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HANDLING INCONSISTENCY OF TRAFFIC COUNTS IN PATH FLOW ESTIMATOR

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Abstract

Path Flow Estimator (PFE) is a one-stage network observer proposed to estimate path flows and hence origin-destination (O-D) flows from traffic counts in a transportation network. Although PFE does not require traffic counts to be collected on all network links when inferring unmeasured traffic conditions, it does require all available counts to be reasonably consistent. This requirement is difficult to fulfill in practice due to errors inherited in data collection and processing. The original PFE model handles this issue by relaxing the requirement of perfect replication of traffic counts through the specification of error bounds. This method enhances the flexibility of PFE by allowing the incorporation of local knowledge, regarding the traffic conditions and the nature of traffic data, into the estimation process. However, specifying appropriate error bounds for all observed links in real networks turns out to be a difficult and time-consuming task. In addition, improper specification of the error bounds could lead to a biased estimation of total travel demand in the network. This paper therefore proposes the norm approximation method capable of internally handling inconsistent traffic counts in PFE. Specifically, three norm approximation criteria are adopted to formulate three L_p-PFE models for estimating consistent path flows and O-D flows that minimize the deviation between the estimated and observed link flows. A partial linearization algorithm embedded with an iterative balancing scheme and a column generation procedure is developed to solve the three L_p-PFE models. In addition, the proposed L_p-PFE models are illustrated with numerical examples and the characteristics of solutions obtained by these models are discussed.

Keywords: Origin-destination estimation, path flow estimator, stochastic user equilibrium, norm approximation, partial linearization method.

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1. INTRODUCTION

Path Flow Estimator (PFE) originally developed by Bell and Shield (1995) is one of the efficient methods for estimating path flows (hence origin-destination (O-D) flows) from traffic counts. The attractiveness of PFE lies on the fact that it is a single level mathematical program in which the interdependency between O-D demand and route choice behavior (congestion effect) is taken into account without the need to employ the bi-level mathematical program (one level estimates the O-D trip table while the other represents the behavioral responses of network users). Network users are assumed to follow the stochastic user equilibrium (SUE) assumption, which allows the selection of non-equal travel time paths due to the imperfect knowledge of network travel times and yields unique path flow estimates. Besides, PFE can perform the estimation using traffic counts collected only on a subset of network links. Nevertheless, these available counts must be reasonably consistent or constitute a consistent system of linear constraints. This requirement is difficult to fulfill in practice due to the errors involved in data collection and processing. If the system of linear constraints is inconsistent, there may not exist any feasible path flow solution that is able to reproduce all traffic counts exactly. This is one source of the inconsistency problem while the other is caused by the capacity constraints used to restrict the estimated link flows on the unobserved links. It is often observed that the total observed flows entering node is greater than the capacity of all exiting links combined or vice versa. In such conditions, no feasible path flow solution is able to satisfy both observation and capacity constraints at the same time.

The original PFE model handles the inconsistency problem by allowing user-specified error bounds (i.e., confidence interval) on the traffic counts. A more reliable traffic count would constrain the estimated link flow within a smaller tolerance, while a less reliable traffic count would allow for a larger deviation. This method enhances the flexibility of PFE by allowing the user (e.g., an experienced traffic engineer who is familiar with the network conditions) to incorporate local knowledge about the network conditions to the estimation process. However, specifying appropriate error bound for every single traffic count in a network of realistic size could be very laborious. One could set a uniform error bound (e.g., a default value of 10% error) across all observations, but this setting might be too loose for the more reliable traffic counts and too tight for the less reliable traffic counts. In addition, setting a uniform error bound could lead to biased estimates of the O-D demand. Typically, the total travel demand of the study area is biased downward due to the minimization of the objective function used in the PFE model (Chootinan et al., 2005a).

Although several preprocessing procedures (Van Zuylen and Branston, 1982; Kikuchi et al., 2000) have been proposed to remove the inconsistency problem among traffic counts prior to the O-D estimation process, it might be more appealing to let the mathematical program to handle this task by itself since the inconsistency of traffic counts is a natural part of the O-D estimation problem. Jornsten and Wallace (1993) formulated an unconstrained stochastic program to simultaneously maximize the entropy objective function and to minimize the expected deviation between the observed and estimated link flows. The requirement of exact replication of the traffic counts was relaxed and explicitly incorporated into the entropy objective function as a penalty term. Similarly, Sherali et al. (1994) proposed a linear PFE model in which two sets of non-negative artificial variables are included into the observation constraints to account for the positive and negative deviations of the link flow estimates from the traffic counts. These artificial variables are concurrently minimized while solving for the deterministic user equilibrium (DUE) path flow pattern. Instead of considering the deviations of link flow estimates directly, Van Aerde et al. (2003) incorporated the first-order necessary conditions of the generalized least squares (GLS) model into the maximum likelihood framework. The first-order necessary conditions were included directly into the objective function as a penalty term to obtain an unconstrained maximum likelihood model, which determines the most likely trip table that produces link flow estimates with the least deviations (similar to least squares) from traffic counts.

In this paper, we propose using the norm approximation method to internally handle the inconsistent traffic counts within the nonlinear PFE model proposed Bell and Shield (1995). Three deviation criteria (i.e., L_∞ norm, L_1 norm, and L_2 norm) for approximating a solution of unsolvable (inconsistent) system of linear equations (i.e., constraint set) are considered in formulating three L_p-PFE models. The organization of this paper is as follows. Section 2 provides the formulations of three L_p-PFE models. Section 3 describes a solution procedure for solving the proposed L_p-PFE models. Section 4 provides numerical results to demonstrate the applicability of the proposed models. Finally, concluding remarks are provided in Section 5.

2. MODEL FORMULATIONS

In this study, we propose a method to internally handle inconsistent traffic counts within the nonlinear PFE model proposed Bell and Shield (1995). Due to measurement errors inherited in traffic counts, there may not exist a path flow solution that can reproduce all traffic counts exactly; however, if measurement errors are allowed in the estimation, a path flow solution may be found to match all traffic with different degrees of deviation between the estimated and observed link flows. This path flow pattern is usually associated with some estimation errors given by:

$$\psi_a = \left| v_a - \sum_{rs} \sum_k f_k^{rs} \delta_{ka}^{rs} \right|, \qquad \forall a \in \mathrm{M}, \tag{1}$$

where

M: Set of links with traffic counts;

v_a: Observed flow on link *a*;

f_k^{rs}: Estimated flow on path *k* connecting origin *r* and destination *s*;

δ_{ka}^{rs}: Path-link indicator: 1 if link *a* is on path *k* between origin *r* and destination *s*, and 0 otherwise;

ψ_a: Error associated with the selected path flow pattern fails to satisfy the observed flow on link *a*.

