An Approximation Algorithm for Quay Crane Scheduling with Non-interference Constraints in Port Container Terminals

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1. Introduction

Nowadays competition between port container terminals, especially between geographically close ones, is rapidly increasing. How to improve the competitiveness of port container terminals is, therefore, an immediate challenge, with which port operators are confronted. In terms of port competitiveness, the makespan of a container vessel, which is the latest completion time among all handling tasks of the container vessel, is a critical success factor (Steenken et al., 2004). In reality, quay crane scheduling significantly affects the makespan of a container vessel since quay cranes are the interface between land side and water side in any port container terminals. Thus this paper aims to study quay crane scheduling problem to enhance the competitiveness of port container terminals.

As illustrated in Figure 1, container vessels are typically divided longitudinally into holds that open to the deck through a hatch. Holds are about eight containers deep, and containers can also be stacked (about six high) on deck (Daganzo, 1989). The interference between quay cranes is that quay cranes cannot cross over each other because they are on the same track. In practice, only one quay crane can work on a hold at any time. Generally, a quay crane can move to another hold until it completes the current one. The average processing time of a hold is about two hours and the travel time of a quay crane between two holds is about one minute. The quay crane scheduling problem in port container terminals is to determine a handling sequence of holds for quay cranes assigned to a container vessel in fulfilling pre-specified objectives and satisfying various constraints. For instance, there are ten holds in a container vessel, and two quay cranes are allocated to handle the container vessel. Table 1 illustrates a feasible quay crane schedule for this instance. It shows the handling sequence of holds for every quay crane, the processing time of each hold and the time schedule for handling every hold.

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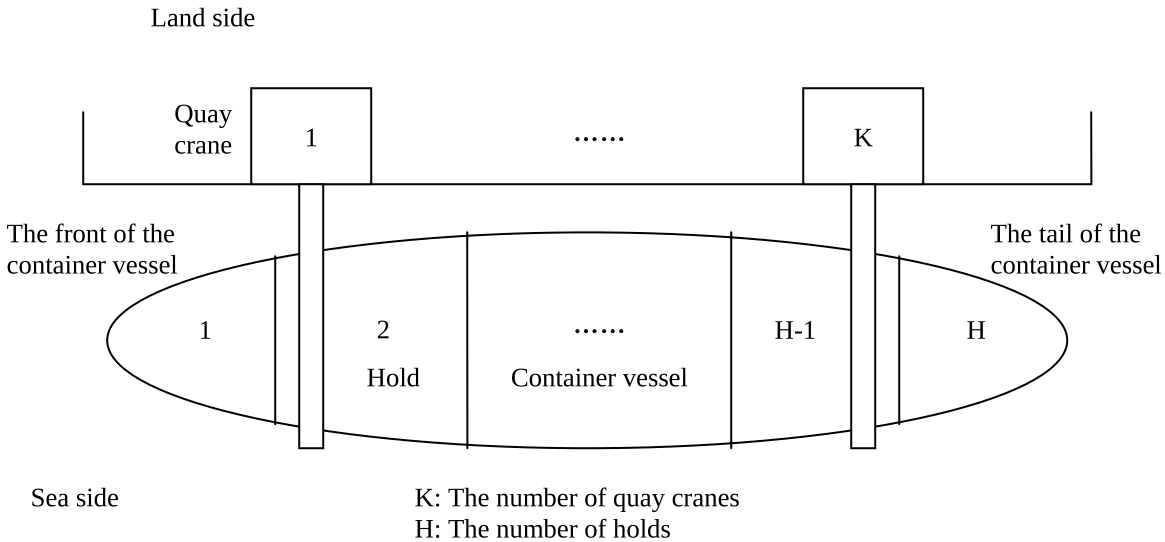

Fig. 1. The illustration of the QCSNIP.

