Extended abstract:
Cost allocation principles in transportation

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Problem formulation

Transportation planning is an important part of the wood flow chain in forestry. Large volumes and relatively long transport distances together with increasing fuel prices and environmental concern makes it important to improve the transportation planning. There are often several forest companies operating in the same region but co-ordination between two or more companies is however rare. In many cases, volumes of the same assortment is transported in opposite directions due to a low level of interaction between the forest companies. Supply, demand and companies are geographically evenly dispersed in the region and there is generally a high potential for coordination of the wood flow. Lately, there has been an increased interest in collaborative planning as the potential savings are large. In many of the case studies the optimized transportation work is compared with the actual transportation work carried out. It is then possible to compute the actual savings. In the case when coordination should be included in the operative planning there are a number of questions arising, such as: How should the potential coordination be computed? How should the saving be divided among the participants? The first question can be approached by using Operations Research methods. The models and methods used in the system FlowOpt (Forsberg et al., 2005) can be used to find the actual saving if all participants co-operate as compared to no coordination. The second question is often not addressed. The reason for this is that wood bartering is often in practice only used between two companies and then only with a fixed volume. When addressing the second question, we suggest to use a *cost allocation method* as a tool for negotiations. That is, we do not split savings, instead we split the common cost among the participants. Further, we do not aim at identifying the "best" cost allocation, but instead, at suggesting and analyzing a number of alternatives.

Transportation planning

A planning tool often used a Linear Programming (LP) model with variables representing the flow from supply point i to demand point j. Constraints are expressed on supply and demand and the objective is to minimize the total cost. In forestry there are several assortments (depending on e.g. species and dimensions) and assortment groups can be used to specify the demands. A demand for an assortment group can then be fulfilled by one or several different assortments, depending on the definition of the assortment group. In the LP model, the cost is based on the fact that the truck drives full from supply to demand point and empty in the other direction. This is the base of standard agreements. However, this gives an efficiency of just 50%. Efficiency would be improved if routes involving several loaded trips were used, i.e. backhauling. Backhauling refers to when a truck that has carried one load between two points, carries another load on its return.

In our case we consider the problem to co-ordinate planning for several companies. It is common that transport costs can be decreased if companies apply bartering. However, this is difficult as planners not want to reveal supply, demand and cost information to competitors. In practice this is solved by deciding on wood bartering of specific volumes. Today this is done in an ad-hoc manner and is mostly dependent on personal relations. In figure 1 we illustrate the potential benefits with wood bartering when two companies are involved. Here we have four mills at two companies (two mills each) together with a set of supply points for each company. In the left part each company operates by itself. The catchment areas are relatively large as compared to the right part where all supply and demand point are used on equal terms.

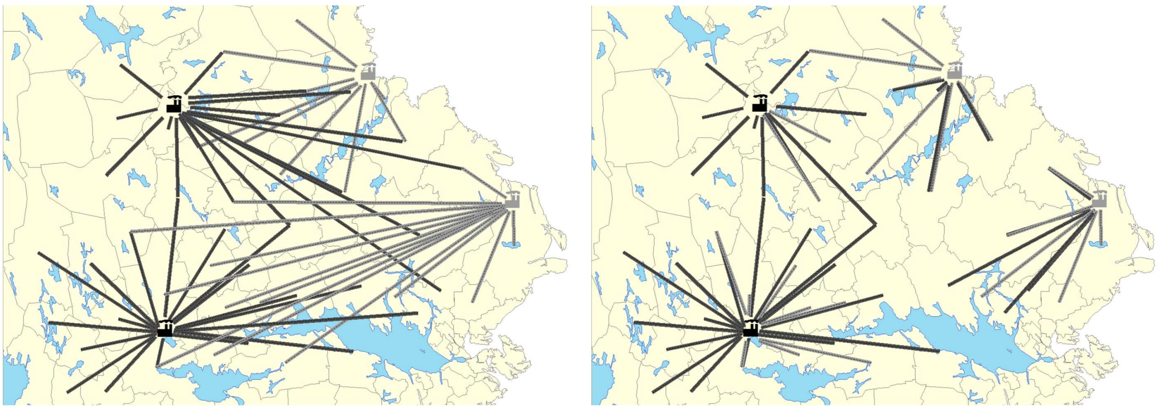

Figure 1: Illustration of wood bartering. In the left part each company operates by itself and in the right part both companies uses all supply points as a common resource.

Case study

The data used in this paper has been taken from a case study done by the Forestry Research Institute of Sweden for eight participating forest companies. The data is taken from transports carried out during one month. It involves all transports from the eight companies and includes information on from/to nodes, volume and assortment. There is a relatively large difference in size between the companies. In order to make a comparison, the same distance table and cost functions are used for both optimized and actual transportation carried out. There are several comparisons that are interesting and we compute the following results.

Real	Single company with real cost.
opt1	Full coordination with direct flows.
opt2	Single company with direct flows.

With the above results we can compare the potential of coordination by comparing (opt1) and (opt2). We can also estimate the overall savings by comparing (Real) with (opt2). We do not choose to use any backhauling in our comparison. This is because the information about the actual transportation does not include any information about the extent to which backhauling were used in practice. In table 1 the results from each of the scenarios is given. We note that the savings from solving for each individual company to a full coordination provides a saving of about 8%. If we also consider the potential savings as compared to the actual transportation we get a saving of about 14%.

Company	Real	opt 1	opt 2
Company 1	1,934	1,541	1,884
Company 2	3,894	3,894	3,778
Company 3	2,103	1,748	2,067
Company 4	333	303	333
Company 5	16,241	12,947	14,785
Company 6	5,084	4,363	4,959
Company 7	10,704	10,500	10,340
Company 8	4,828	4,067	4,742
Total	45,121	39,363	42,888

Table 1: Costs (in kSEK) for the real and optimization runs.

