An Optimization-Based Heuristic for the Split Delivery Vehicle Routing Problem

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In vehicle routing problems (VRPs) a set of customers needs to be served and a fleet of capacitated vehicles is available to do so. The objective is the minimization of costs, which usually means minimizing the total distance traveled. In most VRPs it is assumed that the demand of a customer is less than or equal to the capacity of a vehicle and that each customer has to be served by exactly one vehicle, i.e., there is a single-visit assumption. While it is obvious that when a customer's demand exceeds the vehicle capacity it is necessary to visit that customer more than once, it requires only a little more thought to see that even when all customer demands are less than or equal to the vehicle capacity, it may be beneficial to use more than one vehicle to serve a customer. In the split delivery vehicle routing problem (SDVRP) the single-visit assumption is relaxed and each customer may be served by more than one vehicle.

While the SDVRP has received little attention in the past, compared to other variants of the VRP, it has recently been studied by a number of researchers. The SDVRP was introduced by Dror and Trudeau ([4] and [5]) who defined the problem, derived some structural properties, and proposed a local search heuristic. The computational complexity of the SDVRP was analyzed by Archetti et al. ([2]) while a tight bound on the cost reduction that can be obtained by allowing split deliveries was given by Archetti et al. ([3]).

The purpose of this work is to propose a heuristic solution approach for the SDVRP. At present, no exact methods exist for the SDVRP given to its high complexity. A branch and price exact algorithm has been proposed for the time windows case by Gueguen ([8]) and Gendreau et al. ([6]). In addition to the heuristic local search proposed by Dror and Trudeau ([4] and [5]), a tabu search algorithm can be found in Archetti et al.([1]).

In this work we present a new solution approach that integrates heuristic search with optimization. The proposed approach is based on two main ideas. The first is to use the information provided by a tabu search heuristic to identify parts of the solution space that most likely contain good solutions. The second idea is to explore these parts of the solution space by means of a suitable integer programming model.

An Integer Programming Model

In the SDVRP, a set C of customers has to be served by a fleet M of capacitated vehicles. Each vehicle $v \in M$ has capacity Q and has to start and finish its tour at the depot, which we denote by 0. Each customer $i \in C$ has demand d_i, which can be less than, equal, or greater than the vehicle capacity Q. A customer may be visited more than once. The cost to travel between locations i and j is c_{ij}. We assume that the costs c_{ij} satisfy the triangle inequality. The objective is to serve customers demand at minimum cost.

We present a route-based formulation for the SDVRP. Let R represent a set of routes and let c_r denote the cost of route r. The formulation has two sets of variables. The binary variable x_r takes on value 1 if route r is selected and 0 otherwise. The continuous variable y_r^i represents the quantity delivered to customer i on route r. The integer programming model is presented below.

$$\min \sum_r c_r x_r \tag{1}$$

$$\sum_{i \in r} y_r^i \leq Q x_r \qquad r \in R, \tag{2}$$

$$\sum_{r \in R : i \in r} y_r^i \geq d_i \qquad i \in C, \tag{3}$$

$$x_r \in \{0,1\} \qquad r \in R, \tag{4}$$

$$y_r^i \geq 0 \qquad r \in R,\ i \in C. \tag{5}$$

The objective function (1) minimizes the total cost of the selected routes. Constraints (2) impose that a delivery to a customer i on route r can only take place if route r is selected and that the total quantity delivered on a selected route cannot exceed the vehicle capacity. Constraints (3) ensure that the demand d_i of customer i is completely satisfied.

Model (1)-(5), strengthened by a set of valid inequalities, is implemented in the optimization phase of our solution approach.

A Solution Approach

One of the key ideas underlying our solution approach is that tabu search can identify parts of the solution space that are likely to contain high quality solutions. In particular, we refer to the tabu serch algorithm proposed by Archetti et al.([1]).

The simplest use of this idea is the identification of a set C' of customers which are likely to be served by a single vehicle in high-quality SDVRP solutions. If a customer is never, or rarely, split in the solutions encountered during the tabu search, we interpret this as an indication that it is likely that the customer will be served by a single vehicle in high quality SDVRP solutions (and therefore should be in the set C'). We have implemented this idea as follows. Let S denote the set of all SDVRP solutions encountered during the tabu search. For each customer i, we calculate the *node counter* n_i, the number of times customer i is split in the solutions in S, where we say that a customer is split $k-1$ times if the customer is served by k routes in a solution $s \in S$. Let $n_{max} = \max_i n_i$. We include customer i in C' if $n_i < 0.1 \times n_{max}$ and if i is not split in the final solution of the tabu search.

