capacity does not change. In this paper we propose an approach that also does not require FIFO to hold in the free-moving part, but can take changing outflow capacities into account (so-called *dynamic queue* approach). For example, if the outflow capacity decreases at  $t'$  (e.g., due to spillback, or DTM measures, or even just by changing composition of vehicles in the queue with respect to directions, etc), then the actual exit time will be  $t_{III}$ . Clearly, the true travel time is not known until the time of exiting the link, hence using link travel time functions may yield incorrect travel times that are not consistent with capacity constraints.

Besides approaches splitting links into two parts, other approaches exist, e.g. the *physical queue* approach. This approach is described in models such as the cell transmission model (CTM, [12]), and more recently in the link transmission model (LTM, [13]). As these models rely on FIFO, multiple vehicle types cannot be taken into account, although they have the advantage that shockwaves can be modeled more easily.

### 3 Model framework

Consider a given transport network  $G = (N, A)$  consisting of nodes  $N$  and directed links  $A$  having certain attributes, and a given dynamic vehicle type specific travel demand for each origin-destination (OD) pair  $(r, s)$ , each vehicle type  $m$ , and each departure time  $k$ . The DTA model framework is a typical route-based framework consisting of three main parts: (1) a route set generation model, (2) a route choice model, and (3) a dynamic network loading (DNL) model. The first two models are the same as in the INDY model described in [5] and [9]. We will restrict ourselves to describing the DNL model with dynamic queuing.

The DNL model simulates the route flows (determined by the route choice model) along the links in the network. This model is at the heart of the DTA model and is also the most computationally intensive part. A completely new DNL model has been developed, which has dynamic queuing possibilities and does not have the drawbacks from most DNL models based on link travel time functions (see the previous section). This DNL model consists of a link model and a node model. Both will be discussed in more detail in the next sections.

#### 4 Link model

The link model describes the propagation of the flow through each link, taking into account different speeds for different vehicle types and a dynamic horizontal queue. The main outcomes are the link inflows, queue inflows, queue lengths, and link travel times.

Consider a link  $a \in A$  in network  $G$  for which the following attributes are given: link length  $L_a$  [km], maximum speeds per vehicle type  $\mathcal{G}_{am}^{\max}$  [km/h], and a queue density  $J_a$  [passenger car units (pcu)/km]. Also, an unrestricted inflow capacity  $C_a$  [pcu/h] is given for each link, however this attribute will only be used by the node model, see Section 5. As mentioned in Section 2, we will (artificially) split each link into a free-moving part and a queuing part, conform Figure 2. The lengths of the free-moving part and the queuing part, denoted by  $L_a^f(t)$  and  $L_a^q(t)$  respectively, are variables in the model. Splitting the link into these two parts is also important from a multiple vehicle type point of view. In the queue all vehicle types are assumed to travel at the same speed, while in the free-moving part vehicle types may travel at different speeds and may overtake each other. Hence, first-in-first-out (FIFO) need not be satisfied among different vehicle types, but typically is assumed to hold within each vehicle class.

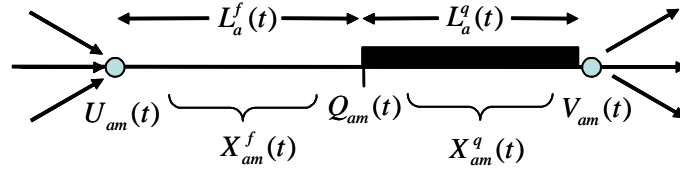


Figure 2: Link variables

The variables  $U_{am}(t)$ ,  $Q_{am}(t)$ , and  $V_{am}(t)$  denote the cumulative link inflow, cumulative queue inflow, and cumulative outflow of link  $a$  at time  $t$ , respectively. In case there is no queue,  $Q_{am}(t) = V_{am}(t)$ . The number of vehicles of each vehicle type in the free-moving part,  $X_{am}^f(t)$ , and the number of vehicles in the queuing part,  $X_{am}^q(t)$ , then are

$$X_{am}^f(t) = U_{am}(t) - Q_{am}(t), \quad \text{and} \quad X_{am}^q(t) = Q_{am}(t) - V_{am}(t). \quad (1)$$

