Implementation of Model for Departure Time Choice

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Abstract

To change departure time is one of the most common responses by car travellers to new network conditions. Yet there are few transportation models taking departure time choice into account. In this paper the project SILVESTER (SImuLation of choice betWEen Starting TimEs and Routes) is presented. Based on stated preference and revealed preference data on travel behaviour of drivers in Stockholm, a departure time and mode choice model has been estimated in a mixed logit framework. In the second, ongoing stage of the project the estimated model is implemented and connected to a dynamic traffic assignment model. Through iterations between the departure time and assignment model the objective is to forecast effects of congestion charges and infrastructure investments on departure time choice.

Keywords: congestion charges, departure time choice, mixed logit, dynamic traffic simulation

1 Introduction

1.1 Background

The expansion of big cities has the last decades led to a considerable increase in traffic. More and more time is spent commuting and driving to leisure activities and shopping centres. Often, part of this travel time is queueing time. This is unfortunate, since time is spent without any benefit for society. Queues are also bad for the environment due to an increase of emissions.

In Stockholm, a trial of congestion charges has recently been carried out (January 3rd to July 31st, 2006) [2]. The aim of the trial was to evaluate if congestion charges can reduce over-all traffic in the city-centre and also to see if a more efficient usage of the transportation system can be achieved. Some roads are heavily congested in the morning and afternoon peak hour, but there is scope for more traffic during other parts of the day. To achieve this desired *peak-spreading effect*, the congestion charges were time-of-day dependent and higher in the peak periods.

Capacity expansion of roads is a more common measure than congestion charges. Increased capacity has however an effect on travel demand: the demand for travel increases with larger capacity. For a heavily congested road, the increased travel demand can outweigh the benefits of a capacity expansion. Furthermore, neglecting the overall extra demand, a capacity expansion also attracts traffic to the peak hour since some drivers switch back to their desired time of travel as travel times are reduced. Therefore, to reduce congestion, charges might be a better alternative than a capacity increase.

Some people will change their departure time to avoid charges. Also in cities without congestion charges, road-users are likely to depart at times other than their preferred departure time to avoid queues. There are however few travel demand models taking departure time choice into account. Even more rare is applications where a demand model including departure time choice and a dynamic assignment model interact to predict how travellers react to new travel conditions.

1.2 Objective

This paper is part of the ongoing project SILVESTER (SImuLation of choice betWEen Starting TimEs and Routes). In the first stage of the project a departure time and mode choice model was estimated by Maria Börjesson in [3] based on *stated preference(SP)* and *revealed preference(RP)* data on travel behaviour of drivers in Stockholm. The objective of the second, ongoing stage is to implement the model and connect it to a dynamic traffic assignment model. The contribution of the resulting application will be its ability to forecast effects of congestion charges and/or road network expansions on departure time choice. Since run times of the traffic assignment model are rather long for the network of Stockholm, a main issue will be to achieve relatively fast convergence.

1.3 Related Projects

The travel demand forecasting tool in use in Sweden is called SAMPERS [1]. The tool contains regional, long distance and international models. As traffic assignment model Emme/2 is used both for car and public transport. On the demand side SAMPERS contains models for car ownership, trip frequency, destination choice and mode choice divided by trip purpose. The long distance model also contains models for departure time choice and ticket type choice. A regional model including departure time choice that can forecast peak spreading effects is however not included in SAMPERS.

Recently (final report May 2005) a transport model called PRISM has been developed for the West Midlands region [14]. The model is based on the software package VISUM in which highway and public transport networks can be integrated and which has an interface to both GIS and the micro simulation model VISSIM. The network included in the model is divided into an external, intermediate and internal area and has a high level of zonal detail especially in the internal urban area (900 zones total and 574 zones for internal area). For the highway assignment in the internal area every junction is modelled and the demand side contain models for car ownership, public transport pass ownership, tour frequency, destination choice, mode choice and time of day choice. The demand models are discrete choice models estimated on data from home or road-side interviews. The time of day choice model is however rather coarse, since the 24 hour modelled period is divided into only four time periods. Thus only large time shifts are modelled, not any kind of peak spreading.

The Department for Transport in UK has developed a software called DIADEM (Development of Integrated Assign and DEmand Models) [6] that connects a variable demand model with a traffic assignment model (either CONTRAM or SATURN) and iterate between the two to come close to an equilibrium. DIADEM accounts for alterations in demand due to changes in choice of frequency, destination, mode and time of day. For time of day choice the choice between broader time periods (typically a couple of hours) can be modelled as part of an incremental nested logit model. There is however no model in DIADEM especially developed for choice between short time periods (fifteen minutes or less).

A project called HADES (Heterogeneous Arrival and Departure times based on Equilibrium Scheduling) has focused on peak spreading [13]. The approach used in HADES was not based on discrete choice modelling, but on a direct equilibration of demand and supply called equilibrium scheduling theory (EST), which follows the work of Vickrey [15]. An advantage of EST is that the demand profile is continuous over time, hence is not dependent of any division into time periods. The conclusion of HADES was however that future peak spreading models should be based on discrete choice methods since the EST implementation faced problems with inconsistencies between travel time gradients at the demand and supply side.

