Introducing asset management to capacitated multicommodity network design

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1. Introduction

In this extended abstract we present the capacitated multicommodity network design problem with asset management considerations (CMNDAM). The problem is an extension of the design balanced fixed charge network design problem (DBCMND), as presented in Pedersen et al. (2006a). DBCMND problems arise in design of transportation networks, where node balance of vehicles has to be ensured. Smilowitz et al. (2003) present a model for multimodal package delivery with node balance constraints for ground vehicles, while Lai and Lo (2004) model ferry service network design with node balance constraints for the ferries. In Barnhart and Schneur (1996), a node-balanced express shipment design for air transport is presented. Andersen et al. (2006) and Pedersen et al. (2006b) present service network design models with node balance constraints for locomotives. While extending DBCMND to CMNDAM we include the design balance constraints and introduce additional issues related to management of assets. For design problems within transportation where specific fleets of vehicles are considered, the available fleets of vehicles restrict the feasible operations. Unfortunately, this aspect has usually been ignored in design studies. In this extended abstract we present a model formulation where the management of assets is introduced explicitly to the design.

Magnanti and Wong (1984) show that the uncapacitated fixed charge network design problem is NP-hard. As the capacitated version is even harder (Balakrishnan et al., 1997), this problem also belongs to the class of NP-hard problems. The introduction of design balance constraints on the nodes further complicates the model.

The outline of this extended abstract is as follows. In Section 2, we define the fixed charge capacitated multicommodity network design (CMND) problem. In Section 3, we introduce management of assets to the design. We first describe the design balance constraints, before introducing other asset management issues from real-world applications. We discuss model solving in Section 4, and end with concluding remarks in Section 5.

2. Fixed charge capacitated multicommodity network design (CMND)

Let the directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ represent the network, where $\mathcal{N}$ is the set of nodes and $\mathcal{A}$ the set of arcs. Without loss of generality we assume that all arcs $(i, j) \in \mathcal{A}$ are design arcs. A set of products, $\mathcal{P} = \{p\}$, should be routed over the network. Each product has a demanded volume w^p that has to be transported from the products unique origin node o^p to its unique destination node d^p.

The set of design variables y_{ij} are binary variables indicating whether an arc is used or not, while the flow variables x_{ij}^p are nonnegative real numbers. Each design arc y_{ij} has an associated capacity u_{ij}, and a fixed cost f_{ij} associated with use of the arc. For each unit of product p there is a flow cost c_{ij}^p for traversal arc $(i, j) \in \mathcal{A}$. For each node we define sets $\mathcal{N}^+(i) = \{ j \in \mathcal{N} : (i, j) \in \mathcal{A} \}$ and $\mathcal{N}^-(i) = \{ j \in \mathcal{N} : (j, i) \in \mathcal{A} \}$ of outward and inward neighbours. Finally, let $b_{ij}^p = \min\{ w^p, u_{ij} \}$.

The problem consists of minimizing the sum of fixed costs and flow costs while satisfying all demand. The arc-based formulation of the CMND problem can now be formulated as follows:

$$Min\ z = \sum_{(i,j)\in\mathcal{A}} f_{ij} y_{ij} + \sum_{(i,j)\in\mathcal{A}} \sum_{p\in\mathcal{P}} c_{ij}^p x_{ij}^p \tag{1}$$

$$\sum_{j\in\mathcal{N}^+(i)} x_{ij}^p - \sum_{j\in\mathcal{N}^-(i)} x_{ji}^p = \begin{cases} w^p, & i = o^p \\ -w^p, & i = d^p \\ 0, & otherwise\ , \end{cases} \qquad \forall i \in \mathcal{N}, p \in \mathcal{P}, \tag{2}$$

$$\sum_{p\in\mathcal{P}} x_{ij}^p - u_{ij} y_{ij} \le 0, \qquad \forall (i, j) \in \mathcal{A}, \tag{3}$$

$$x_{ij}^p - b_{ij}^p y_{ij} \le 0, \qquad \forall (i, j) \in \mathcal{A}, p \in \mathcal{P}, \tag{4}$$

$$x_{ij}^p \ge 0, \qquad \forall (i, j) \in \mathcal{A}, p \in \mathcal{P}, \tag{5}$$

$$y_{ij} \in \{0,1\}, \qquad \forall (i, j) \in \mathcal{A}. \tag{6}$$

The objective (1) minimizes the sum of fixed costs on open design arcs and costs for flow traversal on arcs. Constraints (2) are node balance constraints for freight. Constraints (3) and (4) are weak and strong forcing constraints, defining arc capacities and forcing flow to zero if an arc is not opened. Constraints (4) are redundant in the MIP-formulation, but improve the lower bounds obtained through relaxations (Crainic et al., 2001). The formulation including (4) is the strong formulation, while removal of (4) results in the weak formulation. Constraints (5) state that the flow variables are non-negative real numbers, while the design variables are restricted to binary values in (6).

