Congestion Pricing: Gaining Public Acceptance

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1 Introduction

Congestion pricing has been around for over 80 years and many have since recognized it as an efficient method for regulating congestion. Furthermore, recent advent of electronic tolling makes congestion pricing more practical and many successful implementations exist world wide. On the other hand, getting the public to accept congestion pricing is still an obstacle. Congestion pricing schemes proposed for Hong Kong, Cambridge (England) and Edinburgh were not implemented. Ten or more road pricing proposals were "largely abandoned" in United Kingdom (see [7]).

While there are many reasons why congestion pricing often fails to gain public acceptance, transportation economists (e.g., [5]) have pointed out recently that pricing schemes advocated in the literature such as marginal cost (MC) pricing are "most likely doomed to be political failures" because users will find themselves worse off. Governmental or transportation authorities are generally the only stakeholders who are better off because of the toll revenue they collect.

To make congestion pricing more appealing, we propose an approach less extreme than marginal cost and other forms of first-best pricing. These schemes use tolls as a mean to encourage users to use routes that result in a system optimal flow distribution, i.e., one with the least travel delay possible. As pointed out in [5], this approach creates inequity, in that many users are worse off when compared to the situation without pricing. Our pricing scheme, instead, tries to reduce delay while ensuring some users are better off and no one is worse off. Although such a scheme does not necessarily lead to a system optimal flow pattern, it has a better chance of being accepted (implemented and, eventually, reducing delay) because no one is made worse off.

We assume herein that the travel demands are fixed. (See [8] for the elastic demand case.) For the remainder, Section 2 illustrates with an example the inequity in MC and other forms of first-best pricing schemes. Section 3 introduces notation and defines dominating flow distributions. Under such flow distributions, some users are better off and no one is worse off when compared to the distribution under user equilibrium. In [1] and [4], the authors refer to a dominating flow distribution as a Generalized Braess Paradox and propose models and an algorithm for finding them. Section 4 defines conditions under which a nonnegative toll vector is Pareto-improving and establishes existence conditions for such a vector. Finally, Section 5 gives conclusions.

2 Inequity in First-Best Pricing Schemes

To illustrate the inequity in MC and other forms of first-best pricing, consider the network in Figure 1 (see [4]) in which there is only one OD pair (1, 4) with a demand of 3.6.

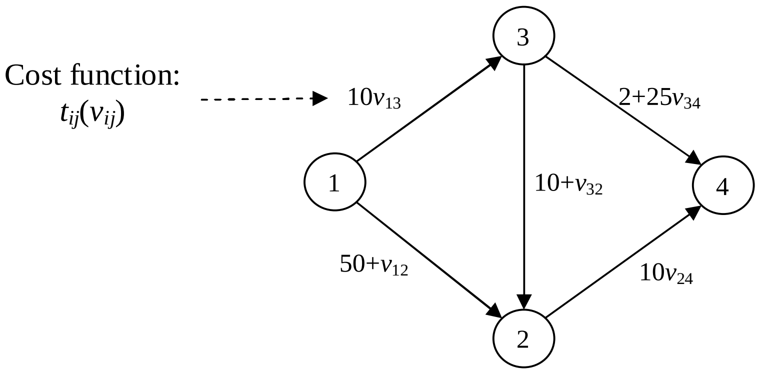

Figure 1: A five-link network from [4]

Table 1 displays the link and path flows under user equilibrium (UE) and MC pricing along with the associated costs. Recall that tolls under MC pricing are of the form $t'_a(v_a)v_a$, where $t'_a(v_a)$ is the derivative of the travel time function, $t_a(\cdot)$, and v_a is the system optimal flow on link a.

	UE		MC Pricing			
Links	Flow	Time	Sys. Opt. Flow	Time	Toll	Gen. Cost
(1, 3)	3.6	36.00	2.0641	20.641	20.64	41.28
(1, 2)	0	50.00	1.5359	51.5359	1.54	53.07
(3, 2)	2.2778	12.28	0.8956	10.8956	0.90	11.79
(3, 4)	1.3222	35.06	1.1685	31.2125	29.21	60.43
(2, 4)	2.2778	22.78	2.4315	24.315	24.32	48.63
Paths						
1-3-4	1.3222	71.06	1.1685	51.85	49.85	101.70
1-3-2-4	2.2778	71.06	0.8956	55.85	45.85	101.70
1-2-4	0	72.78	1.5359	75.85	25.85	101.70
Costs to users		255.82		227.11	139.02	366.13

Table 1: Flow distributions under UE and MC pricing for the five-link network

When implemented, MC pricing forces 1.5359 users to use path 1$\rightarrow$ 2 $\rightarrow$ 4 with a total cost of 101.70, of which 75.85 is the travel time and the rest (25.85) is for tolls (measured in time units). Thus, these 1.5359 users suffer twice, once for having to use a longer route (75.85 instead of 71.06) and the other for having to pay tolls. Overall, the total cost to the 3.6 users under MC pricing is 366.13 which is more than the total cost (225.82) under UE, a cost consisting entirely of time or delay. However, MC pricing yields less total delay (227.11) and generates toll revenue

(139.02) for the transportation authority. Thus, under MC, every user is worse off and the only one better off is the transportation authority.