All projects described above except the PRISM project are trip-based. The argument against the traditional trip-based modelling is that the trip in it self does not derive demand, instead it is the activity at the destination that derive demand for travel. For the network of Portland, Oregon an activity-based model was developed during 1996 and 1997 [4]. The model start with determining a persons activity pattern given household and person variables. The activity pattern is a tour from home to a primary destination and back home again with possible intermediate stops inbetween. Departure time choice is included in the Portland model, but there are only five time periods during the whole day and no variables that capture schedule flexibility are included.

2 Modelling Departure Time Choice

2.1 Step Five

Traditionally, the urban transportation model is called a *four-step model*, since it models travel behaviour in four steps: trip generation, trip distribution, mode and route choice. The model discussed in this paper deals with a fifth step, namely departure time choice. This step describes the time-of-day dimension of travel behaviour and models the re-timing of journeys due to altered travel conditions.

Incremental changes in trip generation and distribution from the base scenario are not considered in the SILVESTER model. The model should therefore be used to forecast short-term projects, where the influence of changes in trip generation and distribution are small and the most common reactions from the travellers are to change route, departure time or mode. The mode choice is in the SILVESTER model included as the option to switch *to* public transport *from* car.

2.2 Model Formulation

The departure time choice model was estimated with the software Biogeme using both SP and RP data. After careful examination of the data, the population was segmented into three groups with respect to schedule flexibility and value of time:

1. commuters with flexible schedule and other trips,

2. commuters with fixed schedule and school trips,

3. business trips.

The representative utility for flexible commuters is:

$$\begin{aligned} U_{CAR,t} &= \beta_1 SDE_t + b_1 SDE730_t + \beta_2 SDL_t + \beta_3 Z_t + b_2 T_t + b_3 \sigma_t + \varepsilon_t, \\ U_{PT} &= C_{PT} + b_4 T_{PT} + b_5 SeasonTicket + \eta + \varepsilon_{PT}, \end{aligned} \tag{1}$$

where $t = 1, \ldots, 14$ is index of time period. The number of time intervals come from a division of the extended morning peak (6:30-9:30) into twelve time slices, each fifteen minutes long, plus one time period before and one after the extended morning peak. There are thus fifteen choices: to go by car and start in one of the fourteen time periods or to go by public transport. The utility function punishes a departure time (DT) different from the *preferred departure time (PDT)* through the variables *schedule deviation early (SDE)*[1] and *schedule deviation late (SDL)*:

$$\begin{aligned} SDE_t &= max(PDT - DT_t, 0), \\ SDL_t &= max(DT_t - PDT, 0). \end{aligned} \tag{2}$$

The other variables are: $SDE730$ - a dummy variable which is 1 if the trip starts in a time interval before 7:30 am, Z - congestion charges, T - car travel duration time, σ - car travel time uncertainty, T_{PT} - public transport travel durations (public transport travel durations are not time slice dependent), $SeasonTicket$ - a dummy variable which is 1 if the car user also has a season ticket for public transport and C_{PT} - an alternative specific constant for the public transport alternative.

The parameter for public transport cost (i.e. ticket cost) was not significant in estimation of the model. One reason for this is that cost is highly correlated with travel time in the public transport alternative.

Parameters labeled β differ in the population, i.e. not all individuals have the same preferences. These parameters are characterized by a distribution with a mean and a standard deviation. A *draw* from the distribution represents an individual of the population. By assuming ramdom parameters for the schedule deviation variables the desired larger correlation between adjacent time intervals is accomplished.

The error components ε are independent and identically distributed Gumbel terms, whereas η is normally distributed. η induces a larger variance in the choice between car and public transport than between two car alternatives. This is similar to dividing the alternatives into different nests, as is done in a nested logit model.

The utility functions for commuters with fixed working hours and for business trips look almost the same as Eq.1, except that for commuters with fixed working hours the extra disutility of early departure is before 7:00 am, and for business trips only a handful of respondents chose the public transport alternative and U_{PT} was therefore excluded. Of course, the values of the estimated parameters also differ between the three segments.

2.3 Mixed Logit

The departure time model is implemented in a *mixed logit* framework. Similarly to the *multinomial logit*, the mixed logit model determines probabilities for choice of various alternatives, in our case the departure time intervals (Eq.3). Mixed logit models contain

[1] The common term is *schedule delay early (late)*, but in this paper *deviation* will be used instead of *delay* since *delay* is a misleading word. SDE does not imply a delay, it implies departing or arriving earlier than what you prefer under ideal, free-flow, conditions. The concepts SDE and SDL thus deal with deviation not delay.

integrals that can not be solved analytically, i.e. no closed form exists. The integrals are instead solved through simulation. Simulation may require long run times depending on number of draws. This problem is however diminishing with the faster computers available today.

$$P_{\hat{a}} = \int \left(\frac{e^{V_{\hat{a}}}}{\sum_{a=1}^{15} e^{V_a}} \right) f(x)dx, \tag{3}$$

where V_a is the representative utility of alternative a, i.e. the utility function in Eq.1 but without gumbel terms, and $f(x)$ is a mixing distribution. It is common to use the normal distribution as mixing distribution, since it is easy to implement. It is however symmetric and allows positive values, which is unsuitable for travel cost parameters. Instead Johnson's S_B distribution is used in this project. Johnson's S_B distribution has rarely been used in demand modelling before. Also rare is the use of mixed logit for forcasting purposes.