Economic models

In this section we describe a few economic concepts or models that have been used to distribute costs in various industrial areas. Each solution concept that can provide us with a cost allocation is said to satisfy a number of properties, i.e. fairness criteria. Below we list some of the most commonly used properties. We denote by a *coalition* S a subset of participants, and by the *grand coalition* N all participants. It is assumed that all participants have the opportunity to form and cooperate in coalitions. When coalition S co-operates, the total (or common) cost $c(S)$ is generated. In terms of co-operative game theory this cost function is called the characteristic cost function and each participant is called a player. We say that the cost allocation problem is formulated as a co-operative game.

A cost allocation method that splits the total cost, $c(N)$, among the participants $j \in N$ is said to be *efficient*, that is $\sum_{j \in N} y_j = c(N)$, where y_j is the cost allocated to participant j. A cost allocation is said to be *individual rational* if no participant pays more than its "stand alone cost", which is the participant's own cost, when no coalitions are formed. Mathematically, this property is expressed as $y_j \leq c(\{j\})$. The *core* of the game is defined as those cost allocations,

y, that satisfy the conditions

$$\begin{aligned} \sum_{j \in S} y_j &\leq c(S), \;\; S \subset N \\ \sum_{j \in N} y_j &= c(N) \;\; (\text{ efficiency}) \end{aligned}$$

That is, no single participant or coalition of participants should together be allocated a cost that is higher than if the individual or coalition would act alone. A cost allocation in the core is said to be *stable*.

Volume weighting

A straight forward and simple allocation is to distribute the total cost of the grand coalition, $c(N)$, among the participants according to a volume or a cost weighted measure. This is expressed by the formula $y_j = w_j c(N)$, where w_j is equal to participant j's share of the total transported volume. From table 2 we can see that the savings varies a lot between companies. In the column "Individual" the companies' costs of operating alone are shown. In the preceding columns we give the cost allocations obtained by the concepts and methods. For each computed cost allocation, the savings as compared to the individual costs are given. It is easy to understand the operational problems when the two largest companies saves 0,6% and 9,7%, respectively.

Company	**Individual**	**Volume**	**Savings**
Company 1	1884	1628	13,6
Company 2	3777	3424	9,3
Company 3	2066	1970	4,6
Company 4	333	285	14,4
Company 5	14785	13354	9,7
Company 6	4959	3953	20,3
Company 7	10339	10274	0,6
Company 8	4742	4195	11,5
Sum	42885	39083	8,9

Table 2: Distribution of costs based on volume distribution.

The Shapley value

The Shapley value is a solution concept that provides us with a unique solution to the cost allocation problem. The computation formula stated below expresses the cost to be allocated to participant j, and is based on the assumption that the grand coalition is formed by entering the participants into this coalition one at a time. As each participant enters the coalition, he is allocated the marginal cost, by which his entry increases the total cost of the coalition he enters. The amount a participant receives by this scheme depends on the order in which the participants are entered. The Shapley value is just the average marginal cost of the participants, if the participants are entered in completely random order. Using this concept we get the result given in table **??**. The differences in savings are not as large as with volume weighting. However, there are still large differences.

Company	**Individual**	**Volume**	**Savings**
Company 1	1884	1562	17,1
Company 2	3777	3597	4,8
Company 3	2066	1876	9,2
Company 4	333	315	5,4
Company 5	14785	13407	9,3
Company 6	4959	4501	9,2
Company 7	10339	9755	5,6
Company 8	4742	4070	14,2
Sum	42885	39083	8,9

Table 3: Distribution of costs based on volume distribution.

Equal Profit Method

In a negotiation situation it would be beneficial to have an initial allocation where the relative savings are as similar as possible for all participants. The relative savings of participant i is expressed as

$$\frac{c(\{i\}) - y_i}{c(\{i\})} = 1 - \frac{y_i}{c(\{i\})}$$

By the assumption, that a cost allocation is stable, we have that $c(\{i\}) \geq y_i$. Thus, the difference in relative savings between two participants, i and j, is equal to

$$\frac{y_i}{c(\{i\})} - \frac{y_j}{c(\{j\})}$$

We suggest a new method which is motivated by finding a stable allocation, such that the maximum difference in pairwise relative savings is minimized. We call this the Equal Profit Method (EPM). Using this concept we get the result given in table 4. This is the most equal possible.

Company	**Individual**	**Volume**	**Savings**
Company 1	1884	1712	9,1
Company 2	3777	3534	6,4
Company 3	2066	1878	9,1
Company 4	333	303	9,0
Company 5	14785	13439	9,1
Company 6	4959	4508	9,1
Company 7	10339	9398	9,1
Company 8	4742	4311	9,1
Sum	42885	39083	8,9

Table 4: Distribution of costs based on EPM.

Concluding remarks

Situations when several companies are cooperating will be more important in order to improve the transportation efficiency. Today, there exist systems that can establish coordinated plans. However, the coordination is limited to few companies that agrees on bartering volumes but not how to distribute savings. As more companies will be a part of the coordination it is not viable to agree only on volumes. We have studied a number of economic models, beside the methods above, in how the savings can be distributed taking various properties into account. In the paper we also perform studies in how a negotiation may work and be organised.

References

[1] Forsberg, M., M. Frisk and M. Rönnqvist. FlowOpt - a decision support tool for strategic and tactical transportation planning in forestry, *International Journal of Forest Engineering* **16**, 101-114, 2005.