The Capacitated Team Orienteering and Profitable Tour Problems

Claudia Archetti[(1)] *Dominique Feillet*[(2)] *Alain Hertz*[(3)] *M. Grazia Speranza*[(1)]

[(1)]Universitá di Brescia, Brescia, Italy

[(2)] Université d'Avignon, Avignon, France

[(3)]École Polytechnique and GERAD, Montréal, Canada

{archetti, speranza}@eco.unibs.it

alain.hertz@gerad.ca

dominique.feillet@univ-avignon.fr

Extended abstract

A huge number of papers appeared in the literature which study the well known Traveling Salesman Problem (TSP) and its generalizations to the case of multiple vehicles known as Vehicle Routing Problems (VRPs). While there exists one and only one TSP, many problems belong to the class of VRPs (see Toth, Vigo (2002)). In the TSP and in the VRPs all customers need to be visited. This means that in the situations modeled all customers are known at the time the optimization model is run and all require service. While this is indeed the case in many practical problems, there are many other practical problems where this assumption is not valid.

Let us consider the following situation where a set of customers is given and only a subset has to be selected and served. Nowadays it is more and more frequent that shippers post their demands for transportation service on the web, usually in specific databases, and carriers identify some of those demands and offer their service to the shippers. Usually a carrier has a fleet of vehicles and a set of regular customers that have to be served. If the capacity of the vehicles is not fully used, the carrier may wish to look for spot customers on the web. In this case, the carrier has to select, within the set of potential customers, those that are most convenient for him. The carrier will take into account in the decision process the customers that, as traditionally, need to be served.

When a set of customers need to be selected and a single tour organized, the optimization problems become variants of the TSP. A profit is associated to each customer which makes customers more or less profitable. On the other hand, the traveling cost and/or time need to be taken into account. A constraint on the traveling time of vehicles may force the selection of a subset of customers, while the traveling cost may make beneficial the selection of only a subset of the potential customers. The problem becomes to identify the most profitable set of customers. A recent survey by Feillet, Dejax, Gendreau (2005) defines those problems as the TSPs with profits. The objective function may be the maximization of the collected total profit (Orienteering Problem), the minimization of the total traveling cost (Prize-Collecting TSP) or the optimization of a combination of both (Profitable Tour Problem). While some of the TSPs with profit have been investigated by a number of researchers, such as the Orienteering Problem (OP), and for some others little can be found in the literature, very few papers are available for any of the extensions of the TSPs with profits to the case of multiple tours. We call this class of problems the VRPs with profits. The list of papers in which multi-vehicle routing problems with profits are addressed can be found in Feillet, Dejax, Gendreau (2005). All such papers assume that the vehicles are un-capacitated.

In this paper we investigate the capacitated versions of two known VRPs with profit. The first is the extension to the case of multiple tours of the OP. In the OP, given a set of potential customers with associated profit and given the distances between pairs of customers, the objective is to find the subset of customers for which the collected profit is maximum, given a constraint on the total length of the tour. The OP is also called the Selective Traveling Salesman Problem (STSP). The extension of the Orienteering Problem to the case of multiple tours is known as the Team Orienteering Problem (TOP). In the TOP there is a time constraint on each tour. The TOP appeared in the literature in a paper by Butt, Cavalier (1994) under the name Multiple Tour Maximum Collection Problem, while the definition of TOP was introduced by Chao, Golden, Wasil (1996). Two recent papers by Tang, Miller-Hooks (2005) and by Archetti, Hertz, Speranza (2006) proposed metaheuristics for the solution of the TOP. We study the Capacitated version of the TOP (CTOP). The second problem we study is the capacitated version of the Profitable Tour Problem (PTP) which we will refer to as Capacitated PTP (CPTP). In the PTP (see Feillet, Dejax, Gendreau (2005)) a single vehicle is available and the objective is to maximize the difference between the total collected profit and the cost of the total distance traveled. In the CPTP, the PTP is extended to the case where a fleet of capacitated vehicles is available.

Let us define the CTOP and the CPTP formally. We consider a complete undirected graph $G = (V, E)$, where $V = 1, \ldots, n$ is the set of vertices and E is the set of edges. Vertex 1 is the depot which is the starting and ending point of each tour. Each vertex $i = 2, \ldots, n$ represents a potential customer. An edge $(i, j) \in E$ represents the possibility to travel from vertex i to vertex j. Each customer has a demand d_i. A nonnegative profit p_i is associated with each customer ($p_1 = 0$). A symmetric travel time t_{ij} and a cost c_{ij} are associated with each edge $(i, j) \in E$. A set of m vehicles is available to visit the customers. Each vehicle has a capacity Q and can visit any subset of the potential customers without exceeding the capacity Q. The profit of each customer i can be collected by one vehicle at most.

In the Capacitated Team Orienteering Problem (CTOP) a subset of the potential customers has to be selected in such a way that the constraint on the capacity Q of each vehicle is satisfied and the duration of the route of each vehicle does not exceed a time limit T_{max}. The objective is to maximize the total collected profit.

In the Capacitated Profitable Tour Problem (CPTP) a subset of the potential customers has to be selected in such a way that only the constraint on the capacity Q of each vehicle is satisfied. The objective in this case is to maximize the difference between the total collected profit and the cost of the total distance traveled.

Among the metaheuristics proposed for the solution of combinatorial optimization problems, the tabu search (see, for example, Gendreau, Hertz, Laporte, 1994) has been shown to be very effective for a variety of vehicle routing problems. Another interesting metaheuristic is the variable neighborhood search (see Mladenovic and Hansen, 1997). In this paper the effectiveness of these metaheuristics is confirmed also on the VRPs with profit. We propose, both for the solution of the CTOP and of the CPTP, two variants of a tabu search algorithm and a variable neighborhood search algorithm. These algorithms extend ideas presented in Archetti, Hertz, Speranza (2006) for the TOP. The heuristics for the CTOP are compared with an exact method based on column generation (see Boussier, Feillet, Gendreau (2006) for an exact method for the TOP) on a set of instances of the CTOP. A similar exact method for the CPTP has also been implemented and computational experiments have been carried out on a set of instances of the CPTP. While the exact algorithm can solve, for each of the two problems, instances of small size, the results show that the heuristics obtain very good results both for the CTOP and for the CPTP, in terms of solution quality, within a reasonable amount of time.

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