An alternative to the mixed logit model could be to use an OGEV model. The OGEV model is suitable for choice of departure time since it allows the alternatives to be ordered and the closer the alternatives are the larger the correlation between them is. One benefit of the OGEV model is that a closed form exists and simulation is not required. The SP data used when estimating the SILVESTER model include several answers from the same respondent. It proved to be important to assume that, for the same individual, unobserved factors affecting departure time choice are correlated. This correlation could not be taken into account with a model from the GEV-family and an OGEV model could thus not be used.

For further reading on mixed logit models see Chapter 6 in [10].

3 Implementation

3.1 Equilibrium Between Demand and Supply

The model for departure time choice (Sec.2.2) is connected to an assignment model (Sec.3.3) and changes in departure times are calculated in an iterative procedure. An overview of the iterative process is shown in Fig.1.

The iterations between demand and supply are controlled by a monitoring program that skims and saves data and evaluates a convergence criterion. The aim is to approach equilibrium between demand and supply. To further illustrate how the simulation program works the different steps are shown in a flowchart (Fig.2).

3.2 Departure Time Model Implementation

Both the monitoring program and the departure time choice model is implemented in Matlab.

Input

In order to calculate the probabilities for choosing the different time periods and the public transport alternative we need to know the variables included in the representative

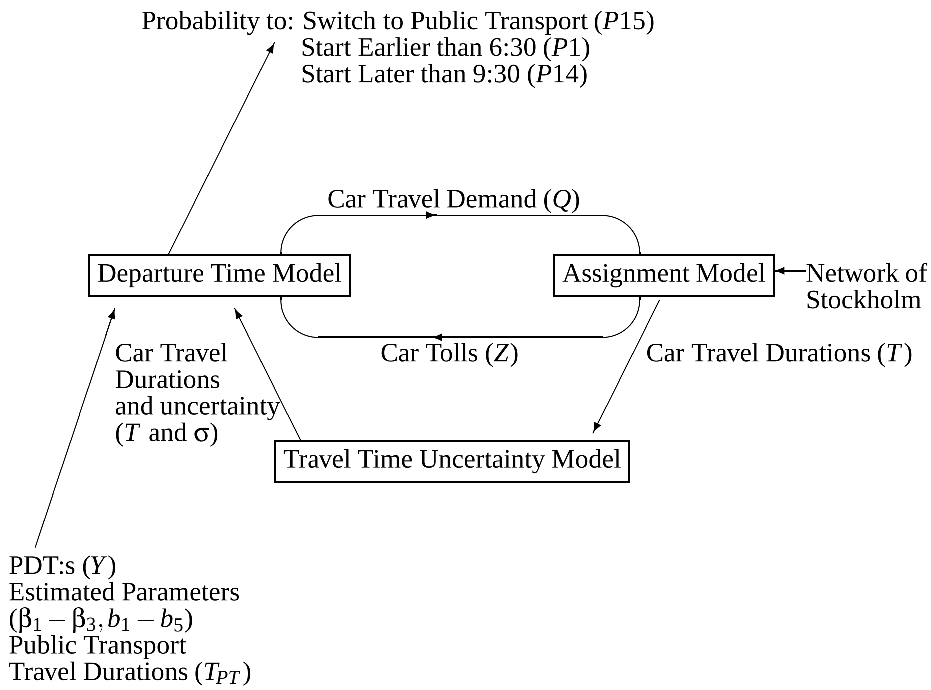

Figure 1: Schematic figure of departure time-assignment interaction.

utility functions (Eq.1). Once the preferred departure times are known, the size of the schedule deviation variables are given. The variable for early departure is determined by time period only. The season ticket variable is determined from a large travel survey carried out before and during the Stockholm trial [5], as the ratio of car travellers who also possess a season ticket. Public transport travel durations are constant over demand-supply iterations and are given from the Emme/2 public transport model for Stockholm. Car travel durations and tolls are on the other hand skimmed from the assignment model after each demand-supply iteration.

Output from the assignment model are travel durations and tolls for *routes*. These are converted to OD-level:

$$T_{dct} = \frac{\alpha_1 T_{r_1 t} + ... + \alpha_R T_{r_R t}}{R},\ Z_{dct} = \frac{\alpha_1 Z_{r_1 t} + ... + \alpha_R Z_{r_R t}}{R}, \tag{4}$$

where route $r_1, ..., r_R$ are the routes used by vehicles belonging to OD-pair d and user class c, and α_r is the ratio of vehicles choosing route r.