3 Dominating Flow Distributions

Instead of trying to achieve a system optimal distribution, we consider a flow distribution with less delay than that under UE. We say that a flow distribution "dominates" the UE distribution if it strictly improves a measure of system efficiency and allows all users to use routes that are no longer (in travel time) than those under UE. (For brevity, we also say that a flow distribution is "dominating" if it dominates the UE distribution.) Below, we consider total delay as the measure of system efficiency (see [8], for other measures).

3.1 Feasible Flow Distributions

Let V^F be a set consisting of all feasible flow distributions, each of which is denoted as v. In particular, $v \in R^n$ and $V^F \subseteq R^n$, where n is the number of links in the transportation network. The set V^F can be described using either path or link flow variables. Using the former, let f_r^w and d_w denote the amount of flows on path (or route) r and the demand for OD pair w, respectively. Then,

$$V^F = \{v : v_a = \sum_w \sum_{r \in P^w} \delta_{ar} f_r^w; \sum_{r \in P^w} f_r^w = d_w, \forall w; f_r^w \geq 0, \forall w, r\},$$

where P^w is the set of paths for OD pair w and δ_{ar} (equals 0 or 1) indicates whether arc a is on path r. Alternatively, let A be the node-arc incidence matrix for the network and E^w denote a vector in R^m, where m is the number of nodes. The vector E^w has exactly two non-zero components, one has a value 1 in the component corresponding the origin node of OD pair w and the other has a value -1 in the component for the destination. Then, V^F can be written as

$$V^F = \{v : v = \sum_w x^w, Ax^w = E^w d_w, x^w \geq 0, \forall w\}$$

where $x^w \in R^m$ is a vector whose components are link flows for OD pair w.

3.2 Finding Dominating flow Distributions

Mathematically, we say that a flow distribution $v \in V^F$ dominates a given UE distribution or is dominating if v and its path flows f_r^w satisfy the following conditions for every OD pair w:

$$\sum_a \delta_{ar} t_a(v_a) \leq c_w^{UE}, \quad \forall r \in P_+^w(v) \tag{1}$$

$$t(v)^T v < t(v^{UE})^T v^{UE} \tag{2}$$

where c_w^{UE} denotes the travel time for OD pair w under UE, $t(\cdot)$ denotes a vector of link travel functions, $t_a(\cdot)$, $P_+^w(v) = \{r : r \in P^w, f_r^w > 0\}$ is the set of utilized paths associated with v, and v^{UE} is the UE flow distribution, i.e., v^{UE} satisfies $t(v^{UE})^T(v - v^{UE}) \geq 0, \forall v \in V^F$. In particular, (1) ensures that all users are not worse off and (2) guarantees that some are better off. If v is dominating, then (2) implies that there must exist at least one utilized path, say $r' \in P_+^w$ for some OD pair w, such that $\sum_a \delta_{ar'} t_a(v_a) < c_w^{UE}$, i.e., users of path r' are better off. Otherwise, $\sum_a \delta_{ar} t_a(v_a) = c_w^{UE}$ for every w and $r \in P_+^w$, implying that $t(v)^T v = t(v^{UE})^T v^{UE}$. The latter contradicts the fact that v is dominating and satisfies (2).

To illustrate, Table 2 lists the UE and dominating flow distributions for the five-link network in Figure 1. The latter are in columns labeled DF-F-1, DF-F-2, and DF-F-3. Observe that the lengths of the longest utilized path for the three dominating distributions are no greater than the one for UE. Among the three, the longest path for DF-F-1 is the shortest while DF-F-3 yields the smallest total delay. In addition, a convex combination of DF-F-2 and DF-F-3 is also dominating. Thus, the number of dominating flow distributions for the five-link network is infinite.