The link inflow rate  $u_{amp}^{rs}(t)$  is determined by the corresponding outflow rate  $v_{a'mp}^{rs}(t)$  of the previous link  $a'$  on route  $p$ , or (in case link  $a$  is the first link on route  $p$ ) given by the route flow rate  $f_{mp}^{rs}(t)$ . This leads to the following flow conservation constraint:

$$u_{amp}^{rs}(t) = \begin{cases} v_{a'mp}^{rs}(t), & \text{if } a' \text{ is the previous link on route } p, \\ f_{mp}^{rs}(t), & \text{if } a \text{ is the first link on route } p. \end{cases} \quad (2)$$

Then, the vehicle type  $m$  specific cumulative inflows  $U_{am}(t)$  (and similar for  $V_{am}(t)$ ) can be computed as

$$U_{am}(t) = \sum_{(r,s)} \sum_{p \in P_m^{rs}} U_{amp}^{rs}(t), \quad \text{with} \quad U_{amp}^{rs}(t) = \int_{\omega=0}^t u_{amp}^{rs}(\omega) d\omega, \quad (3)$$

It is important to note that the link outflow rates  $v_{a'mp}^{rs}(t)$  are determined by the node model (see next section) taking capacity constraints into account, and are not determined by a flow propagation constraint as usual in a link model using link travel time functions. As mentioned in Section 2, using flow propagation based on link travel time functions may violate capacity constraints. In our model, only the outflow out of the free-moving part is determined by a flow propagation constraint, which does not corrupt any existing capacity constraints at the end of the link. Instead of determining a travel time (for the free-

moving part) at the time of link entrance, only a speed is computed per vehicle type. This is due to the fact that the travel time in the free-moving part will be unknown, since the future position of the tail of the dynamic queue is unknown. By choosing the speeds  $\mathcal{G}_{am}(t) = \mathcal{G}_{am}^{\max}$ , FIFO will hold within each vehicle type.

Assume a given link-specific queue density  $J_a$  (pcu/km). The total number of pcu's in the queue is defined by  $\sum_m \rho_m X_{am}^q(t)$ , where  $\rho_m$  denotes the vehicle type specific pcu-value. The length of the free-moving part,  $L_a^f(t)$ , can then be determined as

$$L_a^f(t) = L_a - L_a^q(t), \quad \text{where } L_a^q(t) = \frac{\sum_m \rho_m X_{am}^q(t)}{J_a}. \quad (4)$$

Note that if  $L_a^q(t) = L_a$ , spillback will occur to previous link(s) by restricting the inflow into link  $a$ , see the node model in Section 5. A vehicle of type  $m$  entering link  $a$  at time  $\omega$  with speed  $\mathcal{G}_{am}(\omega)$  will reach the tail of the queue at time  $t$  if  $(\omega + L_a^f(t) / \mathcal{G}_{am}(\omega)) \leq t$ . Hence, the cumulative queue inflow is given by

$$Q_{amp}^{rs}(t) = \int_{\omega \in \Omega(t)} u_{amp}^{rs}(\omega) d\omega, \quad \text{with } \Omega(t) = \left\{ \omega \mid \omega + \frac{L_a^f(t)}{\mathcal{G}_{am}(\omega)} \leq t \right\}. \quad (5)$$

Then, the queue inflow rates  $q_{amp}^{rs}(t)$  and the total cumulative queue inflow  $Q_{am}(t)$  can be computed as

$$q_{amp}^{rs}(t) = \frac{dQ_{amp}^{rs}(t)}{dt}, \quad \text{and } Q_{am}(t) = \sum_{(r,s)} \sum_{p \in P_m^{rs}} Q_{amp}^{rs}(t). \quad (6)$$

