MULTI-CARRIER TRAIN SCHEDULING FOR FREIGHT TRANSPORT

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1. INTRODUCTION

The problem of multi-carrier, freight train scheduling for regular demand (i.e. demand units that are to be shipped at regular intervals of time) within a corridor containing multiple service routes is addressed in this paper. Specifically, a train slot generation optimization model based on concepts of multicommodity network flows is developed for determining the optimal multi-carrier, multi-line train timetable and an exact column generation technique is proposed for its solution. The train slot generation model seeks to minimize operational costs and delays in delivery from scheduled arrival times (referred to herein as delay). Carriers are assumed to share the track capacity within the corridor.

Solutions to the train slot generation model are tested in a mesoscopic simulation platform (Mahmassani, et al., 2006). Feedback in terms of delay from the simulation is used by the train slot generation model to make further improvements to the timetable. Given an initial timetable from the train slot generation model and the origin-destination (O-D) demand table for shipping within and across the region, a mode choice model is employed that estimates the amount of demand (i.e. number of containers or railcars) that will use the corridor. The simulation platform simulates the operations of the corridor for the given timetable and demand levels, allowing estimation of delays at terminals, classification yards and border crossings. The estimated delay is fed back to the train slot generation model and a new train timetable is obtained. This process is repeated until termination criteria are met.

Related freight train scheduling works from the literature are briefly reviewed in section 2. The assumptions made in developing the train slot generation model and relevant solution techniques are provided in section 3. The train scheduling problem is formulated as an integral multicommodity network flow problem, referred to herein as the train slot generation model, in section 4. A method for generating feasible input for the model is proposed in section 5. An integer programming solution technique based on the concepts of column generation for solving

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the problem is proposed in section 6. A case study based on a corridor in Europe is described in section 7.

2. LITERATURE REVIEW

The majority of the rail scheduling literature has focused on modeling single-line operations (for example, Szpigel, 1973; Assad, 1978; Petersen et al, 1986; Kraft, 1987; Carey and Lockwood, 1995; Brannlund et al., 1997; Higgins et al, 1996; Nou, 1997; Cordeau et al., 1998). Single-line operations may involve single or double tracks between two yards and junctions or other significant points. Thus, the network over which the trains are assumed to operate is very simple (i.e. forming a directed, simple and elementary path). Few works address problems with multi-line operations (for example, Jovanvic and Harker, 1991; Odijk, 1996; Kwon et al., 1998; Newman and Yano 2000). Odijk (1996) and Jovanvic and Harker (1991) focused on modifying existing schedules to increase reliability. Kwon et al. (1998) proposed a combined routing and scheduling model to minimize the total delay. They tested their solution approach for their model in a T-shape network. Newman and Yano (2000) proposed a model in which on-time delivery is required to minimize the operating costs.

None of the existing works in the literature considers the objectives of both the shipper and the carrier. Nor does any of these works consider multiple carriers or elasticity in demand. The multi-line, multi-carrier, multi-objective freight train scheduling problem is addressed herein. The solution approach is embedded in an iterative framework where the demand is re-estimated in light of the train schedule (and resulting estimates of level-of-service) that results from solution of this freight train scheduling problem. The scheduling problem is resolved in response to changes in demand estimates and the procedure repeats until termination criteria are met. Thus, this framework allows solution of the freight train scheduling problem with elastic demand.

3. PROBLEM ASSUMPTION

Important assumptions have been made in developing the train slot generation model, simulation platform (developed externally: Mahmassani et al., 2006), and relevant solution techniques. Such assumptions are necessary where relevant data is unavailable or to create mathematically tractable problem formulations. These assumptions involve the definition of a train slot with reference to time; prioritization of passenger over freight traffic within the corridor; availability of an O-D demand table; predetermined classification and train make-up policies;

delay estimates at the terminals, classification yards or border crossings; and carrier preferences for the O-D pairs. The problem of empty car distribution is not considered in this work.