Table 1 An illustration of a quay crane schedule

Quay Crane 1				Quay Crane 2			
Operation Sequence	Hold Number	Processing Time of a Hold (min)	Completion Time of the Quay Crane (min)	Operation Sequence	Hold Number	Processing Time of a Hold (min)	Completion Time of the Quay Crane (min)
1	1	98	98	**1**	2	81	81
2	3	119	217	**2**	5	178	259
3	4	52	269	**3**	10	171	430
4	9	101	370	**4**	8	162	592
5	7	114	484				
6	6	81	565				

Daganzo (1989) studied the static and dynamic quay crane scheduling problems for multiple container vessels. Daganzo (1989) assumed that container vessels were to divide into holds, and only one quay crane could work on a hold at a time. Quay cranes could be moved freely and quickly from hold to hold, and container vessels could not depart until all their holds had been handled. The objective was to serve all these container vessels, while minimizing their aggregate cost of delay. Exact and approximate solution methods for quay crane scheduling were presented in Daganzo (1989). Furthermore, Peterkofsky and Daganzo (1990) developed a branch and bound solution method for the static quay crane scheduling problem. Nevertheless, both papers did not consider the interference between quay cranes, which means the quay cranes could unrealistically cross over each other.

Lim et al. (2004) augmented the static quay crane scheduling problem for multiple container vessels by taking into account non-interference constraints. They assumed that containers from a given area on a container vessel were a job, and there was a profit value when a job was assigned to a quay crane. The objective was to find a crane-to-job matching which maximized the total profit. Dynamic programming algorithms, a probabilistic tabu search, and a squeaky wheel optimization heuristic were proposed in solving the problem. However, it is difficult to define a profit value associated with a crane-to-job assignment in practice, and hence this research cannot be applied in port container terminals easily.

Kim and Park (2004) discussed the quay crane scheduling problem with non-interference constraints in which only single container vessel was considered. Kim and Park (2004) defined a task as an unloading or loading operation for a collection of adjacent slots on one single container vessel. The objective was to minimize the weighted sum of the makespan of the container vessel and the total completion time of all quay cranes. Kim and Park (2004) proposed a branch and bound method and a heuristic algorithm called 'greedy randomized adaptive search procedure (GRASP)' for the solution of the quay crane scheduling problem. Nonetheless, Kim and Park (2004) did not discuss computational complexity of the studied problem to justify why the heuristic algorithm was adopted.

This paper focuses on the Quay Crane Scheduling with Non-Interference constraints Problem (QCSNIP) for any one single container vessel. This work was stimulated from Kim and Park (2004). Section 2 provides a more concise mathematical model than Kim and Park (2004) for the QCSNIP. Moreover, Kim and Park (2004) did not discuss computational complexity of the QCSNIP, but this paper discusses it and proves that the QCSNIP is NP-complete in Section 3. Because there exists no polynomial time algorithm for the exact solution of the QCSNIP, Section 4 proposes an approximation algorithm rather than GRASP of Kim and Park (2004) to obtain its near optimal solution and conducts worst case analysis of the approximation algorithm. The results of computational experiments in Section 5 show that the proposed approximation algorithm is effective and efficient in solving the QCSNIP.

2. Model formulation

This section proposes a mixed integer programming model for the QCSNIP. According to configuration of container vessels, one single container vessel is divided into holds. Figure 1 illustrates the QCSNIP and shows that both quay cranes and holds are arranged in an increasing order from the front to the tail of the container vessel. The following assumptions are imposed in formulating the QCSNIP:

1. Quay cranes are on the same track and thus cannot cross over each other.
2. Only one quay crane can work on a hold at a time until it completes the hold.
3. Compared with processing time of a hold by a quay crane, travel time of a quay crane between two holds is small and hence it is ignored.

In order to formulate the QCSNIP, the following parameters and decision variables are introduced:

Parameters:

K the number of quay cranes;

H the number of holds;

p_h the processing time of hold h by a quay crane ($1 \le h \le H$);

M a sufficiently large positive constant number;

Decision variables:

$X_{h,k}$ 1, if hold h is handled by quay crane k; 0, otherwise ($1 \le h \le H,\ 1 \le k \le K$);

$Y_{h,h'}$ 1, if hold h finishes no later than hold h' starts; 0, otherwise ($1 \le h,\ h' \le H$);

C_h the completion time of hold h ($1 \le h \le H$).