Note that no travel times need to be computed for the flow propagation. However, the link travel times are usually an important output of the model, hence we will compute them afterwards. Since FIFO per vehicle type holds, the link travel time  $\tau_{am}(t)$  for type  $m$  vehicles entering link  $a$  at time  $t$  is given by

$$\tau_{am}(t) = V_{am}^{-1}(U_{am}(t)) - t. \quad (7)$$

To illustrate the flow propagation in the link model with dynamic queuing, consider two consecutive links, where the first link has length  $L_a = 10$  [km], a queue density of  $J_a = 200$  [pcu/km], and where the outflow capacity of the first link is bounded from above by the capacity of the second link being 4,000 [pcu/h]. Consider two vehicle types, cars ( $m=1$  with  $\rho_1=1$ ) and trucks ( $m=2$  with  $\rho_2=2.5$ ), having fixed speeds  $\mathcal{G}_{a1}(t) = 120$  [km/h] and  $\mathcal{G}_{a2}(t) = 80$  [km/h] in the moving part of the link. The computation of the outflow rates given current capacities will be explained in more detail in the node model in the next section. In this example, the node model and capacity constraints are very simple. Concentrating on the first link, the actual outflow rates will be simply bounded from above by the outflow capacity.

The travel demand for both vehicle types is given in Figure 3(a), where the travel demand for trucks is assumed to be 20% of the travel demand for cars. Also indicated is the total travel demand in terms of pcu, where we assume the inflow capacity into the first link is sufficient to accommodate the travel demand for each time instant. However, the outflow capacity (4,000 pcu/h) may prevent the flow from entering the second link. Furthermore, we assume that after an hour (at  $t^* = 3600s$ ) the outflow capacity drops to 2,000 pcu/h, e.g. due to an accident on the second link.

The results are shown in Figures 3(b)-(d). Just before  $t = 1000s$  the entering flow reaches 4,000 pcu/h (see Figure 3(a)). This flow will reach the end of the link after the free-flow travel time elapses, i.e. after 300s and 450s for cars and trucks, respectively. Hence, a queue builds up some time after  $t = 1000s$ , see Figure 3(c). Clearly, the queue length is immediately affected whenever the outflow rate drops at  $t^* = 3600s$ . The link travel times for cars and trucks are shown in Figure 3(d), where it should be pointed out that time instant  $t$  is the time of link entrance. This picture makes the difference between the instantaneous queue and the dynamic queue very clear. Already around link entrance time  $t = 2500s$  there is (correctly) a strong increase in the link travel time, even though the drop in capacity does not happen till  $t^* = 3600s$ . At  $t = 2500s$  the travel time for cars is approximately 1100s, hence at  $t = 3600s$  they reach the end of the link, exactly when the capacity drops to 2,000 veh/h. In models using an instantaneous

queue this increase in travel time would not occur until  $t = 3600s$ , which clearly deviates significantly from the correct time instant. Further note that the travel time of cars is always less than (or equal to) the travel time of trucks. In free-flow conditions, the travel time of trucks is 50% higher than the car travel time. During congestion the two travel times converge, as the travel time inside the queue is the same for all vehicle types. The differences in travel time in the moving part are not the main reason for the differences in travel time during congestion, as vehicles have to wait in the queue anyway till they can exit the link. The difference in travel time is mainly explained by the number cars overtaking the trucks, causing extra delay for the trucks. Note that after  $t = 4000s$  the travel times for both cars and trucks are (more or less) the same, since the travel demand becomes very low such that the number of overtaking cars does not cause a significant extra delay for trucks.