	Link Flows			
Links	UE	DF-F-1	DF-F-2	DF-F-3
(1, 3)	3.60	1.90	2.24	2.24
(1, 2)	0.00	1.70	1.36	1.36
(3, 2)	2.28	0	0.45	0.61
(3, 4)	1.32	1.90	1.79	1.63
(2, 5)	2.28	1.70	1.81	1.97
Cost of the Longest Utilized Path	71.06	68.7	69.4	71.06
System Cost or Total Delay	255.82	247.1	241.17	234.99

Table 2: UE and dominating flow distributions

To find a dominating flow distribution, Abrams and Hagstrom [1] formulate an optimization problem equivalent to the following:

$$\begin{array}{lll} \text{DF-F:} & \min & t(v)^T v \\ & \text{s.t.} & v \in V^F \\ & & f_r^w(\sum_a \delta_{ar} t_a(v_a) - c_w^{UE}) \le 0, \quad \forall w, \text{ and } r \in P^w. \end{array}$$

Note that the last constraint in the above problem ensures that, if path r is utilized, its travel time, $\sum_a \delta_{ar} t_a(v_a)$, must not be larger than the travel time under UE. As formulated above, DF-F is an optimization problem with complementarity constraints, a difficult class of problem to solve optimally (see, e.g., [9]). However, there is a practical method (see [1]) for finding a local optimal solution to DF-F by allowing only routes utilized in the UE distribution to have positive flows.

Let $\widetilde{v}$ be an optimal solution to the DF-F problem. If $t(\widetilde{v})^T \widetilde{v} < t(v^{UE})^T v^{UE}$, then (2) holds and the complementarity constraints ensures that (1) also hold as well. Thus, $\widetilde{v}$ must be dominating. When $t(\widetilde{v})^T \widetilde{v} = t(v^{UE})^T v^{UE}$, there is no $v \in V^F$ that satisfies both (1) and (2), i.e., no dominating distribution exists. Note that for the five-link network, DF-F-3 in Table 2 solves the DF-F problem and the other two do not.

4 Nonnegative Pareto-Improving Tolls

In this section, we assume that a dominating distribution is given and investigate whether there exists a Pareto-improving toll vector that can induce the distribution. As mentioned previously, our focus is on nonnegative toll vectors.

Let $\widetilde{v}$ denote a given dominating distribution. With respect to the travel costs under UE, a nonnegative toll vector, τ, is 'Pareto-improving' if it satisfies the following conditions:

$$\begin{aligned} \sum_a \delta_{ar}(t_a(\widetilde{v}_a)+\tau_a) &= \lambda_w, \quad \forall w \text{ and } r \in \widetilde{P}_+^w(\widetilde{v}) && (3)\\ \sum_a \delta_{ar}(t_a(\widetilde{v}_a)+\tau_a) &\geq \lambda_w, \quad \forall w \text{ and } r \in \widetilde{P}_0^w(\widetilde{v}) && (4)\\ \tau &\geq 0 && (5)\\ \lambda_w &\leq c_w^{UE}, \quad \forall w && (6) \end{aligned}$$

Similar to before, $\widetilde{P}_+^w(\widetilde{v}) = \{r : r \in P^w, \widetilde{f}_r^w > 0\}$ and $\widetilde{P}_0^w(\widetilde{v}) = \{r : r \in P^w, \widetilde{f}_r^w = 0\}$ are the sets of utilized and non-utilized paths associated with $\widetilde{v}$, respectively. For each OD pair, the first two conditions ensure that τ is a "valid" toll vector (see, e.g., [6]), in that they force the utilized paths associated with $\widetilde{v}$ to have the same generalized travel cost that is no greater than those for the non-utilized paths. The fourth condition, (6), ensures that the generalized cost for each OD pair is no greater than the user equilibrium travel time. Without the nonnegativity requirement in (5), setting $\tau = -t(\widetilde{v})$ trivially satisfies conditions (3) - (6) and the resulting generalized travel cost is zero for all paths. Thus, $\tau = -t(\widetilde{v})$ is Pareto-improving. When τ must be nonnegative, a Pareto-improving toll vector may not exist. For example, the counterexample in [2] implies that a nonnegative toll vector does not exist when $\widetilde{v}$ contains a directed cycle.

The following theorem from [2] provides a necessary and sufficient condition for the existence of a toll vector satisfying the first three Pareto-improving conditions, (3) - (5). We restate it here for convenience.

Theorem 1 *There is a toll vector, τ, satisfying conditions (3) - (5) if and only it $\widetilde{v}$ satisfies the following variational inequality (the bounded user equilibrium problem with fixed demands or BUE-F):*

$$t(\widetilde{v})^T(v-\widetilde{v}) \geq 0, \quad \forall v \in \overline{V}^F$$

where $\overline{V}^F = \{v : v \in V^F, v_a \leq \widetilde{v}_a\}$.