4. TRAIN SLOT GENERATION MODEL FORMULATION

A formulation of the train slot generation model as an integral multicommodity network flow problem with block-angular structure required for column generation is proposed in this section. The formulation relies on a train slot representation of the track capacity of each route, where a train slot is defined as the use of a route from shipment origin to shipment destination during a given period of time. The formulation is a path-based one in which each column is a binary variable that represents whether or not a time slot *t* of route *r* will be operated by carrier *c*.

Notations

x_{rt}^{c} : binary variable that indicates whether or not the train slot t of route r is operated by carrier c.

δ_{rt}^{c} : shipment delay (penalty cost) on train slot t of route r operated by carrier c.

θ_{rt}^{c} : train operational cost on train slot t of route r operated by carrier c.

α^{r} : number of train slots needed to be operated to complete shipments on route r.

β_{k}^{c} : number of train slots between a O-D pair (k) that a carrier c would like to operate on.

T^{rc} : set of train slots of route r operated by carrier c.

C : set of carriers.

K : set of origin and destination (O-D) pairs.

R^{kc} : set of routes of a O-D pair k operated by carrier c.

Model Formulation

$$\text{(P) Min } z(x) = \sum_{c\in C}\sum_{r\in R^{kc}}\sum_{t\in T^{rc}}\delta_{rt}^{c}x_{rt}^{c} + \sum_{c\in C}\sum_{r\in R^{kc}}\sum_{t\in T^{rc}}\theta_{rt}^{c}x_{rt}^{c} \quad (1)$$

subject to

$$\sum_{c\in C}\sum_{t\in T^{rc}}x_{rt}^{c} = \alpha^{r}, \ \forall\, r\in R^{kc} \ \ \forall\, k\in K\ \forall c\in C \quad (2)$$

$$\sum_{r\in R^{kc}}\sum_{t\in T^{rc}}x_{rt}^{c} \le \beta_{k}^{c}, \ \forall c\in C \ \ \forall\, k\in K \quad (3)$$

$$x_{rt}^{c} \in \{0,1\}\ ,\ \forall c \in C \quad \forall\, t \in T \ \ \forall\, r \in R^{kc} \ \ \forall\, k \in K \tag{4}$$

The objective given in equation (1) seeks to minimize the total delay incurred along the corridor and total operational costs required to complete the shipments within the corridor. The decision-maker's preferences with respect to delay and cost minimization can be reflected by including appropriate weights on the delay and cost components of the objective function. This formulation assumes that the decision-maker's preference function can be represented through an additive function of delay and cost. Constraints (2) ensure that the total number of train slots available on a route is equivalent to the number of train slots, α^{r}, that may be operated by the carriers to complete the shipments required for the route. Constraints (3) ensure that for a specific O-D pair over all routes, the number of train slots operated by any specific carrier does not exceed the number of train slots, β_{k}^{c}, available to that carrier. Binary integral requirements of the decision variables are given in constraints (4).

5. PREPROCESSING METHOD TO GENERATE THE MODEL INPUT

A preprocessing method is proposed to generate a set of potential feasible train slots for multiple carriers that can be used as an input for the train slot generation model. Given the O-D demand table and a set of potential externally defined routes that comprise the service corridor, a set of train slots that repeat on a weekly basis are assigned to carriers based on carrier preferences through the use of this preprocessing method. The resulting set of train slots must not, if operated concurrently, create conflicts and only one carrier can operate a particular route in a particular train slot. It is assumed that any preferences for operating between an O-D pair are presented by the carriers prior to running this preprocessing method. Thus, for each O-D pair, train slots can be randomly assigned to carriers that are interested in operating such slots.

Many different combinations of train slots and train slot assignments to carriers can be generated through the use of this method. The resulting set of potential feasible train slots and assignments provide necessary input to the column generation technique proposed for determining the optimal train timetable that is described next. The technique can be repeated on different initial sets of potential feasible train slots and the best solution from these runs can be employed.