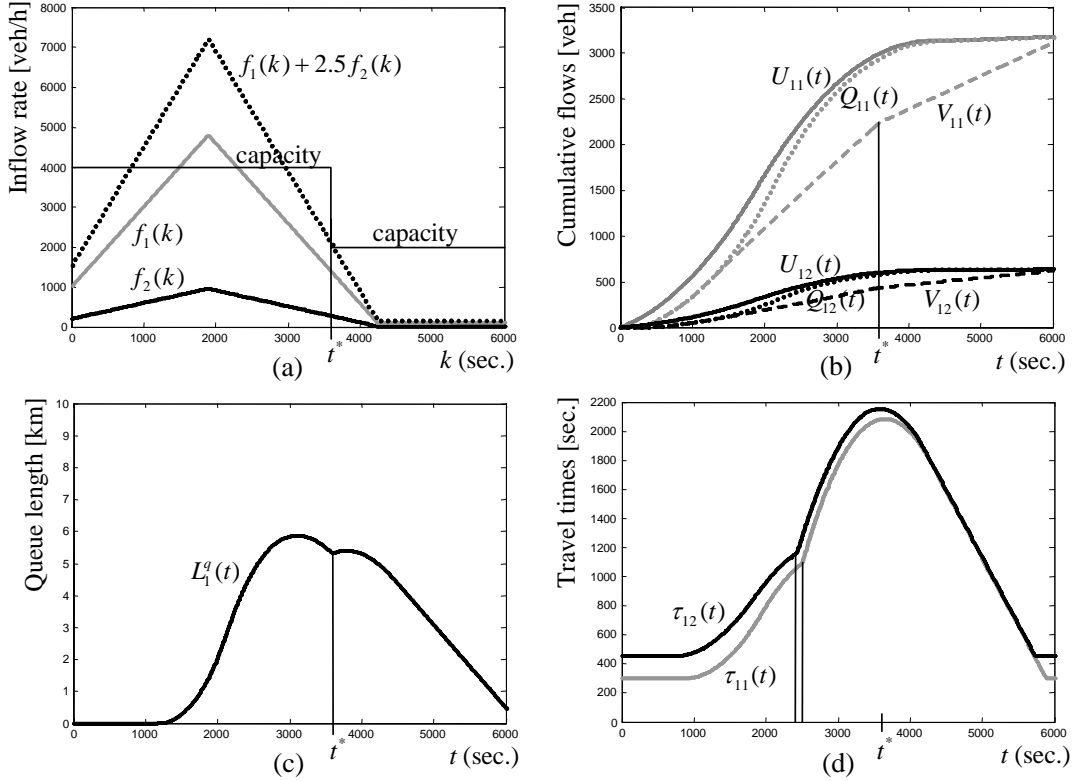


Figure 3: Application of the link model to a simple bottleneck

## 5 Node model

The node model relates the inflows into and outflows out of each node, taking into account the inflow capacities into outgoing links, and the potential outflow rates from incoming links. The capacity constraints of links are therefore completely managed by the node model, not the link model. The outcomes of the node model are the OD-, path- and vehicle type dependent dynamic outflow rates  $v_{amp}^{rs}(t)$ .

Consider a node  $n \in N$  in network  $G$  having a set  $B^{\text{in}}(n)$  of incoming links and a set  $B^{\text{out}}(n)$  of outgoing links, see Figure 4. Each incoming link  $a \in B^{\text{in}}(n)$  has some potential outflow rates  $\bar{v}_{amp}^{rs}(t)$ , consisting of vehicles arriving at the end of the link (queued or not queued) at time  $t$ . However, the actual outflow rates  $v_{amp}^{rs}(t)$  may be smaller than these potential outflow rates due to capacity restrictions. Each outgoing link  $b \in B^{\text{out}}(n)$  has a certain inflow capacity  $C_b^{\text{in}}(t)$  which can change over time due to queue spillbacks or DTM measures. The potential outflow rates in the direction of  $b$  share this limiting capacity.