The QCSNIP can be formulated as follows:

Minimize:

$$\max_h \ C_h \tag{1}$$

Subject to:

$$C_h - p_h \ge 0 \quad \forall 1 \le h \le H \tag{2}$$

$$\sum_{k=1}^{K} X_{h,k} = 1 \quad \forall 1 \le h \le H \tag{3}$$

$$C_h - (C_{h'} - p_{h'}) + Y_{h,h'} M > 0 \quad \forall 1 \le h,\ h' \le H \tag{4}$$

$$C_h - (C_{h'} - p_{h'}) - (1 - Y_{h,h'}) M \le 0 \quad \forall 1 \le h,\ h' \le H \tag{5}$$

$$M(Y_{h,h'} + Y_{h',h}) \ge \sum_{k=1}^{K} k X_{h,k} - \sum_{l=1}^{K} l X_{h',l} + 1 \quad \forall 1 \le h < h' \le H \tag{6}$$

$$X_{h,k},\ Y_{h,h'} = 0 \text{ or } 1 \quad \forall 1 \le h,\ h' \le H,\ \forall 1 \le k \le K \tag{7}$$

The objective function (1) minimizes the makespan of handing one single container vessel, which is the latest completion time among all holds. Constraints (2) define the property of the decision variable C_h. Constraints (3) ensure that every hold must be performed only by one quay crane. Constraints (4) and (5) define the properties of decision variables $Y_{h,h'}$: Constraints (4) indicate that $Y_{h,h'} = 1$ if $C_h \le C_{h'} - p_{h'}$, which means $Y_{h,h'} = 1$ when hold h finishes no later than hold h' starts; Constraints (5) indicate that $Y_{h,h'} = 0$ if $C_h > C_{h'} - p_{h'}$, which means $Y_{h,h'} = 0$ when hold h finishes after hold h' starts. Finally, the interference between quay cranes can be avoided by imposing Constraints (6). Suppose that holds h and h' are performed simultaneously and $h < h'$, then this means that $Y_{h,h'} + Y_{h',h} = 0$. Note that both quay cranes and holds are arranged in an increasing order from the front to the tail of the container vessel. Thus, if quay crane k handles hold h and quay crane l handles hold h', then $k + 1 \le l$.

3. Proof of NP-completeness

This section discusses computational complexity of the QCSNIP to justify why heuristic algorithms are adopted. As well known, if a problem is proved to be NP-complete, then there exists no polynomial time algorithm for its exact solution. Hence heuristic algorithms are needed to obtain near optimal solutions for the problem. In this section, the proposed QCSNIP is proved to be NP-complete.

With respect to computational complexity, the decision version of a problem is as hard as the corresponding optimization version; the decision version of a problem has a natural and formal counterpart, which is a suitable object to be studied in a mathematically precise theory of computation. Consequently the theory of NP-completeness is designed to be applied only to the decision version (Garey and Johnson, 1979). The optimization version of the QCSNIP is presented in Section 2, and the decision version is defined as follows:
Parameter:
Z^+ the set of positive integer.
Instance: There are H holds and K quay cranes. The processing time of hold h by a quay crane is $p_h \in Z^+$ ($1 \le h \le H$). There is a given number $C \in Z^+$.
Question: Is there a quay crane schedule for these K quay cranes handling these H holds such that no interference between quay cranes exists and the makespan of the quay crane schedule $\le C$?

The decision version of the QCSNIP is proved to be NP-complete as the following four steps:
Theorem 1: QCSNIP is NP-complete.
Proof:
***Step 1*: Showing that the QCSNIP is in NP.**
If a quay crane schedule for the QCSNIP is given, its feasibility can be checked in polynomial time. Checking whether the quay crane schedule satisfies the non-interference constraints can be done in $O(H^2)$ time. Checking whether the makespan of the quay crane schedule $\le C$ can be done in $O(H)$ time. Therefore, the QCSNIP is in NP.