Remaining is the travel time uncertainty. This is the variable which is the hardest to get a hold of. Recently camera measurements of travel times have been made for a number of stretches in Stockholm, which has made it possible to estimate a relationship between *link* travel time uncertainty and travel duration and length of the link [11]:

$$\sigma_{lt} = e^{-1.92+0.086 Late+0.24 Speed70_l} \frac{T_{lt}^{1.2}}{L_l^{0.3}} \sqrt{\frac{T_{lt}}{T_{FF,l}} - 1}. \tag{5}$$

Late is a dummy for entering the link after 8:30, $Speed70_l$ is a dummy for links with speed limit 70km/h, T_{lt} is the travel duration of link l in time period t, L_l is the length of link l and $T_{FF,l}$ is the free flow duration of link l. Using this relationship the travel

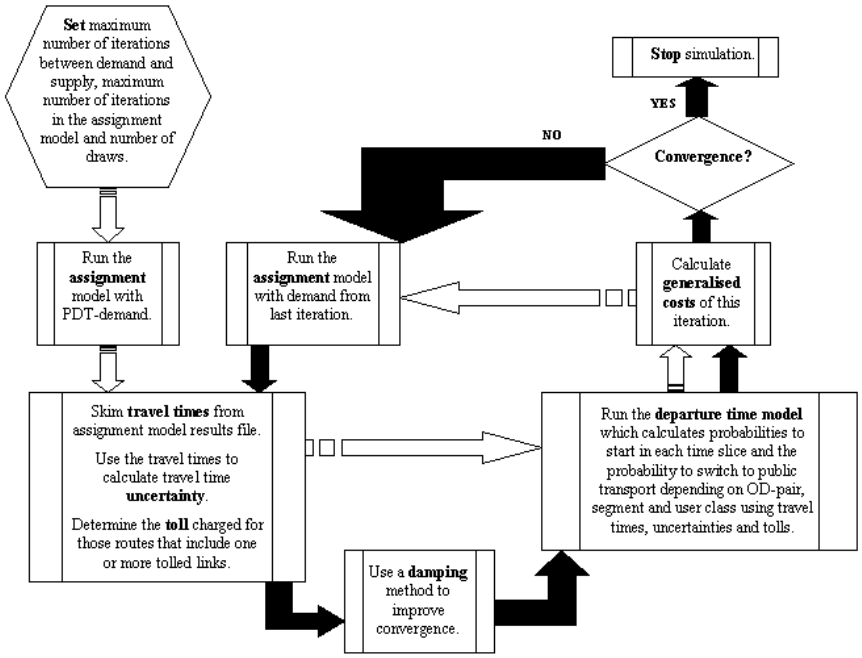

Figure 2: Flowchart of the whole simulation procedure.

time uncertainty is close to zero for uncongested situations (travel durations close to free flow durations) and grows as $T_{lt}^{1.7}$ for heavily congested situations (travel durations $\gg$ free flow durations).

The same report [11] also argues that the fault made when assuming that link travel times are uncorrelated is acceptable. In order to get travel time uncertainties for routes we thus assume that the link travel times are uncorrelated. The uncertainty for route r in time period t can then be calculated as:

$$\sigma_{rt} = \sqrt{\sigma_{l_1 t}^2 + \ldots + \sigma_{l_L t}^2}, \tag{6}$$

where $l_1, \ldots, l_L$ are the links used by route r. The travel time uncertainty is then converted to uncertainty for a certain OD-pair and user class in the same way as for travel durations and tolls (Eq.4).

Output

Each of the three departure time models calculate fifteen probability matrices $P1$ to $P15$. Element (dc, y) in $P1$ contains the probability of choosing to depart before 6:30 am, given OD-pair d, user class c and preferred departure time interval y. $P15$ contains the probabilities for the public transport alternative.

In the assignment model there are twelve time periods between 6:30-9:30 each fifteen minutes long. The first and last time period in the departure time choice model should thus be interpreted as before and after the peak in a broader sense than just fifteen minutes. Row dc of the demand matrix Q_p is calculated as:

$$Q_{pdc} = \left[\sum_{y=1}^{14} \left(P2_{pdcy} Y_{pdcy} \right), \ \dots, \sum_{y=1}^{14} \left(P13_{pdcy} Y_{pdcy} \right) \right],$$

where Y_{pdcy} is a matrix containing, for each OD-pair d, user class c and segment p, the number of *cars per hour* that has time slice y as preferred departure time interval. Before demand is sent to assignment summation is done over the three segments:

$$Q = \sum_{p=1}^{3} Q_p.$$

3.3 Assignment Model

The dynamic route choice model CONTRAM will be used as assignment model [8]. The assignment model needs to be dynamic, since duration of a trip that departs in some time interval may depend on traffic conditions in an earlier or later time interval. CONTRAM is a *mesoscopic model* , i.e. intermediate between *macroscopic* and *microscopic*. It combines a macroscopic model that describes the network state through aggregate quantities such as average arrival rate on a link with a microscopic model that routes packets through a detailed network in which traffic signals, roundabouts, etc. are coded.

A packet consists of demand that have the same origin and destination, belong to the same user class and start in the same time period. All demand in one packet will use the same route. Since the best route for one packet depends on flows on the links, i.e. decisions of other packets, the process of assigning packets to the network must be repeated in order to approach a user equilibrium.