In the above theorem, if $\widetilde{v}$ solves BUE-F then the Lagrangian multipliers, $\widetilde{\tau}$, associated with the upper bounds on v_a are nonnegative and naturally satisfy (3) - (5). However, these multipliers are not unique and solving the following problem can identify a Pareto-improving toll vector, if one exists.

$$\begin{array}{lll} \min & \sum_w s_w & \\ \text{s.t.} & \sum_a \delta_{ar}(t_a(\widetilde{v}_a)+\tau_a) = \lambda_w & \forall w \text{ and } r \in \widetilde{P}_+^w \\ & \sum_a \delta_{ar}(t_a(\widetilde{v}_a)+\tau_a) \geq \lambda_w & \forall w \text{ and } r \in \widetilde{P}_+^w \\ & \tau \geq 0 & \\ & \lambda_w - s_w \leq c_w^{UE}, & \forall w \end{array}$$

If the optimal objective value is zero, then the τ^* component of an optimal solution, (τ^*, λ^*, s^*), to the above problem is a Pareto-improving toll vector. Alternatively, the theorem below provides a sufficient condition under which the Lagrangian multipliers from BUE-F also satisfy (6). (See [8] for the proofs of the results stated below.)

Theorem 2 *Let $\widetilde{v}$ be a dominating distribution that solves the BUE-F problem. If there exists a vector of multipliers, $\widetilde{\tau}$, associated with the constraints $v_a \leq \widetilde{v}_a$ such that, for every OD pair w, $\sum_a \delta_{ar}\widetilde{\tau}_r = 0$ for some $r \in \widetilde{P}_+^w$, then $\widetilde{\tau}$ is a Pareto-improving toll vector.*

When there is only one origin (or, equivalently, one destination) in the problem, Theorem 2 can be strengthened. In this case, the problem reduces to a network problem with one commodity and V^F can be written as $\{v : Av = \sum_w E^w d_w, v \geq 0\}$.

Lemma 3 *Assume that there is only one origin and $t_a(\cdot)$ is an increasing function for all a. If $\widetilde{v}$ solves the DF-F problem, then $\widetilde{v}$ also solves the BUE-F problem.*

Corollary 4 *If there is only one origin and $t_a(\cdot)$ is an increasing function for all a, then there is always a Pareto-improving toll vector associated with a dominating distribution that solves the DF-F problem*

To illustrate, consider the five link example in Figure 1. Table 3 displays the UE distribution and DF-F-2, a dominating distribution in Table 2.

	UE		Pareto-Improving Tolls			
Links	Flow	Time	DF-F-3 ($\widetilde{v}$)	Time ($t_a(\widetilde{v})$)	τ^1	Gen. Cost
(1, 3)	3.6	36.00	2.24	22.4	0.00	22.40
(1, 2)	0	50.00	1.36	51.36	0.00	51.36
(3, 2)	2.2778	12.28	0.45	10.45	18.51	28.96
(3, 4)	1.3222	35.06	1.79	46.75	0.31	47.66
(2, 4)	2.2778	22.78	1.81	18.1	0.00	18.10
Paths						
1-3-4	1.3222	71.06	1.79	65.15	0.31	69.46
1-3-2-4	2.2778	71.06	0.45	50.97	18.51	69.46
1-2-4	0	72.78	1.36	69.46	0.00	69.46
Costs to users		255.82		241.17	8.88	250.06

Table 3: UE Distribution and Pareto-Improving Tolls

To determine Pareto-improving tolls in Corollary 4, construct the longest path tree using the link cost $t_a(\widetilde{v})$ in Table 3. Based on this longest path tree, Dial's algorithm [3] sets the tolls on link (3, 2) and (3, 4) to be $\tau_{32} = 18.51$ and $\tau_{34} = 0.31$. Doing so produces the toll vector $\tau^1 = [0.0, 0.0, 18.51, 0.31, 0.0]^T$. Under τ^1, everyone (users, transportation authority, and society) is better off. The generalized cost (see the last column in Table 3) of every path equals 69.46, 1.6 units less than that (71.06) under UE. Of the 3.6 users, 2.24 users who pay tolls actually get to use routes (with travel times of 65.15 and 50.97) shorter than the toll-free route (69.46). From the last row, the transportation authority collects 8.88 in toll revenue and the total delay decreases from 255.82 (under UE) to 241.17, approximately 6% more than the least possible (227.11).

5 Conclusion

The talk focuses on finding "Pareto-improving" tolls, a congestion pricing scheme that makes some user better off and no one worse off, when compared to the situation without pricing. In addition, a Pareto-improving scheme induces a dominating flow distribution, a concept equivalent to a Generalized Braess Paradox introduced earlier in [4]. Because they do not always exist, we provide sufficient conditions for the existence of a nonnegative Pareto-improving toll vector. In particular, we show that a Pareto-improving toll vector always exists when there is only one origin or destination.

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