Intuitively, the best approximate path flow pattern is the solution that keeps such deviation as small as possible. However, there are several ways to define the deviation L_p norm defined below.

$$\|\mathbf{\Psi}\|_p = \left(\sum_{a \in \mathbf{M}} \psi_a^p \right)^{1/p} . \tag{2}$$

In practice, three different norms (i.e., different p values) - namely the L_∞, L_1, and L_2 norms - are considered for evaluating the approximate solutions. They constitute the criteria that aim to (i) minimize the maximum absolute error ($p \Rightarrow \infty$), (ii) minimize the average absolute error (p=1), and (iii) minimize the average squared error (p=2), respectively. The question as to which criterion should be adopted is not trivial and depends upon, for examples, the nature of errors causing the inconsistency problem, the required characteristic of the approximate solution, etc. As discussed by Chvatal (1983), minimizing the L_1 norm leads to the most robust approximate solution. Here, the robustness is defined by the insensitivity to the outliers (e.g., flawed data). Minimizing the L_∞ norm, on the other hand, tends to minimize gross discrepancies between the observed and adjusted values (e.g., accommodate all the data points as much as possible), thus it is quite sensitive to the outliers (i.e., less robust). Lastly, minimizing the L_2 norm can be shown to be suitable for the applications in which the errors causing the inconsistency tend to be small and follow the normal distribution. Using the above norm approximations, we develop three modified L_p-PFE formulations to determine the stochastic user equilibrium (SUE) path flow pattern that minimizes the estimation errors.

2.1 L_∞ Approximation

Let us consider the maximum absolute error defined below.

$$\psi_\mathrm{o} = \underset{\forall a \in \mathrm{M}}{\mathrm{Max}} \{ \psi_a \}. \tag{3}$$

Since ψ_0 is the maximum absolute error among all observations, the following condition must hold.

$$-\psi_\mathrm{o} \le v_a - \sum_{rs} \sum_k f_k^{rs} \delta_{ka}^{rs} \le \psi_\mathrm{o}, \qquad \forall a \in \mathrm{M}. \tag{4}$$

A feasible path flow solution can be defined by two inequalities in Equation (4), which represents the lower and upper limits of the estimated link flow. One can view the maximum absolute error (ψ_0) as a flow on the virtual path, which traverses through all measured links. It plays the role of setting the boundaries acceptable for the link flow estimates. Since the virtual path does not really exist, intuitively it should not be used very often. In addition, flow on the virtual path should be small, but large enough to ensure the existence of a feasible path flow solution. Hence, the modified L_∞-PFE formulation is to search for a SUE path flow pattern that produces a link flow pattern within the maximum absolute error as follows.

Minimize: $$Z_{L_\infty} = \sum_a \int_0^{x_a} t_a(w)dw + \frac{1}{\theta}\sum_{rs}\sum_k f_k^{rs}(\ln f_k^{rs} - 1) + \frac{1}{\theta}\psi_o(\ln\psi_o - 1) + \rho_o\psi_o \quad (5a)$$

subject to:

$$x_a \geq v_a - \psi_o, \quad \forall a \in \mathrm{M}, \quad (5b)$$

$$x_a \leq v_a + \psi_o, \quad \forall a \in \mathrm{M}, \quad (5c)$$

$$x_a \leq C_a, \quad \forall a \in \mathrm{U}, \quad (5d)$$

$$x_a = \sum_{rs}\sum_k f_k^{rs}\delta_{ka}^{rs}, \quad \forall a \in \mathrm{A}, \quad (5e)$$

$$q_{rs} = \sum_{k \in K_{rs}} f_k^{rs}, \quad \forall\, rs \in \mathrm{RS}, \quad (5f)$$

$$f_k^{rs} \geq 0, \quad \forall\, k \in K_{rs}, rs \in \mathrm{RS}, \quad (5g)$$

$$\psi_o \geq 0, \quad (5h)$$

where

U: Set of links without traffic counts;

A: Set of links in the network, A = M∪U;

RS: Set of O-D pairs;

K_{rs}: Set of paths connecting O-D pair *rs*;

θ: Dispersion parameter;

ρ_o: Penalty cost;

x_a: Estimated flow on link *a*;

C_a: Capacity of link *a*;

$t_a(\cdot)$: Travel time function of link *a*;

q_{rs}: Estimated flow of O-D pair *rs*.

The objective function (5a) of the modified L_∞-PFE formulation is to minimize the travel costs and path entropies for both physical and virtual paths. The entropy of the virtual path is treated in the same manner as those of the physical paths while the travel cost of the virtual path is treated as a penalty term. The penalty cost (ρ_o) should be chosen judiciously such that the maximum deviation is minimized. Equations (5b) and (5c) define the lower and upper limits of the estimated link flows, respectively. These two constraints restrict the estimated link flows (derived from the physical path flow estimates) to be within the boundaries defined by the traffic counts and the maximum absolute error (ψ_o). For the unobserved links, the estimated link flows are constrained not to exceed their capacities defined by Eq. (5d). Equations (5e) and (5f) are definitional constraints to obtain link flows and O-D flows from the path flow solution. Equations (5g) and (5h) ensure the non-negativity of both physical and virtual path flows.

The Lagrangian function of the modified L_∞-PFE formulation and its first partial derivatives with respect to path-flow variables can be expressed as follows:

$$\begin{aligned} L(\mathbf{f}, \Psi, \ell, \mathbf{u}, \mathbf{d}) = Z_{L_\infty} &+ \sum_{a \in M} \ell_a \cdot (v_a - \sum_{rs} \sum_k f_k^{rs} \delta_{ka}^{rs} - \psi_o) + \\ &\sum_{a \in M} u_a \cdot (v_a - \sum_{rs} \sum_k f_k^{rs} \delta_{ka}^{rs} + \psi_o) + \\ &\sum_{a \in U} d_a \cdot (C_a - \sum_{rs} \sum_k f_k^{rs} \delta_{ka}^{rs}) \end{aligned} , \tag{6}$$

$$\frac{\partial L}{\partial f_k^{rs}} = 0 \Rightarrow \frac{1}{\theta} \ln f_k^{rs} + \sum_{a \in A} t_a(x_a) \delta_{ka}^{rs} - \sum_{a \in M} \ell_a \delta_{ka}^{rs} - \sum_{a \in M} u_a \delta_{ka}^{rs} - \sum_{a \in U} d_a \delta_{ka}^{rs} = 0 , \tag{7}$$

$$\frac{\partial L}{\partial \psi_o} = 0 \Rightarrow \frac{1}{\theta} \ln \psi_o + \rho_o - \sum_{a \in M} \ell_a + \sum_{a \in M} u_a = 0 . \tag{8}$$

The optimality conditions lead to the analytical expressions of flows for both physical paths and virtual path as follows:

$$f_k^{rs} = \exp\left(\theta(-\sum_{a \in A} t_a(x_a) \delta_{ka}^{rs} + \sum_{a \in M} (\ell_a + u_a) \delta_{ka}^{rs} + \sum_{a \in U} d_a \delta_{ka}^{rs}) \right), \ \forall\, k \in K_{rs}, rs \in \mathrm{RS} \tag{9}$$

$$\psi_o = \exp\left(\theta(-\rho_o + \sum_{a \in M} (\ell_a - u_a)) \right), \tag{10}$$

where ℓ_a, u_a, and d_a are the dual variables of constraints (5b), (5c), and (5d), respectively. The values of u_a and d_a are restricted to be non-positive while the value of ℓ_a must be nonnegative. ℓ_a and u_a can be viewed as the corrections in link costs, which bring the estimated path flows into agreement with the observed link volumes. d_a is related to link delay when the estimated link flow reaches its capacity.