***Step 2*: Selecting a known NP-complete problem.**
PARTITION is a known NP-complete problem (Garey and Johnson, 1979). The decision version of the PARTITION is defined as follows:
Instance: There are H elements in a finite set $S = \{s_1, s_2, \cdots, s_H\}$. For each element $s_h \in S$, $s_h \in Z^+$ and the sum of all elements $\sum_{s_h \in S} s_h = D$.
Question: Can the set S be partitioned into two disjoint subsets S_1 and S_2 such that $\sum_{s_h \in S_1} s_h = \sum_{s_h \in S_2} s_h = D/2$?

A numerical example of the PARTITION is provided as follows. There is a finite set $S = \{95, 71, 136, 114, 192, 75, 123\}$ and the sum of all elements $\sum_{s_h \in S} s_h = D = 806$. The answer to **Question** is **Yes** because the set S can be partitioned into two disjoint subsets $S_1 = \{95, 123, 71, 114\}$ and $S_2 = \{75, 136, 192\}$ such that $\sum_{s_h \in S_1} s_h = \sum_{s_h \in S_2} s_h = D/2 = 403$.

***Step 3*: Constructing a transformation from the PARTITION to the QCSNIP.**

The PARTITION is transformed to the QCSNIP as follows. A QCSNIP instance corresponding to an arbitrary PARTITION instance has K quay cranes and $H + K$ holds; the given number C is set as D; the following Equations (8)-(10) indicate the processing time of each hold which means the processing time of Hold 1 and Hold $H + 2$ is set as $D/2$, the processing time of Hold 2 to Hold $H+1$ is set as s_1 to s_H respectively, and the processing time of Hold $H+3$ to Hold $H + K$ is set as D. Figure 2 illustrates this transformation. It shows K quay cranes, $H + K$ holds and the processing time of each hold.

$$p_1 = p_{H+2} = D/2 \tag{8}$$

$$p_{h+1} = s_h \quad \forall 1 \le h \le H \tag{9}$$

$$p_h = D \quad \forall H + 3 \le h \le H + K \tag{10}$$

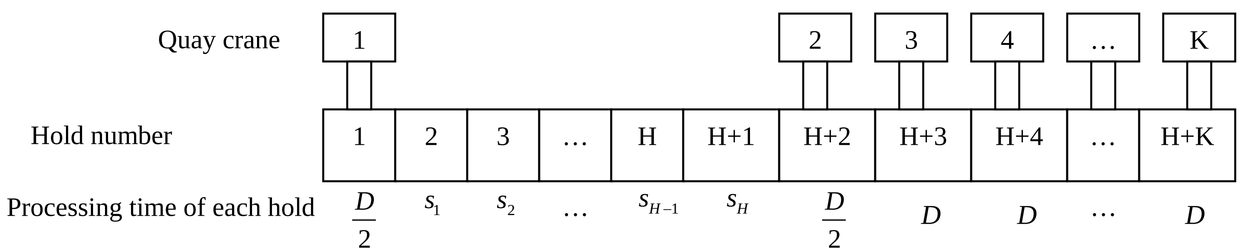

Fig. 2. The illustration of the transformation from the PARTITION to the QCSNIP.

Then, it must be proved that the set S can be partitioned into two disjoint subsets S_1 and S_2 such that $\sum_{s_h \in S_1} s_h = \sum_{s_h \in S_2} s_h = D/2$ if and only if all the $H + K$ holds can be completed by K quay cranes in D time without interference between quay cranes.

First, suppose that the set S can be partitioned into two disjoint subsets S_1 and S_2 such that $\sum_{s_h \in S_1} s_h = \sum_{s_h \in S_2} s_h = D/2$. Then K quay cranes can be scheduled without interference as follows: Quay Crane 1 handles all the Holds $h+1$, where $s_h \in S_1$ and then Hold 1; Quay Crane 2 handles Hold $H + 2$, and then all the Holds $h+1$, where $s_h \in S_2$; Quay Cranes 3 to Quay Crane K handle Hold $H+3$ to Hold $H + K$, respectively. Obviously, there is no interference in this schedule and the latest completion time among all holds is D. Hence, if the set S can be partitioned into two disjoint subsets S_1 and S_2 such that

$\sum_{s_h \in S_1} s_h = \sum_{s_h \in S_2} s_h = D/2$, all the $H+K$ holds can be completed by K quay cranes in D time without interference between quay cranes.