The use of this idea in the identification of the set R of promising routes is more involved. For each edge $\{i,j\}$, we calculate n_{ij}, the number of times edge $\{i,j\}$ appears in any of the routes of the solutions in S. We will refer to n_{ij} as the *edge counter* of edge $\{i,j\}$. Again, we interpret a large value n_{ij} as an indication that it is likely that edge $\{i,j\}$ will be included in high quality SDVRP solutions. The edge counters n_{ij} guide the construction of a set of *promising routes* $\bar{R}$. The procedure which constructs the set $\bar{R}$ starts from a set B of base edges which includes those edges $\{i,j\}$ with an edge counter that is greater than or equal to a given percentage p_B of the maximum edge counter, i.e., $\{i,j\} \in B$ if $n_{ij} \geq p_B \times \max_{\{i,j\}} n_{ij}$. For each edge $\{i,j\} \in B$ a set of routes is generated. The routes in $R'_{\{i,j\}}$ are generated as follows. We start with a path P consisting only of the base edge $\{i,j\}$ and then extend the path from both endpoints until the depot is encountered, in which case we have identified a route. More precisely, we look at the edges incident to an endpoint u of P, i.e., edges $\{u,v\}$ with $v \notin P$. Whenever the edge counter of an edge $\{u,v\}$ is greater than some threshold L or the edge $\{u,v\}$ belongs to at least one route of an improving solution found during the tabu search (where an improving solution is a solution that improved the current best solution), we extend the path with edge $\{u,v\}$. The threshold is calculated as a percentage of the maximum counter of an edge incident to u, i.e., $L = p_I \times \max_v n_{uv}$. There is one additional precaution. When the total demand of a partial path starting at i or j exceeds $Q + \frac{\delta}{2}$, where δ is the average customer demand, we connect the other endpoint of the partial path to the depot.

The set $\bar{R}$ is not used directly in the route optimization IP, because it is usually too large, but the route optimization IP is solved several times with subsets R of $\bar{R}$.

The identification of good subsets is based on three ideas. First, we always include the routes of the best known solution. That way it is possible for the route optimization IP to improve just a portion of the best solution, i.e., to perform a local improvement. Second, we include the routes with a positive value in the solution to the linear programming relaxation of the route optimization IP over the entire set $\bar{R}$. As the linear programming relaxation considers the entire set of routes $\bar{R}$, it may be able to identify sets of complimentary routes from a global perspective. Finally, we include routes based on a desirability criterion (we tested different desirability criteria).

Let r_{max} be the maximum number of routes we allow in R and let n_{max} be the maximum number of IPs we want to solve. Parameter r_{max} is set so as to ensure that the route optimization IP will solve in a reasonably short time. In effect, preliminary tests have shown that it is possible to reach better solutions by solving many "small" IPs than solving only one "big" IP. The routes of the best known solution are always included in R. The first IP includes also the routes with positive values in the linear programming relaxation of the route optimization IP over the entire set $\bar{R}$. These routes are complemented with desirable routes that have not been used before, i.e., starting from the first not yet used desirable route and following the order determined by the desirability criterion, routes are added until the maximum number of routes r_{max} is reached (or all the desirable routes have been used). An overview of the proposed route optimization can be found in Algorithm 1.

Algorithm 1 Route optimization

```
$counter_{ip} = 1$
while the elapsed time is less than $T_{max}$ and $counter_{ip} \leq n_{max}$ do
  $R \leftarrow \emptyset$.
  Add the routes of best known solution to $R$.
  If $counter_{ip} = 1$ add the routes with a positive value in the LP relaxation to $R$.
  Add desirable routes to $R$ until $|R| = r_{max}$ (or no more desirable routes exist).
  Delete the selected desirable routes from the set of desirable routes.
  Solve the route optimization IP over $R$.
  if the solution found improves the best known solution $s^*$ then
    Update $s^*$.
  end if
  $counter_{ip} \leftarrow counter_{ip} + 1$
end while
```

Computational Results

To evaluate the merits of the proposed optimization-based heuristic, we tested it on a set of 42 instances with varying demand characteristics. The instances are derived from seven basic instances; the same instances used to test the tabu search algorithm of Archetti et al. ([1]). These basic instances vary in terms of the number of customers (ranging from 50 to 199) and in terms of vehicle capacity (ranging from 140 to 200). Five additional set of instances are created by changing the demand of the customers in the basic instances (following Dror ([4])), but keeping all other characteristics the same. Each of the new sets of instances is characterized by a lower bound and an upper bound on the demand at the customers, expressed as a fraction of the vehicle capacity Q.

The computational results we have obtained are encouraging and validate the interest in non-traditional uses of integer programming. The proposed optimization-based heuristic was able to find an improved solution for all but one instance (the only exception is basic instance p11). The average improvement is a little over 0.5 percent. Even though the improvements are relatively small, we believe this is primarily because the tabu solutions are already very good. Moreover, we observe very small gaps between the value of the solutions found and the value of the linear relaxation. Although the value of the linear program over the entire set of promising route is not a true lower bound, as we are not optimizing over the entire set of routes, it is likely to be very close to a true lower bound and this substantiate the fact that the solutions obtained are likely to be close to optimal. Recently, Golden et al. ([7]) also proposed a solution approach for the SDVRP that incorporates heuristic as well as integer programming components.

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