In each iteration in CONTRAM several convergence measures are calculated:

$$\begin{aligned} RMS &= \sqrt{\frac{1}{LT} \sum_{l=1}^{L} \sum_{t=1}^{T} (q_{ltk} - q_{ltk-1})^2}, \qquad (7) \\ AAD &= \frac{1}{LT} \sum_{l=1}^{L} \sum_{t=1}^{T} |q_{ltk} - q_{ltk-1}|, \\ \%AAD &= \frac{100}{LT} \sum_{l=1}^{L} \sum_{t=1}^{T} \frac{|q_{ltk} - q_{ltk-1}|}{q_{ltk-1}}, \end{aligned}$$

where q_{ltk} is flow on link l in time period t and iteration k. The available stability criteria are in the order listed in Eq.7: route mean square change ($< 5veh/h$), average absolute difference ($< 1veh/h$) and percentage relative average absolute difference ($< 1\%$). Also supplied is the percentage of links whose flows change with less than five percent ($> 95\%$). Minimum requirements for convergence are given inside brackets.

CONTRAM includes time-dependent queuing methods to realistically model the build up and dissipation of queues. The queues start at the stop line of a junction and build up vertically. So called blocking back effects are modelled by comparing the length of the queue with the storage capacity of the link. When 80% of the link storage capacity is reached, CONTRAM reduces the throughput capacity of the upstream links. The queuing model is thus relatively advanced. One thing it does not handle though is shock waves. This means that in the model there is a free space at the upstream node of the link exactly when a packet leaves the downstream node, whereas in reality the free space moves backwards like a wave and it takes some time before it reaches the entrance of the link.

Input to CONTRAM is a network with appropriate characteristics and a demand matrix (Q). For the CONTRAM network, without the departure time choice model, a demand matrix has been estimated using traffic counts. The generalised cost function used during this estimation was:

$$C_{ijtc} = 60T_{ijtc} + 0.5D_{ijtc} + Z_{ijtc}, \tag{8}$$

where T is travel duration time, D distance and Z congestion charge. Z was zero during this matrix estimation, since the demand matrix was calibrated for the situation without charges.

4 Performance and Run Time

For an application to be user-friendly the run time should be not be much longer than fifteen hours (it is then possible to start a simulation run at the end of the day and it will be finished when you come back next morning). With a restriction on the run time we want to achieve best performance possible. There are several factors that affect performance, e.g. the number of: *OD-pairs*, *user classes*, *draws in the mixed logit model*, *iterations in CONTRAM* and *iterations between demand and supply*. Increasing any of these factors will (presumably) make the application results more accurate, but it will also increase the run time. The different factors will be described further below.

4.1 Reducing Number of OD-pairs Using the Fratar Method

The original demand matrix consists of 90770 OD-pairs. This very large OD-matrix gives rise to long run times in CONTRAM (about 8 hours for seven iterations). Furthermore this is just for one user class. If we want to have more user classes (see Sec.4.2 to why we want that) all OD-pairs will be counted one time for each class.

The monitoring program also has a run time which is dependent on number of OD-pairs. This is because of the calculation of travel time uncertainty for each OD-pair. For 15970 OD-pairs this run time is 15.6 minutes, but just as for CONTRAM, the increase in run time due to more OD-pairs is faster than linear.

There is thus a lot to gain in run time by reducing number of OD-pairs. Many of the entries in the demand matrix are very small and with a relatively small threshold OD-pairs can be removed while still keeping most of the demand (Tab.1).
One must however be careful to remove the small entries and have to spread out the removed demand in a clever way, since many links have a flow with contributions from many small demand sources. There is otherwise a risk that these links get zero flow. It is also important to check that the trip distance distribution remains approximately the same.

First one method is tested in which OD-pairs are removed, and the demand spread out by multiplying the whole matrix by number such that the total number of trips is still the same as before the reduction. That is, we use a uniform growth factor and apply it to all elements in the OD-matrix. This method turned out not to be so good, since trip origins and destinations move from zones in the inner city with little demand to zones in the outer city with large demand. This will thus change trips from short to long and move traffic from small inner city streets to highways.

Trip Threshold	♯ Active OD-pairs	Percent of Trips Removed
0,2	70531	0,7
0,5	50637	3,2
1	35120	7,3
2	21906	14,3
3	15970	19.8
4	12400	24,4
5	10010	28.4

Table 1: OD-pairs with number of trips starting during the whole morning period less than the trip threshold are removed.

Instead we use the Fratar-method to adjust the new matrix, such that origin and destination sums will coincide almost with the original ones. This is the method of doubly constrained growth factors. The procedure is iterative and either the row or the column sums can meet their target perfectly, at the same time as the other one is kept close to the target. We choose to meet column sum targets perfectly and to keep row sums within 1% of their target values.

The reduction of OD-pairs will sometimes result in that no trips start and/or end in some zones. For link flows that are made up of demand from these zones the Fratar method does not help. These zones should therefore be as few as possible. Tab.2 shows the number of times demand from or to a zone has become zero after demand have been spread out using the Fratar method, this for different number of active OD-pairs. In brackets are the total demand (in number of trips) disregarded when the start/end zones become zero.