2.2 L_1 Approximation

In the L_1 approximation, instead of using only one virtual path to absorb the residuals of all link flow estimates, there are as many virtual paths as the number of observed links. In other words, there is one virtual path for each observation. Likewise, there is one cost penalty for each estimation error (ρ_a), which reflects the reliability (confidence) of each observation. It should be noted that the L_1 norm gives the mean absolute error (MAE) commonly used as a statistical measure to evaluate the closeness of link flow replication. Hence, the modified L_1-PFE formulation is to search for a SUE path-flow pattern that produces a link flow pattern with the minimum MAE as follows.

$$\text{Minimiz}\, Z_{L_1} = \sum_a \int_0^{x_a} t_a(w)dw + \frac{1}{\theta}\sum_{rs}\sum_k f_k^{rs}(\ln f_k^{rs} - 1) + \frac{1}{\theta}\sum_{a\in \mathrm{M}} \psi_a(\ln \psi_a - 1) + \sum_{a\in \mathrm{M}} \rho_a \psi_a \quad (11a)$$

subject to

$$x_a \geq v_a - \psi_a, \qquad \forall a \in \mathrm{M}, \quad (11b)$$

$$x_a \leq v_a + \psi_a, \qquad \forall a \in \mathrm{M}, \quad (11c)$$

$$x_a \leq C_a, \quad \forall a \in \mathrm{U}, \quad (11d)$$

$$x_a = \sum_{rs}\sum_k f_k^{rs}\delta_{ka}^{rs}, \quad \forall a \in \mathrm{A}, \quad (11e)$$

$$q_{rs} = \sum_{k\in K_{rs}} f_k^{rs}, \quad \forall\, rs \in \mathrm{RS}, \quad (11f)$$

$$f_k^{rs} \geq 0, \quad \forall\, k \in K_{rs}, rs \in \mathrm{RS}, \quad (11g)$$

$$\psi_{\mathrm{a}} \geq 0, \qquad \forall a \in \mathrm{M}. \quad (11h)$$

Following the same derivation as the modified L_∞-PFE formulation above, the solution to the L_1-PFE formulation is given by:

$$f_k^{rs} = \exp\left(\theta(-\sum_{a\in A} t_a(x_a)\delta_{ka}^{rs} + \sum_{a\in M}(\ell_a + u_a)\delta_{ka}^{rs} + \sum_{a\in U} d_a \delta_{ka}^{rs})\right), \ \forall\, k \in K_{rs}, rs \in \mathrm{RS}, \quad (12)$$

$$\psi_a = \exp\big(\theta(-\rho_a + \ell_a - u_a)\big), \quad \forall a \in M. \quad (13)$$

As can be seen above, the analytical expression of path-flow estimates of the L_1-PFE formulation, Eq. (12), and of the L_∞-norm formulation, Eq. (9), are exactly the same, although the analytical expressions of their virtual paths are different. Besides the difference in the number of virtual paths, the single virtual path flow in the L_∞-norm formulation is controlled by the magnitude of all dual variables for all observation constraints, while each virtual path flow in the L_1-norm formulation is controlled by the magnitude of two dual variables of each observed link: the lower and upper limits (ℓ_a and u_a). These dual variables represent the difficulty in replicating each observed link volume, reflecting the magnitude of flow on each virtual path (i.e., residual).

2.3 L_2 Approximation

Similar to the L_1 approximation, the L_2 norm is also a statistical measure known as the root mean squared error (RMSE). The modified L_2-PFE formulation is to search for a SUE path flow pattern that produces a link flow pattern with the minimum RMSE as follows.

$$\text{Minimize } Z_{L_2} = \sum_a \int_0^{x_a} t_a(w)dw + \frac{1}{\theta}\sum_{rs}\sum_k f_k^{rs}(\ln f_k^{rs} - 1) + \frac{1}{\theta}\sum_{a\in \mathrm{M}} \psi_a(\ln \psi_a - 1) + \sum_{a\in \mathrm{M}} \rho_a \psi_a^2 \quad (14a)$$

subject to

$$x_a \geq v_a - \psi_a, \qquad \forall a \in \mathrm{M}, \quad (14b)$$

$$x_a \leq v_a + \psi_a, \qquad \forall a \in \mathrm{M}, \quad (14c)$$

$$x_a \leq C_a, \quad \forall a \in \mathrm{U}, \quad (14d)$$

$$x_a = \sum_{rs}\sum_k f_k^{rs}\delta_{ka}^{rs}, \quad \forall a \in \mathrm{A}, \quad (14e)$$

$$q_{rs} = \sum_{k\in K_{rs}} f_k^{rs}, \quad \forall\, rs \in \mathrm{RS}, \quad (14f)$$

$$f_k^{rs} \geq 0, \quad \forall\, k \in K_{rs}, rs \in \mathrm{RS}, \quad (14g)$$

$$\psi_{\mathrm{a}} \geq 0, \qquad \forall a \in \mathrm{M}. \quad (14h)$$

The first partial derivatives of the Lagrangian function of the modified L_2-PFE formulation with respect to path-flow variables can be expressed as follows:

$$\frac{\partial L}{\partial f_k^{rs}} = 0 \Rightarrow \frac{1}{\theta}\ln f_k^{rs} + \sum_{a\in A} t_a(x_a)\delta_{ka}^{rs} - \sum_{a\in M}\ell_a\delta_{ka}^{rs} - \sum_{a\in M} u_a\delta_{ka}^{rs} - \sum_{a\in U} d_a\delta_{ka}^{rs} = 0, \quad (15)$$

$$\frac{\partial L}{\partial \psi_a} = 0 \Rightarrow \frac{1}{\theta}\ln\psi_a + 2\rho_a\psi_a - \ell_a + u_a = 0, \quad (16)$$

These optimality conditions lead to the analytical expressions of flows on the physical paths and the virtual paths as follows:

$$f_k^{rs} = \exp\left(\theta(-\sum_{a\in A} t_a(x_a)\delta_{ka}^{rs} + \sum_{a\in M}(\ell_a + u_a)\delta_{ka}^{rs} + \sum_{a\in U} d_a\delta_{ka}^{rs})\right), \ \forall\, k \in K_{rs}, rs \in \mathrm{RS}, \quad (17)$$

$$\psi_a = \exp\big(\theta(-2\rho_a\psi_a + \ell_a - u_a)\big), \quad \forall a \in M \,. \tag{18}$$

Similarly, the analytical expression of path-flow estimates of the L_2-PFE formulation, Eq. (17), is the same as those of the L_1-PFE formulation, Eq. (12), and the L_∞-norm formulation, Eq. (9). In terms of the expression of the virtual paths in Eq. (18), it is quite different. In the L_2-PFE formulation, ψ_a appears on both sides of the equation due to the quadratic penalty term in the objective function. However, it can be solved numerically in an iterative solution procedure to be discussed in the next section.