The inflow capacity  $C_b^{\text{in}}(t)$  depends on whether the queue on link  $b$  is spilling back or not (or if a DTM measure changes the inflow capacity). If there is no spillback (i.e.,  $L_b^q(t) < L_b$ ), then the inflow capacity is equal to the unrestricted capacity  $C_b$ . On the other hand, if there is spillback, then the inflow capacity is set to the current outflow out of that link. Mathematically,

$$C_b^{\text{in}}(t) = \begin{cases} C_b, & \text{if } L_b^q(t) < L_b, \\ \sum_{(r,s)} \sum_m \sum_{p \in P_m^{rs}} \rho_m v_{bmp}^{rs}(t), & \text{otherwise.} \end{cases} \quad (8)$$

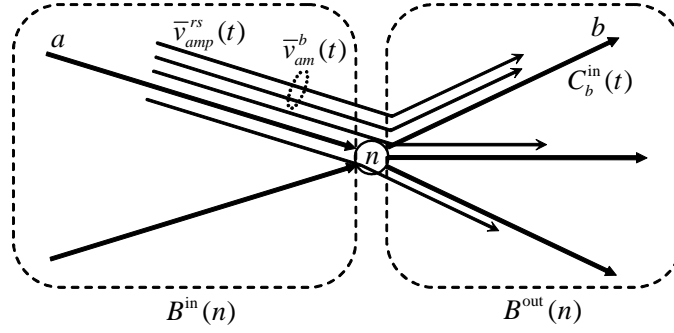


Figure 4: Node variables

In order to compute the potential outflow rates  $\bar{v}_{amp}^{rs}(t)$ , we have to determine the flow rates at the head of the queue. In case there is no queue, the potential outflow rates are simply equal to  $q_{amp}^{rs}(t)$ . However, if there is a queue, it will be assumed that all lanes will be used by vehicles in the queue and that the total potential outflow is equal to the capacity of the link,  $C_a$ . The potential outflow rates are then functions of earlier queue inflow rates  $q_{amp}^{rs}(t_a^*(t))$ , where  $t_a^*(t)$  is the time instant in which the vehicles now at the head of the queue entered the tail of the queue. Since FIFO holds in the queuing part, this time instant can easily be determined by  $t_a^*(t) = Q_a^{-1}(V_a(t))$  with  $Q_a(t) = \sum_m \rho_m Q_{am}(t)$  and  $V_a(t) = \sum_m \rho_m V_{am}(t)$ . Of importance is that the outflow rate proportions are conserved between each OD-pair, each path, and each vehicle type (again, due to FIFO). Therefore, the proportions between the queue inflow rates  $q_{amp}^{rs}(t_a^*(t))$  should transfer to proportions of the potential outflow rates sharing the link capacity  $C_a$ . Hence, the potential outflow rates are determined by

$$\bar{v}_{amp}^{rs}(t) = \begin{cases} q_{amp}^{rs}(t), & \text{if } L_b^q(t) = 0, \\ \frac{q_{amp}^{rs}(t_a^*(t))}{\sum_{(r',s')} \sum_{m'} \sum_{p' \in P_m^{rs}} \rho_m q_{am'p'}^{r's'}(t_a^*(t))} C_a, & \text{otherwise.} \end{cases} \quad (9)$$

Since multiple paths can use the same outgoing link, the *directional* potential outflow rates  $\bar{v}_{am}^b(t)$  describing the (vehicle type specific) outflow from link  $a$  to link  $b$  will be used (see also Figure 4):

$$\bar{v}_{am}^b(t) = \sum_{(r,s)} \sum_{p \in \{P_m^{rs} | b \in p\}} \bar{v}_{amp}^{rs}(t). \quad (10)$$

Knowing the boundaries on the inflows and outflows, the actual outflow rates  $v_{amp}^{rs}(t)$  should be determined. The determination of these actual outflow rates is not trivial for a general node with multiple incoming and outgoing links and with multiple vehicle types. In the cell transmission model in [14], these outflow rates are computed for a simple merge and a simple diverge. These computations are essentially based on a linear programming (LP) problem in which the throughput of the node is maximized subject to capacity constraints and proportion conservation constraints. Adopting this idea, and extending it to a general node with multiple in- and outgoing links and multiple vehicle types, leads to the following LP formulation:

$$\max_{v_{am}^b(t)} \sum_{a \in B^{\text{in}}(n)} \sum_{b \in B^{\text{out}}(n)} \sum_m v_{am}^b(t) \quad (11)$$