♯Active OD-pairs	♯ start zones (trips)	♯ end zones (trips)
70531	0 (0)	0 (0)
50637	2 (3.0)	2 (5.2)
35120	3 (18.0)	2 (5.2)
21906	4 (41.2)	3 (11.2)
15970	5 (45.6)	5 (58.7)
12400	10 (263.2)	6 (147.0)
10010	19 (944.2)	7 (193.1)

Table 2: Number of zones with zero trips starting and ending respectively. Affected ♯trips in brackets.

With the Fratar method, as opposed to the uniform growth method, the total number of trips is not exactly reproduced. The total number of trips in the original matrix was 267497. Using the Fratar method the total number of trips will decrease with the number of trips disregarded at the end zones, this since we have chosen to meet the trip end targets exactly (except for the disregarded demand).

4.2 User Classes Depending on Value of Time

If there are several routes from an origin to a destination and they have similar costs, then demand for travel will split between these routes. In a system with no congestion charges this does not imply any problems. Since the routes are perceived almost equally good to the drivers, the travel times are also very similar and we can use a mean value of the route travel times as travel time for that OD-pair in the departure time model.

However, if charges are added to the system the situation becomes a little bit more tricky. The generalised cost for route choice (Eq.8) still has about the same value for all used routes, but the travel duration times do not have to be similar anymore. On the contrary, the situation can be one where some drivers choose a route which is free but has a long travel time, whereas some choose a tolled but fast route. The choice of route thus depends on each drivers value of time (VOT). To use a mean value of route travel times would in this situation be a coarse calculation.

Instead the three departure time models (flex, fixed and business) are modified such that they calculate probabilities to start in a certain time period depending on user class. Since the cost parameter differs in the population, each draw from the cost parameter distribution belongs to a user class depending on its value of time:

$$VOT = 60\frac{b_2}{\beta_3^n}, \tag{9}$$

where b_2 is the deterministic coeffient for travel time and β_3^n is a draw from the distribution for the cost parameter. Multiplication by 60 is done to get VOT per hour instead of per minute. The distribution of value of time is shown in Fig.3. The draws can be

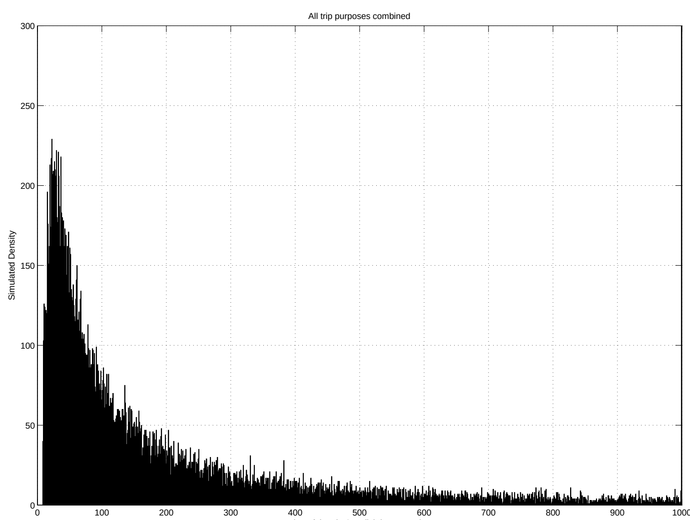

Figure 3: Value of time distribution for the three segments combined.

seen as representing different individuals. In the iterations between demand and supply an equilibrium should be sought with respect to the same individuals. We should therefore not generate new random numbers[2] (individuals) in each iteration, rather the random numbers of the first iteration need to be used in comming iterations.

[2]These are pseudo random numbers generated with the command `randn` in Matlab and transformed from the normal distribution to $Johnson's\ S_B$ distribution.

There are a portion of the population with very high value of time who in this case can be said to be price insensitive: they will not change departure time or mode for any of the amounts in question for a congestion charge. It is important to capture this by having atleast two user classes. Otherwise the mean travel time and mean toll for a cheap but slow route and a tolled but fast route will be a coarse approximation to the network conditions for vehicles travelling from a certain origin to a certain destination. The benefit of more than two user classes is more uncertain.

4.3 Draws in the Simulated Mixed Logit Model

As mentioned in Sec.2.3 the departure time choice model calculates choice probabilities through simulation and the more draws the more accurate the result becomes. However, run times of the departure time choice model increase with number of draws (Tab.3 and Fig.4) and a trade off between accuracy and speed has to be made. The simulations are run on a Dual 3.0 GHz Intel Pentium 4 with 1527 MB memory.

Number of Draws	Run Time (min)
10	0.72
30	1.82
50	2.92
70	4.01
Least squares approx.	$t = 0.055 \cdot d + 0.18$

Table 3: Departure time choice model run times for different number of draws. Commuters with flexible schedule and other trips.

As can be seen from Fig.4 the increase in run time is linear. How long the run time is for a given number of draws is determined by number of OD-pairs, which here was 15970 (one user class with 15970 OD-pairs, i.e. the trip threshold of three trips during the whole morning was used). The models for the three groups will have to be run one after the other, thus approximately trebling the run time. For the business group the run time is somewhat shorter since this model does not include the public transport alternative.