3. Introducing asset management to the design

Among the practical applications that may result in CMND models are service network design problems. In these models, the fixed charges represent a cost that must be paid for each network arc that is opened, while the network is capacitated because several flows need to share capacities of the arcs that are opened. The purpose of service network design is to plan services and operations in such a way that demand is met while profitability is ensured (Crainic, 2000). Service network design models are increasingly being made more sophisticated, and with increased computational capacity and improved solution techniques, larger instances can be solved to optimality or near-optimal solutions may be found. Due to these improvements, it is also possible to incorporate more aspects in one model for simultaneous consideration. Compared to sequential solutions, this gives possibilities for more coherent solutions of tasks that are interrelated, but which have been treated separately in an attempt to maintain tractability. For applications within transportation, assignment of vehicles to services have usually not been considered in the service network design, but rather been carried out a posteriori. Such an approach might result in inefficient vehicle utilization. In this section we elaborate on a few important issues related to vehicles that may be incorporated to service network design models, which necessitate extensions to the CMND model of section 2. We start by introducing design balance constraints before introducing other aspects of asset management that we will incorporate to the model that we evaluate in our computational study. These issues are collected from real-world planning problems.

Design-balanced capacitated multicommodity fixed charge network design (DBCMND)

DBCMND problems arise in design of transportation networks, where node balance of vehicles has to be ensured. Smilowitz et al. (2003) present a model for multimodal package delivery with node balance constraints for ground vehicles, while Lai and Lo (2004) model ferry service network design with node balance constraints for the ferries. In Barnhart and Schneur (1996), a node-balanced express shipment design for air transports is presented. Andersen et al. (2006) and Pedersen et al. (2006b) present service network design models with node balance constraints for locomotives. To obtain the DBCMND model, we add node balance constraints (7) to formulation (1)-(6):

$$\sum_{j \in \mathcal{N}^+(i)} y_{ij} - \sum_{j \in \mathcal{N}^-(i)} y_{ji} = 0, \qquad \forall i \in \mathcal{N}. \qquad (7)$$

The motivation for these restrictions is clear; in design of transportation services where vehicle scheduling are considered explicitly, one aims at achieving feasible paths of vehicle movements through the network. In rail and ship industries, where the costs for acquiring vehicles is high compared to operating costs, it is of particular importance to establish service networks and schedules that utilize the fleet of vehicles in an efficient manner. There has however been sparse attention to this issue in the literature. Because of the complexity of DBCMND, there is a strong need for research on solution approaches for these formulations.

Fleet size constraints

Network design with node balance constraints (7) on design arcs captures important aspects of real-world design of transportation services. However, if the fleet of vehicles is given, the available fleet constrains the number of simultaneous operations. If a given number of vehicles are available, this number of vehicles is an upper bound on the number of operations that may take place simultaneously. This aspect is not covered by the DBCMND formulation.

Let each node in a time-space network have associated a time period in which the node exists, $T_{IME}(i), \forall i \in \mathcal{N}$. Constraints (8) state that the number of simultaneous realizations (nonzero variable values) is bounded by the available number of assets, V_{MAX} :

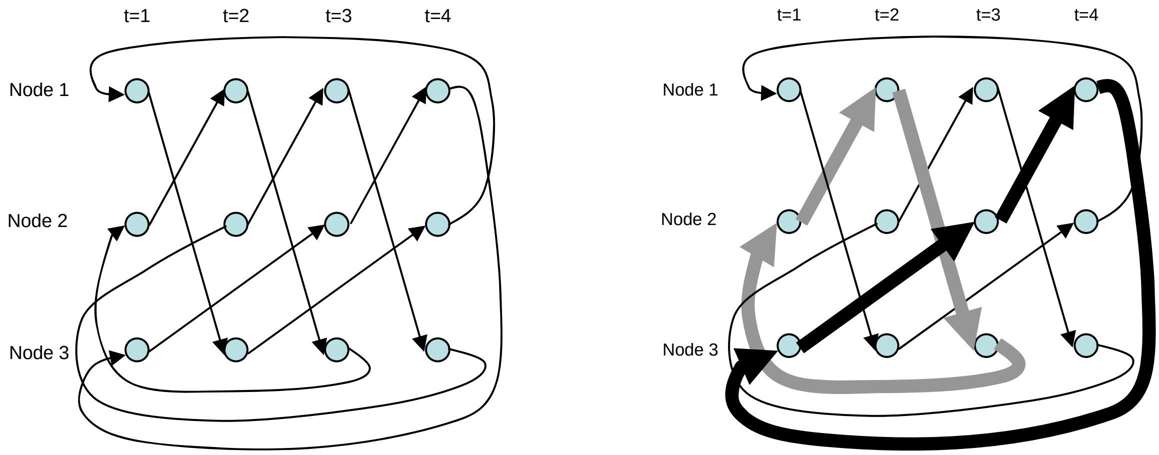
$$\sum_{i,j \in \mathcal{A}: T_{IME}(i) \le t < T_{IME}(j)} y_{ij} \le V_{MAX}, \qquad \forall t \in \mathcal{T}, \qquad (8)$$

In Andersen et al. (2006) constraints (8) were used instead of having fixed costs on design arcs, because it was assumed that the fleet of vehicles was given and the focus was on time minimization. The objective function was thus not affected by how the vehicles were managed, as long as they were able to cover all the services that were selected for operation. However, it might be useful to include an option to not utilize all vehicles in the fleet. By introducing a cost for each vehicle used, we are able to let the model determine the number of vehicles needed. Such costs for use of vehicles can replace fixed costs on design arcs, or be used in addition to the costs on arcs. The idea is not to include a thorough monetary analysis with trade-offs between fixed costs for vehicles and flow costs for products, but rather to investigate whether the fleet size could be reduced.

