Vendor–Managed Inventory Policies for an Integrated Production–Distribution System

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The integration of production, distribution and inventory management is one of the challenges of today's competitive environment. In the last decade the importance of the relations between internal management and external environment has been widely recognized, and the expression "supply chain management", which emphasizes the view of the company as part of the supply chain, has become of common use. Sometimes the expression "coordinated supply chain management" is used to emphasize the coordination among the different components of the supply chain. The availability of data and information tools which derive from the advances in technology and communication systems has created the conditions for the coordination inside the supply chain. Different models for the integration of decisions of the production, inventory and distribution functions have been proposed. We refer to Cohen and Lee (1988) for a strategic stochastic model. Thomas and Griffin (1996) review the coordination issue of the three functional areas at an operational level when deterministic models are used. Blumenfeld et al (1985) analyze the trade-offs between transportation, inventory and production set-up costs over an infinite time horizon. Sarmiento and Nagi (1999) and Erengüç et al (1999) review the integration between production and transportation. Chandra and Fisher (1994) propose a computational study to evaluate the value of coordination between production and distribution planning over a finite time horizon. Hall and Potts (2003) study the problem of minimizing the scheduling and delivering costs in the supply chain. An introduction to both deterministic and stochastic inventory routing problems can be found in Campbell et al (1998). A decomposition approach has been applied to the solution of a complex inventory routing problem by Campbell and Savelsbergh (2004).

The availability of new information technologies has also led in the last years to the development of new forms of relationships in the supply chain. One of these is the so called Vendor–Managed Inventory (VMI), in which the supplier monitors the inventory of each retailer and decides the replenishment policy of each retailer. The supplier is responsible of the inventory level of each retailer and acts as a central decision–maker; therefore, she/he has to solve an integrated inventory–routing or production–inventory–routing problem. The advantage of the application of a VMI policy with respect to the traditional retailer–managed inventory policies relies in a more efficient utilization of the resources: The supplier can reduce its level of inventories maintaining the same level of service or can increase the level of service and can reduce the transportation cost by a more uniform utilization of the transportation capacity. On the other hand, the retailers can devote fewer resources to monitor their inventories and to place orders, having thus the guarantee that no stock-out will occur. We refer to Bertazzi, Paletta and Speranza (2005) for a literature review about applications of the VMI policies to systems with stochastic demand.

We study an integrated model in which several products are produced at a production facility and shipped to several retailers over a finite time horizon by applying a deterministic VMI policy. Shipments from the production facility to the retailers are performed by a fleet of vehicles. Each vehicle has a given transportation capacity. Each item is absorbed by the retailers in a deterministic and time-varying way. Each retailer determines a maximum and a minimum level of the inventory of the products and can be visited several times during the time horizon. The production policy, the retailers replenishment policies and the transportation policy have to be determined so as to minimize the total system cost. The cost includes the fixed and variable production costs at the facility, the inventory costs at the facility and at the retailers and the routing costs. We study two different types of VMI policies: *Maximum Level* (ML) and *Order–Up to Level* (OU). In the former type of policies, if the retailer i is visited at time t, the quantity delivered to i is such that the level of the inventory in i is not greater than its maximum level. In the latter type of policies, if the retailer i is visited at time t, then the quantity shipped to retailer i at time t is such that the level of the inventory in i reaches exactly its maximum level. OU policies are inspired, in a deterministic setting, by the classical stochastic order-up-to level policy, widely studied in inventory theory. We refer to Bertazzi, Paletta and Speranza (2002) for an application of the deterministic order-up-to level policies to an inventory-routing problem and to Bertazzi, Paletta and Speranza (2005) for an application of these policies to an integrated production–distribution system.

The scope of the paper is five-fold. The problem we study is NP-hard, both when the ML and the OU policies are applied, since it reduces to the VRP in the class of instances in which the time horizon is made by one time instant only, the fixed and variable production costs, the inventory costs are zero and all the retailers need to be served. Our first aim is to investigate the computational complexity of the problem when the transportation is outsourced. We prove that in this case the problem with the ML policy can be solved in polynomial time even if all the other cost components are included in the objective function, while the problem with the OU policy is NP-hard even if the inventory cost at the retailers only is included. Our second aim is to show the worst–case performance of the OU policy with respect to the ML policy. We prove that, in the worst case, the ratio between the optimal cost of the former policy and the optimal cost of the latter one tends to infinity. Therefore, the solution obtained by solving the problem with the OU policy can be very suboptimal if used as heuristic solution of the problem with the ML policy. The problem with the OU policy has been heuristically solved in Bertazzi, Paletta and Speranza (2005), while the problem with the ML policy has never been solved. Therefore, our third aim is to solve this problem. Since it is very complex and the exact solution would be impractical in general, we propose a heuristic algorithm to solve it. In particular, we decompose the problem into two subproblems, one concerning the production and one concerning the distribution. We first solve the distribution subproblem and then the production subproblem. The subproblem concerning the production is optimally solved, while the subproblem concerning the distribution is solved by applying a constructive heuristic algorithm in which at each iteration a retailer is inserted in the solution. For each retailer, a problem, which is a generalization of the dynamic lot–size problem with time–varying capacity constraints studied in Baker et al (1978), is optimally solved. The exact algorithm we propose is based on properties of the optimal solution and on feasibility and dominance relations among partial solutions proved in the paper. Finally, the solution obtained by hierarchically solving the two subproblems is improved by applying two procedures which coordinate production and distribution. Our fourth aim is to evaluate the performance of this heuristic algorithm. We implement a branch–and–cut algorithm for the solution of the problem in which one vehicle only can be used at each delivery time instant and compare the optimal solution of this problem with the solution generated by the heuristic on the basis of specific randomly generated problem instances with one vehicle. Finally, we compare the performance of the OU policy with respect to the ML policy. Computation results, obtained on a large set of randomly generated problem instances with several vehicles, show that the ML policy allows us to significantly reduce the total cost with respect to the OU policy.

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