4.4 Iterations in CONTRAM and Iterations Between Demand and Supply

Iterations in CONTRAM are performed to approach a user equilibrium. For each iteration the convergence measures of Sec.3.3 improve, but each additional iteration implies additional run time. The same goes for iterations between demand and supply. The latter is also dependent on CONTRAM convergence, since the application will presumably approach a demand-supply equilibrium faster if the route choice has converged reasonably well.

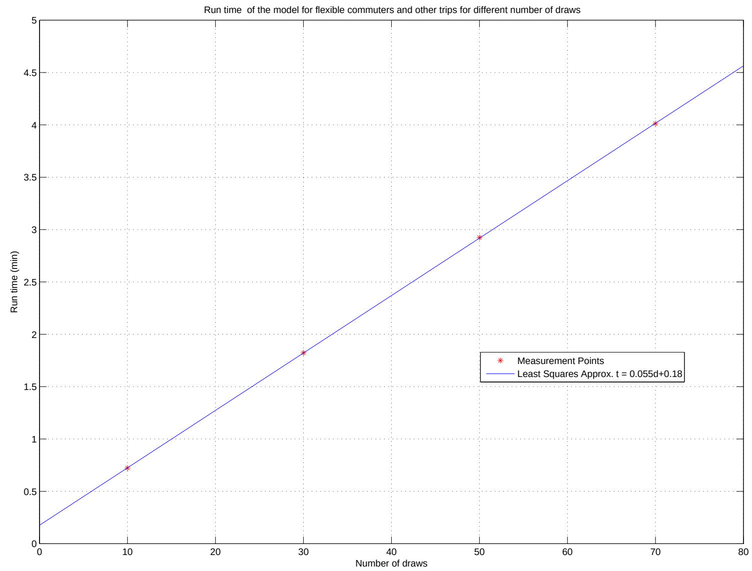

Figure 4: Linear dependence between runtime and number of draws.

4.5 Performance Criterion

We will decide how many user classes, draws in the departure time models, iterations in CONTRAM and iterations between demand and supply to use by comparing the effects of different parameter settings on resulting performance, which we measure by Eq.10:

$$E \quad = \quad 100 \cdot \frac{\sum_{ijtc} Q^I_{ijtp} |C^A_{ijtp} - C^I_{ijtp}|}{\sum_{ijtp} Q^I_{ijtp} C^I_{ijtp}}, \qquad (10)$$

where I is an "ideal" case in which all parameters take on their highest values (Tab.4) and A is an approximate case under evaluation. Eq.11 shows the expression we use for generalised cost:

$$C_{ijtp} = \frac{\bar{\beta}^p_1 SDE_{ijtp} + b^p_1 SDE730_{ijtp} + \bar{\beta}^p_2 SDL_{ijtp} + \bar{\beta}^p_3 Z_{ijtp} + b^p_2 T_{ijtp} + b^p_3 \sigma_{ijtp}}{\beta^p_3}, \qquad (11)$$

where mean values are used for the random parameters β. The generalised cost is thus not dependent on user class, which is important since we want to evaluate the effect of different number of user classes.

From the ideal run we get a reference value C^I to aim towards. Its very important that this ideal run has converged properly and it is possible that we will continue the demand-supply iterations above the highest value (12) if convergence it not satisfactory.

First an approximation where all parameters take on their lowest values is evaluated and then the parameters are varied one at the time to capture the influence of a certain

Parameter	Lowest Value	Highest Value
♯User Classes	2	5
♯Draws	20	200
♯Iterations in CONTRAM	6	10
♯Iterations in D/S	4	12

Table 4: Parameter settings

parameter on resulting performance. For all runs the value on E and the run time will be noted down.

5 Convergence

In the end, the purpose of the application developed in this project is to calculate socio-economic benefits of congestion charges or road investments, taking departure time choice into account. One measure of consumer surplus of a strategy compared to a do nothing scenario is the so called *rule-of-a-half*[3] [7]:

$$\Delta S = \sum_{ijtp} (C^1_{ijtp} - C^2_{ijtp}) \frac{\left(Q^1_{ijtp} + Q^2_{ijtp}\right)}{2} + \sum_{ijtp} Z_{ijtp}, \qquad (12)$$

where C^1 is generalised cost in the do nothing scenario, C^2 is generalised cost in the scenario with congestion charges and Z is amount of collected charges. The collected amount need to be added since it can be used for purposes beneficial to society.

As a measure of closeness to convergence during a simulation run of our program there are several candidates:

$$\begin{aligned} e1_k &= 100 \cdot \frac{\sum_{ijtp} C_{ijtp,k} |Q_{ijtp,k} - Q_{ijtp,k-1}|}{\sum_{ijtp} C_{ijtp,k} Q_{ijtp,k-1}}. \;\; or \qquad (13) \\ e2_k &= 100 \cdot \frac{\sum_{ijtp} Q_{ijtp,k} |C_{ijtp,k} - C_{ijtp,k-1}|}{\sum_{ijtp} Q_{ijtp,k} C_{ijtp,k-1}} \;\; or \\ e3_k &= 100 \cdot \frac{\sum_{ijtp} Q_{ijtp,k} (C_{ijtp,k} - C_{ijtp,k-1})^2}{\sum_{ijtp} Q_{ijtp,k} (C_{ijtp,k-1})^2}. \end{aligned}$$

All three measures are in percent. In DIADEM $e1$ is used and it is called the percentage demand/supply gap (%*GAP*). Recommended level for convergence is a value of %*GAP* which is less than 0.2%. Also recommended is that the consumer surplus, expressed as percentage of total network costs, should be more than ten times larger than %*GAP* [12].