6. SOLUTION TECHNIQUE FOR TRAIN SLOT GENERATION MODEL

Algorithm Overview

In this work, column generation is employed to solve exactly a binary integral CMCNP representation of the multi-line, multi-carrier freight train scheduling problem. Column generation has been successfully applied to solve many large-scale optimization problems in, for example, vehicle routing (Desrosiers et al., 1984), air crew scheduling (Lavoie et al., 1988), and production scheduling for multiple items in a single machine (discrete lotsizing and scheduling) (Cattrysse et al., 1993). Column generation is a price-directive method in which tolls (or prices) are placed on the complicating bundle capacity constraints. The procedure takes advantage of a block-angular structure of the problem formulation in which *r* independent constraints exist (i.e. the demand constraints) and joint constraints (i.e. the bundle constraints) link the commodities. This structure makes it possible to decompose the problem into *r* easier subproblems.

At each iteration of the column generation approach, a master problem is formulated including only a subset of the columns (i.e. variables) of the original formulation. This smaller program is solved to optimality by, for example, the simplex method. The key idea of column generation is never to explicitly list all of the columns given in the original problem formulation, but rather to generate them only "as needed." The algorithm determines whether the solution is optimal for the original program or if additional columns must be added to improve the solution. A subproblem of each commodity is used to generate a new column for the master problem and to assess optimality of the solution resulting from each iteration. The column with the most negative reduced cost in each subproblem will be added to the restricted master program and the process is repeated.

A brief overview of a generic column generation procedure is provided next. See, for example, Ahuja et al. (1993) or Hu (1963) for additional background. Italicized text in bold will be discussed in detail in following subsections.

Generic Column Generation Procedure

Step 1. Solve the ***master problem***.

Step 2. Use the dual variable values resulting from solution of the master problem to update the cost coefficients of the ***subproblem*** associated with each commodity.

Step 3. Solve each single commodity subproblem. Identify the column (variable) in each subproblem with the most negative reduced cost based on the subproblem solution and add a new column to the master problem for each such subproblem. Return to *step 1* if a new column is identified for inclusion. Otherwise, if no column in any subproblem has a negative reduced cost, the procedure is terminated. The optimal solution has been obtained.

master problem

The goal of the master problem is to obtain the value of the dual variables so that the reduced cost for each train slot can be calculated for the subproblems. Since the train slot generation formulation has a block-angular structure, this formulation associated with a smaller set of variables than would be required otherwise can be treated as the master problem

subproblem

The goal of the subproblem is to find the column (train slot) with the minimum reduced cost to be added to the master problem. If the minimum reduced cost is nonnegative, then we can terminate the column generation procedure and the problem is solved to optimality. Let σ^r denote the dual variable corresponding to each route *r* in constraints (2) and w_k^c denote the dual variable corresponding to O-D pair *k* operated by carrier *c* in constraints (3). Thus, the reduced cost, λ_{rt}^c , of the column corresponding to the master problem is given by (5). The value of the reduced cost of each column, λ_{tr}^c can be treated as the benefit (i.e. reducing the train operational and penalty costs) obtained by carrier *c* using a train slot *t* of route *r*.

$$\lambda_{rt}^c = \delta_{rt}^c + \theta_{rt}^c + w_k^c - \sigma^r \quad \forall c \in C, \forall\, t \in T^{rc}, \forall\, r \in R^{kc}, \forall\, k \in K \qquad (5)$$

7. CASE STUDY

The solution technique proposed in this work is applied on a multi-national rail network that bridges the Nordic European region with the south and southeastern European regions via central Europe (known as the REORIENT corridor). Real-world data concerning the network attributes are employed.

The multi-carrier weekly based train timetable is intended for use in making customer commitments. The train timetable includes carrier assignment to time slots and departure and arrival times at terminals, stations, and classification yards. The delay of each shipment is calculated from the difference between the shipment's arrival time resulting from implementing the timetable and the shipper's preferred arrival time. The track capacity utilization within the corridor is analyzed by track segment and day of week. It is hoped that by using this train timetable within the corridor, new markets will open and trade will increase along this corridor, fostering economic growth in the region.

8. REFERENCE

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