$$\text{s.t.} \quad \sum_{a \in B^{\text{in}}(n)} \sum_m \rho_m v_{am}^b(t) \leq C_b^{\text{in}}(t), \quad \forall b \in B^{\text{out}}(n), \quad (12)$$

$$v_{am}^b(t) \leq \bar{v}_{am}^b(t), \quad \forall a \in B^{\text{in}}(n), \forall b \in B^{\text{out}}(n), \forall m, \quad (13)$$

$$\frac{v_{am}^b(t)}{v_{a'm}^b(t)} = \frac{\bar{v}_{am}^b(t)}{\bar{v}_{a'm}^b(t)}, \quad \frac{v_{am}^b(t)}{v_{am}^{b'}(t)} = \frac{\bar{v}_{am}^b(t)}{\bar{v}_{am}^{b'}(t)}, \quad \frac{v_{am}^b(t)}{v_{am'}^b(t)} = \frac{\bar{v}_{am}^b(t)}{\bar{v}_{am'}^b(t)}, \quad \forall a, a' \in B^{\text{in}}(n), \forall b, b' \in B^{\text{out}}(n), \forall m \neq m'. \quad (14)$$

The objective function is the total outflow through node  $n$  at time  $t$ , Eqns. (12) and (13) describe the outflow and inflow constraints, respectively, and Eqns. (14) ensure flow proportion conservation. These

proportion conservation constraints typically hold for freeway traffic if we assume that flow in a capacity constraint direction also constrains flow in other directions (assuming a single queue on each link). It can be shown (see [15]) that solving this LP problem yields the following analytical solution:

$$v_{am}^b(t) = \min_{b' \in B^{out}(n) \setminus \{b \mid \bar{v}_{am}^{b'}(t) \geq 0\}} \left\{ \bar{v}_{am}^b(t), \frac{\bar{v}_{am}^b(t)}{\sum_{a'} \sum_m \rho_m \bar{v}_{a'm}^{b'}(t)} C_b^{in}(t) \right\}, \quad \text{such that } v_{amp}^{rs}(t) = \frac{\bar{v}_{amp}^{rs}(t)}{\bar{v}_{am}^b(t)} v_{am}^b(t). \quad (15)$$

Clearly from this expression, the actual outflow is either the potential outflow if the capacity constraints are not binding, or the actual outflow is a proportion of the available capacity.

As an example, consider a node with two incoming and two outgoing links (see Figure 5). Furthermore, assume that two vehicle types are present. The directional potential outflow rates  $\bar{v}_b^b(t)$  and the inflow capacities  $C_b^{in}(t)$  are given, where the potential outflow rates have already been converted to pcu's for convenience. What can be observed is that link 3 is the bottleneck with a capacity of 2,000 pcu/h, while in total 2,500 pcu/h would like to enter this link. Link 3 will be used at capacity, constraining the outflows out of links 1 and 2 in the direction of link 3. Because the flows from these links are constrained, the flows to link 4 will also be constrained (although 4,500 pcu/h could potentially flow into link 4, only 3,600 pcu/h actually enters link 4).