Conversely, suppose all the $H+K$ holds can be completed by K quay cranes in D time without interference between quay cranes, then all the K quay cranes are fully utilized as the sum of the processing time of all the holds is KD. Thus, the completion time of each quay crane must be D. Furthermore, there is no interference in the above mentioned quay crane schedule. According to it, the sum of the processing time of all the holds except Hold 1 handled by Quay Crane 1 must be $D/2$ and the sum of the processing time of all the holds except Hold $H+2$ handled by Quay Crane 2 must be $D/2$ as well, which means that the set S can be partitioned into two disjoint subsets S_1 and S_2 such that $\sum_{s_h \in S_1} s_h = \sum_{s_h \in S_2} s_h = D/2$. Hence, if all the $H+K$ holds can be completed by K quay cranes in D time without interference between quay cranes, the set S can be partitioned into two disjoint subsets S_1 and S_2 such that $\sum_{s_h \in S_1} s_h = \sum_{s_h \in S_2} s_h = D/2$.

***Step 4*: Proving that the above mentioned transformation is a polynomial transformation.**
The above mentioned transformation can be done in $O(H+K)$ time.

Therefore, $\text{PARTITION} \propto \text{QCSNIP}$, and the theorem is proved.

4. An approximation algorithm

As proved in the previous section, QCSNIP is NP-complete, and thus there exists no polynomial time algorithm for the exact solution of QCSNIP. This paper develops a problem oriented approximation algorithm to obtain near optimal solution which is elaborated as follows:

AT the average working time of a quay crane;
k quay crane number ($1 \le k \le K$);
h_1, h_2 hold number ($1 \le h_1 \le h_2 \le H$).

Step 0: Set $k=1, h_1 = h_2 = 1$.

Step 1: Calculate $AT = \sum_{h=1}^{H} p_h / K$.

Step 2: If $\sum_{h=h_1}^{h_2} p_h \le AT$, then $h_2 = h_2 + 1$ and repeat Step 2; if $\sum_{h=h_1}^{h_2} p_h > AT$, then go to Step 3.

Step 3: If $\left|\sum_{h=h_1}^{h_2} p_h - AT\right| \geq \left|\sum_{h=h_1}^{h_2-1} p_h - AT\right|$, then assign Hold h_1 to Hold $h_2 - 1$ to Quay Crane k, set $h_1 = h_2$, $k = k+1$ and go to Step 4; if $\left|\sum_{h=h_1}^{h_2} p_h - AT\right| < \left|\sum_{h=h_1}^{h_2-1} p_h - AT\right|$, then assign Hold h_1 to Hold h_2 to Quay Crane k, set $h_1 = h_2 + 1$, $h_2 = h_2 + 1$, $k = k+1$ and go to Step 4.

Step 4: If $k \leq K-1$, then go to Step 2; if $k = K$, then assign Hold h_1 to Hold H to Quay Crane K and go to End.

Worst case of the proposed approximation algorithm is analyzed as follows:

c_k the completion time of quay crane k ($1 \leq k \leq K$);

Z^H the solution obtained by the approximation algorithm;

Z^* the optimal solution to the QCSNIP.

Theorem 2: $Z^H / Z^* \leq 2$.

Proof:

Note that $Z^H = \max_k c_k$. Assume the completion time of Quay Crane l is the latest and Hold i to Hold $i+j$ are assigned to Quay Crane l, and thus $Z^H = c_l = p_i + p_{i+1} + ... + p_{i+j-1} + p_{i+j}$. According to the approximation algorithm, $p_i + p_{i+1} + ... + p_{i+j-1} \leq AT \leq p_i + p_{i+1} + ... + p_{i+j-1} + p_{i+j}$, and hence $Z^H \leq AT + p_{i+j}$. From the objective function (1) and Constraints (2), it is clear that $Z^* = \max_h C_h \geq p_h \ \forall 1 \leq h \leq H$, and therefore $p_{i+j} \leq Z^*$. Obviously $AT \leq Z^*$, and thus $Z^H \leq AT + p_{i+j} \leq 2Z^*$. The theorem is proved.