3. SOLUTION PROCEDURE

The modified L_p-PFE formulations presented above can be solved by the partial linearization method (Evans, 1976; Patriksson, 1994). The method consists of two major steps: (i) a direction finding and (ii) a line search. In the direction-finding step, certain part of the objective function is linearized. The solution to the partial linearized subproblem defines a feasible direction. The line search step determines how far the current solution should move in the feasible direction. These two steps are iterated until convergence is reached. A column generation is also implemented to avoid path enumeration for a general transportation network.

Consider the modified L_p-PFE formulations presented in Section 2 without the definitional constraints in vector form as follows.

$$\underset{\mathbf{f}}{\text{Minimize}} \qquad Z(\mathbf{f}) = P(\mathbf{f}) + G(\mathbf{f}) \tag{19a}$$

subject to

$$\mathbf{\Delta}_1\mathbf{f} \geq \mathbf{v}\,,\ \mathbf{\Delta}_2\mathbf{f} \leq \mathbf{v}\,,\ \mathbf{\Delta}_3\mathbf{f} \leq \mathbf{C}\,,\ \text{and}\ \mathbf{f} \geq \mathbf{0}\,, \tag{19b}$$

where **f** is a solution vector to the problem, $\mathbf{f} = (\ldots, f_k^{rs}, \ldots, \psi_a, \ldots)$, (i.e., flows on both physical and virtual paths); **v** is a vector of observed link volumes, **C** is a vector of link capacities; **0** is a vector of zeros; Δ_1, Δ_2, and Δ_3 represent the coefficient matrices of the left-hand side of the constraint set; $P(\mathbf{f})$ (i.e., travel cost and penalty terms) is the part of the objective function to be linearized while $G(\mathbf{f})$ (i.e., entropy terms for both physical and virtual paths) is the remaining part of the objective function.

Suppose at iteration *n-1*, a feasible path flow is given. $P(\mathbf{f})$ is linearized, which amounts to assuming that the travel costs are fixed at their current values. The resulting subproblem defined the search direction is given by

$$\underset{\mathbf{h}}{\text{Minimize}} \qquad \nabla_{\mathbf{f}} P(\mathbf{f}^{n-1})^T \mathbf{h} + G(\mathbf{h}) \tag{20a}$$

subject to:

$$\boldsymbol{\Delta}_1\mathbf{h} \geq \mathbf{v},\ \boldsymbol{\Delta}_2\mathbf{h} \leq \mathbf{v},\ \boldsymbol{\Delta}_3\mathbf{h} \leq \mathbf{C}, \text{ and } \mathbf{h} \geq \mathbf{0}, \tag{20b}$$

where $\nabla_{\mathbf{f}} P(\mathbf{f}^{n-1})$ is the derivative of P with respect to $\mathbf{f}$ evaluated at iteration n-1. This subproblem is a nonlinear program with linear inequality constraints and can be solved by the iterative balancing scheme used in the original PFE model for right angle cost functions (Bell and Shield, 1995; Bell et al., 1997).

Given the brief descriptions above, the solution procedure can be summarized into the following steps:

Step 0 (*Initialization*): Generate an initial feasible path flow solution, $\mathbf{f}^1$.

- Set $x_a^0 = \ell_a^0 = u_a^0 = d_a^0 = 0, \forall a \in \mathrm{A},\ \mathrm{K}_{rs}^0 = \varnothing, \forall r,s$.
- Set iteration counter: n = 1.
- Determine the shortest path for all O-D pairs based on free-flow travel time: $\bar{k}_{rs}^n, \forall r,s$.
- Update path set: $\mathrm{K}_{rs}^n = \mathrm{K}_{rs}^{n-1} \cup \bar{k}_{rs}^n, \forall r,s.$
- Solve the following partial linearized subproblem, (20a) and (20b), for $\mathbf{h} = (..., h_k^{rs}, ..., \tau_a, ...)$:

  $$\underset{\mathbf{h}}{\text{Minimize}} \qquad \nabla_{\mathbf{f}} P(0)^T \mathbf{h} + G(\mathbf{h})$$

  subject to: $\boldsymbol{\Delta}_1\mathbf{h} \geq \mathbf{v},\ \boldsymbol{\Delta}_2\mathbf{h} \leq \mathbf{v},\ \boldsymbol{\Delta}_3\mathbf{h} \leq \mathbf{C}$, and $\mathbf{h} \geq \mathbf{0}$.

  The above partial linearized subproblem is solved using the iterative balancing scheme (see Chootinan (2006) for details). Besides the primal variables (**h**), the dual variables, (…, ℓ_a, …, u_a, …, d_a, …), are also available. Set $\ell_a^n = \ell_a, u_a^n = u_a, d_a^n = d_a$
- Set $\mathbf{f}^1$ = **h**, and update link flows: $x_a^n = \sum_{rs}\sum_k f_k^{rs}(n)\delta_{ka}^{rs}$.

Step 1 (*Column Generation*):

- Set iteration counter: $n = n + 1$.
- Update link costs: $\tilde{t}_a^n = t_a\left(x_a^{n-1}\right) - \ell_a^{n-1} - u_a^{n-1} - d_a^{n-1}, \forall a \in \mathrm{A}$.
- Determine the shortest path for all O-D pairs based on $\tilde{t}_a^n$: $\bar{k}_{rs}^n, \forall r,s$.
- Update path set: $\mathrm{K}_{rs}^n = \mathrm{K}_{rs}^{n-1} \cup \bar{k}_{rs}^n, \forall r,s.$

Step 2 (*Direction Finding*): Solve the following partial linearized subproblem for **h**,

$$\underset{\mathbf{h}}{\text{Minimize}} \qquad \nabla_{\mathbf{f}} P(\mathbf{f}^{n-1})^T \mathbf{h} + G(\mathbf{h})$$

subject to: $\boldsymbol{\Delta}_1\mathbf{h} \geq \mathbf{v},\ \boldsymbol{\Delta}_2\mathbf{h} \leq \mathbf{v},\ \boldsymbol{\Delta}_3\mathbf{h} \leq \mathbf{C}$, and $\mathbf{h} \geq \mathbf{0}$.