Repetitiveness in the operations

In some planning problems there is a need for repetitiveness in the operations. This need could stem from crew members needing to start operations in their hometown every Monday morning, or transport operations where specific vehicles of a fleet should operate the same services in each repetition of the planning horizon. This repetitiveness requirement can be handled by adding constraints stating that each vehicle needs to be in the same locations in all time periods when the planning horizon is repeated. Each vehicle must be addressed explicitly to capture this requirement. In a cyclic time-space representation, the repetitiveness requirement implies that the assets are represented by cycles in the network, as can be seen from Figure 1.

Figure 1 illustrates a time-space network with three physical nodes and four time periods. In Figure 1a), the solid lines represent design arcs that may be chosen for operation. In Figure 1b), six design arcs are opened (bold lines), and these satisfy design balance constraints (7), as all nodes have the same in- and outdegree. We observe that the selected design arcs constitute two cycles in the time-space network. The operations illustrated in Figure 1b) correspond to the use of exactly two vehicles, in line with constraints (8).



1a) 1b)

Figure 1. Time-space diagram for a network with three nodes and four time periods (1a) and example of feasible service plan (1b).

Several solution approaches for CMND models are based on introduction of path variables for the flow, which eliminates the need for node balance constraints for the flow (2). An elaboration of column generation based on path flow variables can be seen in Ahuja et al. (1993). In addition to this idea, we explore an analogous approach to the introduction of path variables for the flow: The repetitiveness requirements for assets imply that each asset can be represented by exactly one cycle. We thus introduce path variables for the design arcs, where the paths actually are cycles in the time-space network. The idea is to remove the difficult node balance constraints for design variables (7) from the formulation.

With the opportunity to have two different representations for the flow variables and two representations of the design variables, we may deduce four different representations of the problem. To distinguish between the models we introduce the following taxonomy for the CMNDAM formulations:

yx-CMNDAM, where
y=a or c, depending of whether design variables are defined in terms of arcs or cycles
x=a or p, depending of whether flow variables are defined in terms of arcs or paths

4. Solution methods and scope of presentation

In the computational study we work with a CMNDAM model incorporating all the aspects that we discussed in Section 3. We evaluate the four alternative model formulations for a range of test cases and compare running times for solution approaches based on the different formulations. For the cycle generation we develop a heuristic for arc aggregation. Strengths and weaknesses of the alternative formulations are discussed.

5. Concluding remarks

In this extended abstract we have presented the capacitated multicommodity fixed charge network design problem with design balance constraints and other asset management considerations. We have presented new ideas for model solving based on cycle representations in time-space networks, where the cycles are clusters of design variables. Four alternative model formulations are solved and compared in the computational study.

References

Ahuja, R.K., Magnanti, T.L. and Orlin, J.B. (1993). Network Flows. Prentice Hall, Upper Saddle River, NJ.

Andersen, J., Crainic, T.G. and Christiansen, M. (2006) Optimizing intermodal transfers. Working paper.

Balakrishnan, A., Magnanti, T.L. and Mirchandani, P. (1997). Network Design. In Dell'Amico, M., Maffoli, F. and Martello, S., editors, Annotated bibliographies in combinatorial optimization, pages 311-334. John Wiley & Sons, New York, NY.

Barnhart, C. and Schneur, R.R (1996) Air network design for express shipment service. *Operations Research*, 44 (6), 852- 863.

Crainic, T.G. (2000) Service network design in freight transportation. *European Journal of Operational Research*, 122(2), 272-288.

Lai, M.F. and Lo, H.K. (2004) Ferry service network design: optimal fleet size, routing and scheduling. *Transportation Research A* 38, 305-328.

Magnanti, T.L. and Wong, R.T. (1984) Network design and transportation planning: models and algorithms. *Transportation Science* 18(1), 1-55.

Pedersen, M.B., Crainic, T.G. and Madsen, O.B.G. (2006a) A Design Balanced Network Design Model for Transportation Service Networks. Working paper.

Pedersen, M.B., Crainic, T.G. and Madsen, O.B.G. (2006b) Optimization of Intermodal Freight Train Service Schedules on Train Canals. Working paper.

Smilowitz, K.R., Atamtürk, A. and Daganzo, C.F. (2003) Deferred item and vehicle routing within integrated networks. *Transportation Research E* 39, 305-323.