As shown in Figure 3, the error bound of 2 is tight for the proposed approximation algorithm in terms of the instance which has K quay cranes and $2K$ holds (assume $K > 3$). The processing time of the leftmost K holds is all $K-1$ and the processing time of the rightmost K holds is all 1. The optimal schedule is to assign two holds to each quay crane, one from the leftmost K holds and the other from the rightmost K holds. The optimal makespan is K. The approximation algorithm is to assign Hold 1 to Hold $K-1$ to Quay Crane 1 to Quay Crane $K-1$ respectively and to assign Hold K to Hold $2K$ to Quay Crane K. The makespan obtained by the approximation algorithm is $2K-1$. Therefore, $Z^H / Z^* = (2K-1)/K \to 2$ as $K \to \infty$.

Hold number	1	…	K	K+1	…	2K
Processing time of each hold	K-1	K-1	K-1	1	1	1

Fig. 3. A tight instance for the approximation algorithm.

5. Computational experiments

In order to evaluate the performance of the proposed approximation algorithm, the lower bound can be calculated firstly by relaxing the non-interference constrains. The mathematical model of the relaxed problem is formulated as follows:

Minimize:

$$\max_k \; c_k \tag{11}$$

Subject to:

$$\sum_{k=1}^{K} X_{h,k} = 1 \quad \forall 1 \le h \le H \tag{12}$$

$$c_k \ge \sum_{h=1}^{H} X_{h,k} \, p_h \quad \forall 1 \le k \le K \tag{13}$$

The objective function (11) minimizes the makespan of handing one single container vessel without considering interference between quay cranes. Constraints (12) ensure that every hold must be performed only by one quay crane. Constraints (13) define the property of the decision variable c_k. Then, the mathematical model of the relaxed problem can be exactly solved by CPLEX (a commercial software for exactly solving integer programming). The objective function value of the optimal solution to the relaxed problem obtained from CPLEX is the lower bound to the original problem.

Twenty computational experiments are conducted to examine the performance of the proposed approximation algorithm that is coded in C++ and executed in a Pentium IV 1.7GHz PC with 256MB RAM. The processing time of a hold is randomly generated from a uniform distribution of $U(30, 300)$. As observed in Table 2, the gaps between solutions obtained from the proposed Approximation Algorithm (AA) and lower bounds are all small (for example the maximum gap among the twenty instances is 11.18%, the minimum gap is 1.59%, and the average gap is 7.08%), and all the computational time of these twenty instances is within one second. Therefore, the proposed approximation algorithm is concluded to be effective and efficient in solving the proposed QCSNIP.

Table 2 The results of computational experiments

Experiment No	Size (holds×cranes)	Lower Bound	AA	Gap [a] (%)
1	16×3	953	990	3.88
2	16×4	754	766	1.59
3	17×3	960	1044	8.75
4	17×4	667	714	7.05
5	18×3	964	1024	6.22
6	18×4	723	795	9.96
7	19×3	906	941	3.86
8	19×4	861	933	8.36
9	20×3	915	998	9.07
10	20×4	686	727	5.98
11	21×3	1134	1181	4.14
12	21×4	850	937	10.24
13	22×3	1453	1487	2.34
14	22×4	1011	1116	10.39
15	23×3	1312	1441	9.83
16	23×4	984	1080	9.76
17	24×3	1372	1476	7.58
18	24×4	1216	1352	11.18
19	25×3	1484	1532	3.23
20	25×4	1113	1204	8.18

[a] Gap = (solution obtained from the proposed AA - lower bound)×100/lower bound

6. Conclusions

The contributions of this paper to the literature are that it has provided a mixed integer programming model for the proposed QCSNIP, proved that the QCSNIP is NP-complete and proposed an approximation algorithm to obtain near optimal solutions for the QCSNIP. In addition, worst case of the proposed approximation algorithm has been analyzed and computational experiments have been performed to examine the proposed approximation algorithm. The results showed that the proposed approximation algorithm has been effective and efficient in solving the QCSNIP.

In this paper, factors such as the travel time of a quay crane between two holds and the handling priority of every hold were not taken into account. The incorporation of these factors into the QCSNIP can be a topic for future research.

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