Step 3 (*Line Search*): Solve the following one-dimension optimization problem for an optimal step size (ϕ).

$$\underset{\phi \in [0,1]}{\text{Minimize}} \quad Z(\mathbf{f}^{n-1} + \phi \cdot (\mathbf{h} - \mathbf{f}^{n-1}))$$

Step 4 (*Solution Update*):

- Update path flows: $\mathbf{f}^{n} = \mathbf{f}^{n-1} + \phi \cdot (\mathbf{h} - \mathbf{f}^{n-1})$
- Update link flows: $x_a^n = \sum_{rs} \sum_{k} f_k^{rs}(n) \delta_{ka}^{rs}$.

Step 5 If a convergence criterion (e.g., maximum change of the path-flow solution between two consecutive iterations is less than a predetermined threshold) is met, stop; otherwise, go to Step 1.

4. NUMERICAL EXAMPLES

4.1 Problem description

To illustrate the application of the modified L_p-PFE formulations in handling inconsistency among traffic counts, a simple grid network is used to study the solution properties as well as the characteristics of the different L_p-PFE models. The grid network depicted in Figure 1 consists of 9 nodes, 14 unidirectional links, and 9 O-D pairs. Nodes 1, 2, and 4 are origin nodes while nodes 6, 8, and 9 are destination nodes (all shaded nodes in Figure 1). For this network, there are a total of 33 paths, all of which will be included in the estimation.

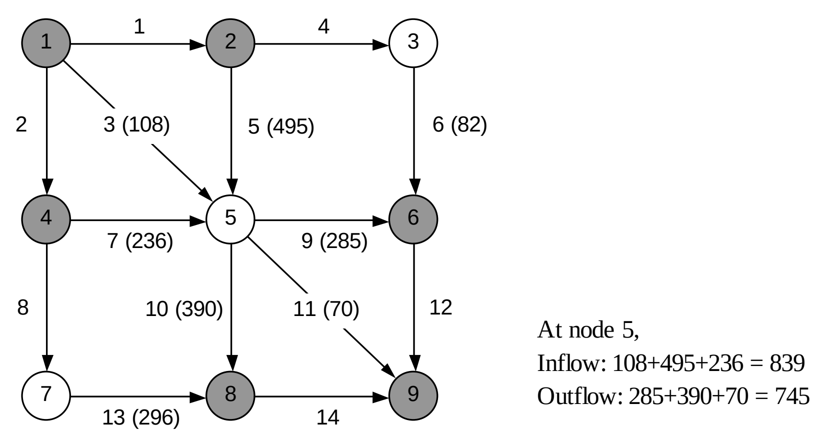


Figure 1 Grid network and observed link volumes

The characteristics of the grid network at the link level are summarized in Table 1 (see also Yang et al. (2001b)). For demonstration purpose, it is assumed that the true O-D trip table is available as shown in Table 2. In addition, this trip table is used to synthesize the observed traffic volumes according to the logit-based SUE model with a dispersion parameter of 1.50. Since the first set of traffic counts reported in Table 1 is a direct result of the logit-based SUE model, they are consistent (i.e., satisfy the conservation of flows at all intermediate nodes). To create inconsistency in traffic counts, it is assumed that the observed traffic volumes are independent Poisson variates with means and variances equal to the link volumes in Set 1. The second set of traffic counts (also see Table 1) is one instance (sample) generated according to this assumption. Observed traffic volumes are assumed available only on links 3, 5, 6, 7, 9, 10, 11, and 13 (8 out of 14 links). Links 6, 9, 10, 11, and 13 were selected to intercept the total demand from all origins to all destinations (see Yang et al. (1991), Yang and Zhou (1998), Yang et al. (2001a), Bierlaire (2002), Chen et al. (2005), and Chootinan et al. (2005a) for a discussion on selecting traffic counts to observe the total demand). Three additional links (links 3, 5 and 7) are included to create nodal-inconsistency at node 5 (see Figure 1).

Table 1: Link characteristics of grid network and observed link volumes

Link	Node		Capacity	Free-flow travel time	SUE link flow (Set 1)	Observed link flow (Set 2)
	From	To				
1	1	2	280.00	2.00	124	-
2	1	4	290.00	1.50	137	-
3	1	5	280.00	3.00	109	108
4	2	3	280.00	1.00	77	-
5	2	5	600.00	1.00	467	495
6	3	6	300.00	2.00	77	82
7	4	5	500.00	2.00	212	236
8	4	7	400.00	1.00	295	-
9	5	6	500.00	1.50	303	285
10	5	8	700.00	1.00	400	390
11	5	9	250.00	2.00	85	70
12	6	9	300.00	1.00	50	-
13	7	8	350.00	1.00	295	296
14	8	9	220.00	1.00	165	-

Table 2: True O-D trip table of grid network

From/To	6	8	9
1	120	150	100
2	130	200	90
4	80	180	110

In this study, two statistical measures, root mean squared error (RMSE) and mean absolute error (MAE) defined below, are considered for assessing the accuracy of link flow estimates:

$$\mathrm{RMSE} = \sqrt{\frac{1}{|\mathrm{M}|}\sum_{a\in\mathrm{M}}(x_a - v_a)^2}\,, \tag{21}$$

$$\mathrm{MAE} = \frac{1}{|\mathrm{M}|}\sum_{a\in\mathrm{M}}|\, x_a - v_a \,|, \tag{22}$$

where |M| is the number of link observations (size of set M), x_a and v_a are the estimated and observed flows on link *a*, respectively. Further note that this study adopts the standard Bureau of Public Road (BPR) function as the link travel time function:

$$t_a = t_a^f \cdot (1 + 0.15 \cdot (x_a / C_a)^4)\,, \tag{23}$$

where t_a^f is the free-flow travel time on link *a*.

4.2 Setting of Experiments

To examine how the original PFE model and the modified L_p-PFE models resolve the inconsistency problem of traffic counts, six experiments using the observed volumes in Set 2 are considered. Experiments A, B, and C are designed to examine the effects of user-specified error bounds in the original PFE model. Experiments D, E, and F are for the three proposed L_p-PFE models, which do not require the user-specified error bounds for the measured links. In experiment A, the user is assumed to know the exact errors for all measured links. In other words, the user knows a set of consistent link volumes with the least deviation from the observed values. Several criteria discussed earlier (e.g., L_1 norm or L_2 norm) could be used to define such a deviation when preprocessing these inconsistent data. The L_1-norm criterion is used to determine the error bounds to obtain consistent link volumes for experiment A1 and the L_2-norm criterion for experiment A2 (see Table 3). An optimal determination of a consistent set of link volumes (i.e., preprocessing method) involves solving a mathematical program that minimizes the deviation between observed and adjusted volumes subject to conservation of flows at intermediate nodes (Kikuchi et al., 2000). It should be mentioned that the preprocessing method for PFE has to take the link capacity constraints (Equation 3) into account as well. The reader may refer to Chootinan (2006) for the detailed descriptions of the preprocessing method employed in this study.