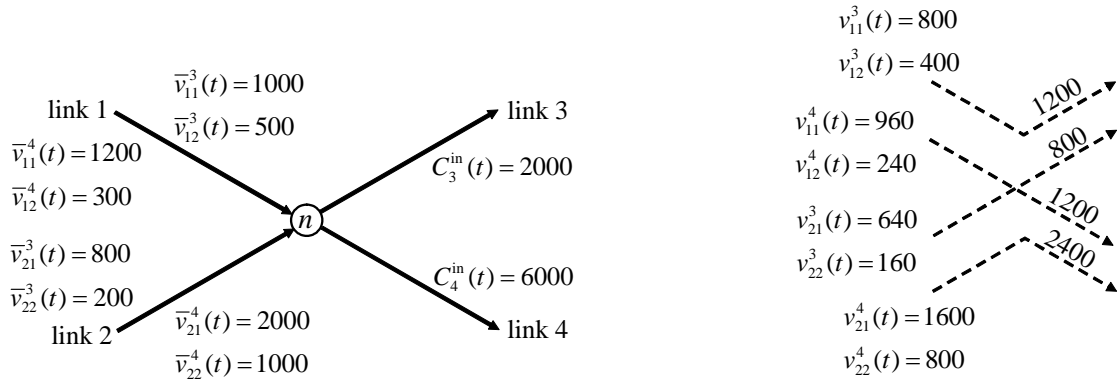


Figure 5: Application of the node model on a two-link in, two-link out node

## 6 Application

In order to illustrate the applicability of the proposed model in a larger real-life network, the model has been implemented (although currently only for one vehicle type) in the INDY DTA software and has been successfully tested on some small hypothesized networks. Queues were correctly formed upstream bottlenecks. The model presented in this paper is defined in continuous time, hence a discretization scheme was needed for implementation. This discretization scheme is not trivial, but it is beyond the scope of this paper to describe the discretization and the algorithm. Details can be found in [15]. Here we will just briefly mention an application on a reasonably large real-life network.

The application involves an evacuation study in the Voorne-Putten area near Rotterdam in The Netherlands, see Figure 7. This area is surrounded by water and has only five exit points (as indicated) to get to the main land. Hence, in case of a flooding, the approximately 150,000 inhabitants of a few smaller cities have to evacuate using one of these five exit points. The aim of the study was to design evacuation plans for all inhabitants (consisting of the appointed exit points, routes, and departure times) that minimize the total evacuation time. The network consists of approximately 1,500 nodes and 6,000 directed links. In total there are 468 zones, of which 5 destinations and 463 origins. Results showed that evacuation of the city of Spijkenisse was the bottleneck. Long queues in the direction of exit point 4 existed in most scenarios (as indicated in the figure on the right-hand side), spilling back in multiple directions. The dynamic queuing model in the DNL model seems to give plausible and realistic outcomes. More on the evacuation study can be found in [16].

## 7 Conclusions

Analytical dynamic network loading models typically have difficulties to correctly deal with flow propagation, queue formation, and spillback. Most of these difficulties are caused by the adoption of link travel time functions, which are not able to capture queue dynamics correctly. Cell-transmission and link-transmission models (CTM/LTM) typically overcome this problem, but are restricted to homogeneous vehicle types and typically assume simple intersection layouts.

In this paper a new analytical dynamic network loading model is proposed, consisting of a link model and a node model. In this new model, multiple vehicle types can be taking into account, queue formation and spillback is correctly handled, and general intersection layouts can be used in the node model. The link model abandons the use of link travel time functions to maintain consistency between the travel times and dynamically changing queues. A general node model has been proposed to enable the distribution of the capacity to the corresponding directions. The model has been implemented in the INDY DTA software and has been successfully run on a fairly large real-life network in the application presented. Special attention needs to be paid to gridlocks, as capacity constrained models typically may suffer from this, and our proposed model is no exception. It should be noted that the proposed model can handle dynamic changes of network attributes (by means of events in INDY), such as capacities, hence traffic lights and dynamic traffic management measures can be simulated.

The proposed model has also some drawbacks. It is not able to describe shockwaves, as it explicitly assumes an artificial split of each link into a single moving part (at the beginning of the link) and a single queuing part (at the end of the link). Currently, research is conducted to include shockwaves and multiple queuing parts into the model. Models based on cell/link-transmission are able to deal with shockwaves explicitly, which may lead to a more realistic description of moving queues and even multiple queuing parts on a single link. However, extending CTM/LTM to include multiple vehicle types (with different speed-density relationships) remains a challenge to researchers that is still unsolved and will be difficult to solve due to the explicit assumption of first-in-first-out (FIFO) of vehicles on a link (which no longer holds in case of multiple vehicle types).

## Acknowledgments

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