The measures $e2$ and $e3$ are however more similar to the expression for consumer surplus (Eq.12). As a measure of how close the simulation is to convergence we will therefore use $e2$. The generalised cost C in Eq.13 is the same as in Eq.11.

In order to reach convergence in a reasonable number of iterations some form of damping, in which the present solution from assignment is combined with the solution of the previous iteration, is needed. The previous solution becomes more and more

[3] In future benefit assessments we will have to evaluate if more sofisticated methods than the rule-of-a-half shall be used.

trustworthy for each iteration. A common way to combine the two solutions is therefore the *method of successive averages* (MSA):

$$\begin{aligned} T_k &= \hat{T}_k \cdot \lambda_k + T_{k-1} \cdot (1-\lambda_k), \\ Z_k &= \hat{Z}_k \cdot \lambda_k + Z_{k-1} \cdot (1-\lambda_k), \\ \sigma_k &= \hat{\sigma}_k \cdot \lambda_k + \sigma_{k-1} \cdot (1-\lambda_k), \\ \lambda_k &= \frac{1}{k}. \end{aligned} \tag{14}$$

There are other damping algorithms as well, but to evaluate them is a whole project in its own. MSA is the most established method and will therefore be used in this implementation.

6 Conclusions

One question important to this paper is whether to model departure or arrival time. Since the respondents choose departure time on the basis of their *preferred arrival time(PAT)* one could argue the PAT should be used instead of PDT. This is however most important to travellers with fixed working hours, which is a small group in our sample. In addition, when travel time is uncertain, the travellers can not choose arrival time, only departure time. Modelling departure time is also easier from an implementation perspective, since the assignment model CONTRAM takes demand partitioned into departure time intervals as input. We have on the basis of what has been said above chosen to model departure time. The PDT:s will have to be adjusted if travel durations change a lot, this to preserve the PAT-distribution.

In Sec.4.1 we saw that if we remove OD-pairs with very little demand and raise the remaining matrix with a percent such that total demand is the same, then demand is shifted from inner city start and end zones to start and end zones in the outer region. Thus, traffic is transfered from small inner city streets to large approach roads. We therefore conclude that the uniform growth method does not work well in this case and the Fratar method shall be used instead, which proved to work well.

When congestion charges are added to the network we cannot use the same mean travel duration for everybody anymore, since the travel durations are no longer similar. We thus conclude that more than one user class is needed. Which user class an individual (draw) belongs to depends on its value of time.

From Tab.3 one can conclude that the implementation of the departure time choice model is fast enough. The increase in run time due to more draws is linear. Since CONTRAM does not calculate travel time uncertainty, we have to build an own model and calculate the uncertainty before it is sent to the departure time choice model. This travel time uncertainty calculation is heavily dependent on number of OD-pairs and the run time can become crusial for large OD-matrices and/or many time value classes.

7 Future Work

7.1 Ideal Starting Times

In Sec.3.2 it was not explained how to calculate the matrix of preferred departure times Y. For a congested situation they cannot be observed, since it is not certain that they are equal to actual starting times. Another option would be to ask travellers about their preferred time of travel, but this kind of survey is both expensive and unreliable. Instead the PDT:s will be estimated using a reverse engineering approach. The approach is described for one OD-pair in [9]. In this project the reverse engineering approach will be applied to the whole network of Stockholm. The application of reverse engineering to so many OD-pairs has not been done before.

The reverse engineering idea is as follows: for a base scenario use observed number of departures in each time period together with a departure time model (for example the one described in this paper) and calculate ideal starting times ”backwards”. The ideal starting times can be revealed from the observed ones since the departure time model contains information on how travellers trade-off travel time and deviation from ideal starting time. The next step is to use the ideal starting times in order to calculate actual starting times in an updated scenario, for example one with congestion charges added to the base situation.

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Nomenclature

Variable	Description
b	Deterministic coefficients
β	Random coefficients
C	Generalised cost
D	Distance
e	Deviation between iterations
ε	Unobservable part of utility, Gumbel distributed
η	Unobservable part of utility, Normally distributed
P	Probability
Q	Traffic flow matrix (demand matrix)
Z	Toll (congestion charge)
S	Socio-economic-benefit
SDE/SDL	Schedule deviation early/late
σ	Travel time uncertainty
T	Travel duration time
U	Random utility
V	Observable part of utility

Table 5: Description of Variables

Index	Description
a	alternative
$\hat{a}$	a certain alternative
c	class
i	origin
j	destination
k	iteration
l	link
d	OD-pair
n	decision maker
p	trip purpose
r	route
s	scenario
t	time interval
y	preferred time interval

Table 6: Description of Indices