In experiment B, the user is assumed to have a rough idea on the quality of traffic counts and uses the average error determined by the L_2-norm criterion as the default error bound for all measured links (i.e., uniform error bound). In fact, the minimum uniform error bound, which still results in a solvable system of equations, could be determined by the L_∞-norm criterion. The minimum uniform error bound used for this data set is

5.39 percent. Experiment C examines the effect of mis-specified (i.e., unnecessarily large) error bound on the estimation results.

Table 1 Consistent (preprocessed) observed link volumes for grid network

Link	Observation	L_1-norm Criterion			L_2-norm Criterion		
		Consistent flow[a]	Absolute Error	% Error	Consistent flow[a]	Absolute Error	% Error
3	108	108.00	0.00	0.00	92.33	15.67	14.51
5	495	491.80	3.20	0.65	479.33	15.67	3.17
6	82	82.00	0.00	0.00	82.00	0.00	0.01
7	236	184.78	51.22	21.71	220.33	15.67	6.64
9	285	285.00	0.00	0.00	300.67	15.67	5.50
10	390	394.00	4.00	1.03	405.67	15.67	4.02
11	70	105.58	35.58	50.84	85.67	15.67	22.39
13	296	296.00	0.00	0.00	296.00	0.00	0.00
			Average	9.28		Average	7.03

[a] - one pattern among many possibilities (non-unique)

4.3 Handling Traffic Inconsistency with Different PFE Models

Table 4 summarizes the estimation results of the experiments discussed in Section 4.2 by solving different PFE models. Several numerical indices, for instance, two statistical measures (MAE and RMSE) of link-flow estimates, PFE objective value, and computational requirements, are provided for comparison purpose (see also Figure 2 for a graphical comparison of these measures). Table 5 also provides the estimation of individual O-D demands and total demand for each of the six experiments. With a proper setting of the error bounds, the original PFE model can perform the estimation quite well. A well performance here is indicated by the ability to produce a close estimation of observed link volumes (quantified by MAE and RMSE) and the ability to capture the total demand of the network. As expected, in experiment A1 when the error bounds were determined by the L_1-norm criterion, the MAE of link-flow estimates is minimized (compared to all other cases being discussed later). On the other hand, when the L_2-norm criterion is used as in experiment A2, the RMSE of link-flow estimates is minimized instead. In both experiments (A1 and A2), the total demand estimate do not deviate much from the true value (1,160 units) despite that the estimated demands of individual O-D pairs are quite different from the known values (see Table 5). This characteristic (i.e., the under-determinate nature) is common for the O-D estimation problem when only traffic counts (even on all network links) are used.

Table 4 Estimation results for the grid network

Experiment	Max. Err	MAE	RMSE	PFE Obj.	Norm Obj.[a]	Penalty[b]	No. of Iterations		
							Total	Main (n)	Avg. Inner (m)
A1	51.22	11.75	22.12	5,967.29	-	-	1,207	9	134
A2	15.67	11.75	13.57	6,007.24	-	-	480	10	48
B (Avg. error)	34.80	15.07	18.69	5,792.24	-	-	500	10	50
C1 (5.39%)	29.38	14.55	16.85	5,873.17	-	-	564	12	47
C2 (10.0%)	49.50	23.60	27.22	5,577.17	-	-	296	10	30
C3 (12.5%)	61.88	26.56	31.63	5,401.02	-	-	210	10	21
D (L_∞-norm)	15.67	15.67	15.67	5,828.77	2,370.43	150.10	4,467	15	298
E (L_1-norm)	45.49	11.75	20.38	5,711.35	1,216.68	11.27	5,441	19	286
F (L_2-norm)	21.60	13.73	14.84	5,820.61	604.11	0.27	110,428	474	233

[a] – virtual path entropy term in the modified L_p-PFE objective function
[b] – penalty parameter, ρ

Table 5 O-D trip tables estimated by different PFE models

O-D Pair	Ref. Demand	Experiment								
		A1	A2	B	C1 (5.39%)	C2 (10.0%)	C3 (12.5%)	D (L_∞)	E (L_1)	F (L_2)
(1,6)	120.00	47.42	45.42	46.12	48.30	41.35	40.20	44.81	35.94	43.11
(1,8)	150.00	84.56	80.40	84.42	84.03	84.36	81.74	79.14	68.16	77.37
(1,9)	100.00	49.07	42.36	42.45	42.71	41.76	40.88	41.99	32.73	39.93
(2,6)	130.00	203.01	205.60	191.58	198.82	175.34	170.47	193.40	206.00	198.29
(2,8)	200.00	193.12	192.03	192.59	190.61	194.50	188.29	191.97	195.25	191.61
(2,9)	90.00	150.45	137.07	126.84	127.59	124.83	122.17	134.42	131.26	132.99
(4,6)	80.00	49.65	61.87	59.86	62.82	53.37	51.89	61.87	58.15	60.51
(4,8)	180.00	288.73	303.71	291.58	293.09	285.90	277.51	291.97	299.68	296.41
(4,9)	110.00	96.57	101.53	96.01	96.81	93.89	91.73	99.09	95.85	98.38
Total	1,160.00	1,162.58	1,170.00	1,131.44	1,144.78	1,095.30	1,064.87	1,138.67	1,123.01	1,138.60

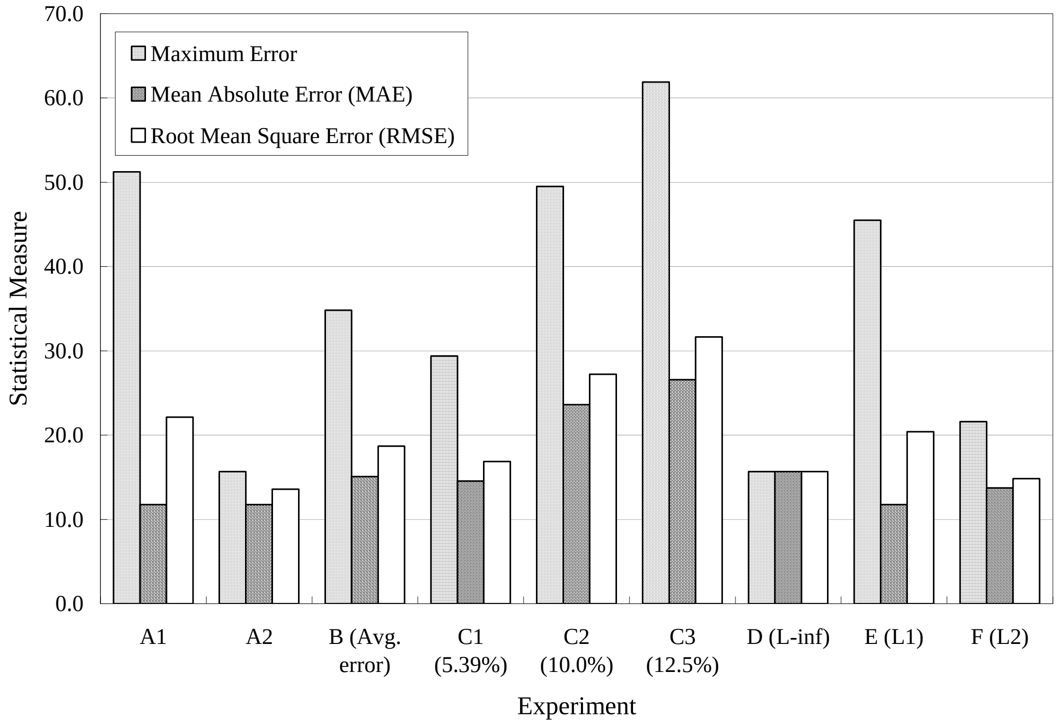


Figure 2 Statistical measures of link-flow estimates by different PFE models

In experiment B when the average error is specified for all measured link flows (i.e., uniform error bound), the performance of the original PFE model deteriorates as indicated by the increased value of both statistical measures (MAE and RMSE) – link-flow estimates are farther away from the observed values. Experiment C examines the effects of specifying different uniform error bounds for all measured links. As mentioned earlier, the smallest uniform error bound required to resolve the inconsistency of this data set is 5.39 percent (experiment C1). If a smaller error bound (e.g., <5.39%) is used, there is no feasible solution. When the uniform error bound (in the original PFE model) is set larger than 5.39 percent (see experiments C2 – 10% and C3 – 12.5%), there could be several feasible (consistent) link-flow patterns within the specified error bounds. The original PFE model will select a path-flow pattern that gives the lowest objective value despite that the solution has a higher MAE/RMSE value (a lower quality of link flow replication). This is reasonable because neither MAE nor RMSE is a quantity to be minimized in the original PFE objective function. This phenomenon, as discussed earlier, will lead to the underestimation of the total travel demand. That is, if the error bounds are specified too loose, the original PFE model will select a solution with a lower objective value (i.e., lower travel cost and path entropy), thus a lower total demand. From Table 4 (among the three C experiments), it is also observed that PFE is likely to find a solution faster (e.g., less iterations) when the error bounds are large.

When the modified L_p-PFE models are applied to this data set, they are expected to resolve the inconsistency problem differently according to the criterion incorporated into the model. As expected, in experiment D, the L_∞-norm model can minimize the maximum error among all observations, the L_1-norm model can minimize the average error (MAE) as in experiment E, and the L_2-norm model can minimize the root mean squared error (RMSE) as in experiment F. However, by comparing the results of experiments F and A2, they are quite different (i.e., RMSE in case A2 is lower than that in case F) even though they are based on the same criterion for handling data inconsistency (L_2-norm). This is due to the difficulty in setting the cost penalty (ρ) in the modified L_p-PFE models. Basically, the cost penalty in the L_2-norm model could not be increased to the level at which the RMSE will be minimal without causing any numerical difficulty (i.e., ill-conditioned problem). Based on the results of the grid network, this issue only occurs with the L_2-norm model because of the quadratic cost function (penalty term) in the modified objective function. This explanation is supported by the computation required to solve the L_2-norm model, which is generally higher than those required by the other models. Among the three proposed L_p-PFE models and the original PFE model with uniform error bounds, the L_2-PFE model however gives the lowest RMSE (14.84), which indicates that such a modification still inherits the property of L_2-norm criterion in handling the inconsistent traffic counts.

Another observation is that experiments A1 and E obtain the estimations with the lowest MAE (11.75). In both experiments, the L_1-norm criterion is used either for preprocessing inconsistent data or for modifying the PFE formulation. However, the detailed solutions (e.g., individual link flow estimates) obtained from both experiments are different (see also other indices in Table 4). This observation is due to the non-uniqueness property of the mathematical formulation (linear program) for preprocessing traffic counts (Kikuchi et al., 2000). In other words, the predetermined error bound provided in Table 3 is one solution among many possibilities. In general, the modified L_p-PFE models require a higher computational time to internally resolve the inconsistency problem of traffic counts. Figure 3 compares the link volumes estimated by different models. The data points along the 45° line represent an accurate estimate while the data points under (above) this line represent an underestimate (overestimate) of the observed link volume.

As can be seen, most of the link-flow estimates obtained by the L_1-norm model lie almost exactly on the 45-degree line with the exception of a few data points while those obtained by the L_∞-norm and L_2-norm models cluster around the 45° line. When unnecessarily large uniform error bounds (12.5%) are used in the original PFE model, a majority of the estimated volumes (6 out of 8 data points) lie on the lower limit (-12.5%), which results in a lower PFE objective value. The distributions of link-flow estimates obtained by the modified L_p-PFE models are consistent with the characteristics of different norms discussed earlier. Namely, the L_1-norm model aims to reproduce most of the data points by disregarding a few points, which are sometimes believed to be outliers. On the other hand, the L_∞-norm and L_2-norm models distribute the amount of underestimated and overestimated flows (i.e., the number of data points below and above the 45° line) such that the maximum error and the RMSE are respectively minimized.

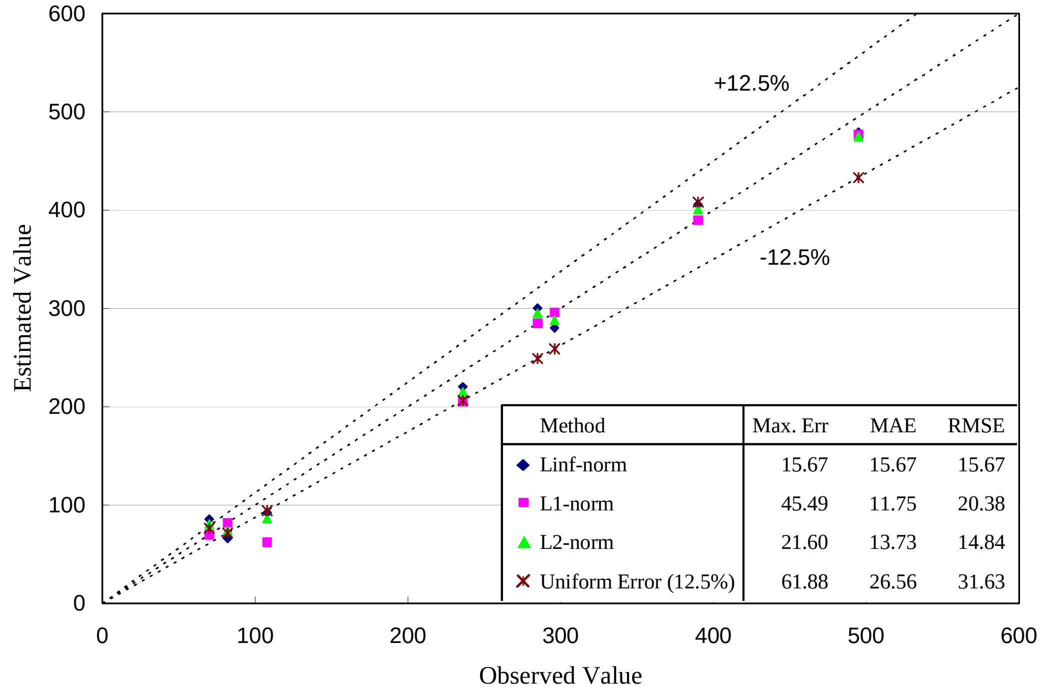


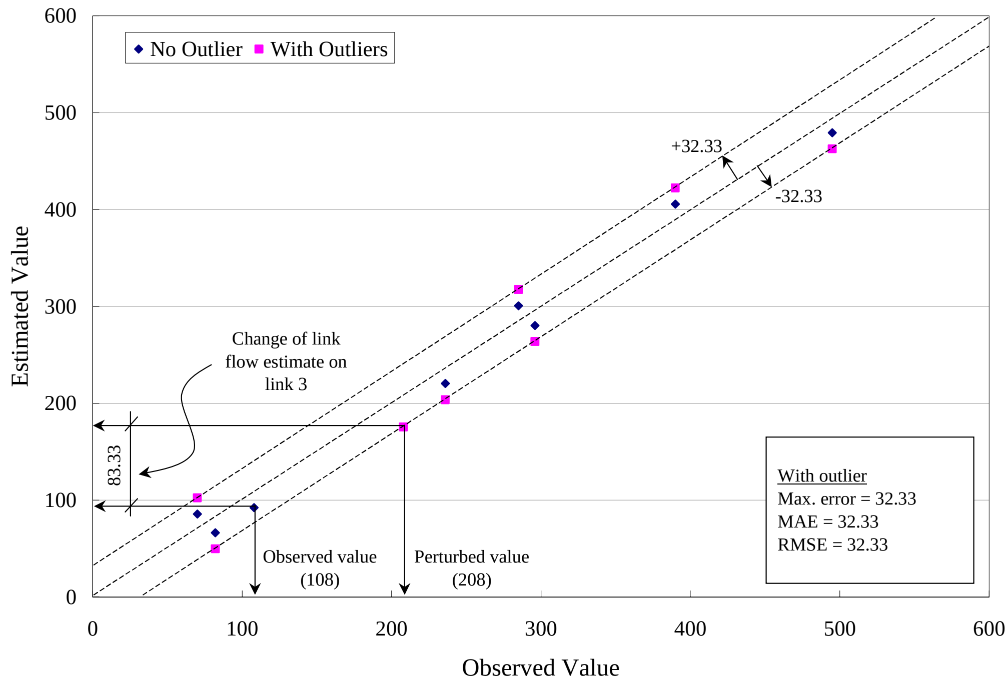
Figure 3 Comparison of link-flow estimates.

4.3 Effects of an Outlier in the Observed Link Volumes

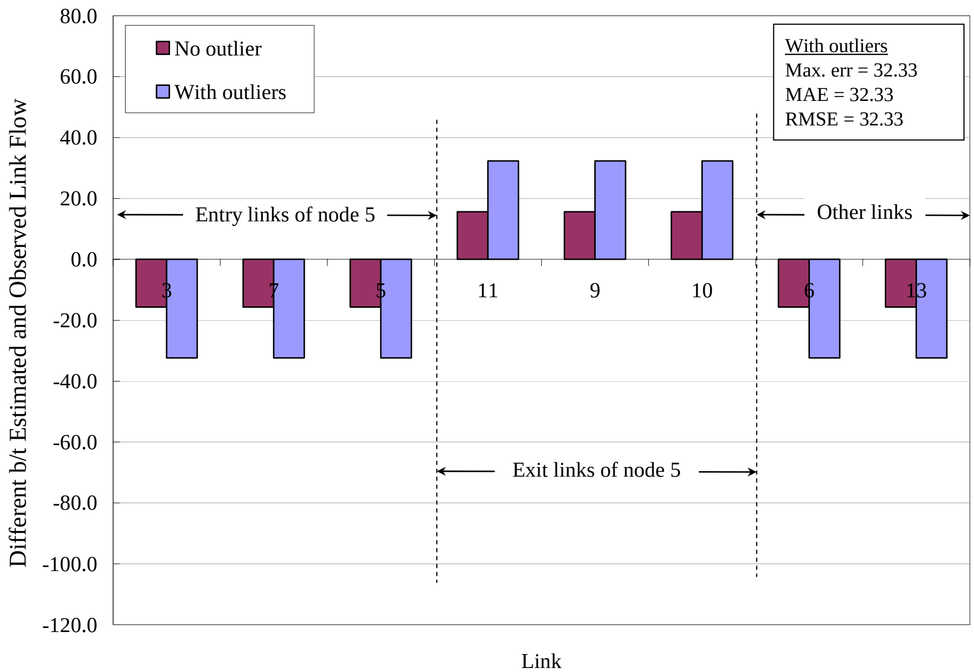
This section is provided to investigate the effects of an outlier in the observed link volumes on the performance of the modified L_p-PFE models. The observed traffic volume on link 3 is intentionally perturbed from 108 units to 208 units to create an outlier in the observed link volumes. The estimations using the proposed L_p-PFE models are repeated on this perturbed data set in which a high inconsistency of traffic counts is expected. Figures 4, 5, and 6 compare the link volumes using the original and perturbed data sets estimated by the L_∞-norm, L_1-norm, and L_2-norm models, respectively. Figures 4a, 5a, and 6a respectively show the scatter plots of the observed and estimated link volumes of both data sets for each of the three L_p-PFE models. Figures 4b, 5b, and 6b further show how each model adjusts the estimated link volumes by link types surrounding node 5 (e.g., entry links, exiting links, and other links). As can be seen, each model has to redistribute the link-volume estimates in order to minimize the corresponding objective value (e.g., Max. error, MAE, or RMSE) in the presence of an outlier in the observed traffic counts. However, such a re-distribution is more pronounced in the L_∞-norm and L_2-norm models, but not in the L_1-norm model. As can be seen in Figures 5a and 5b, a majority of the link volumes (exit links of node 5 and other links) estimated by the L_1-norm model remains unchanged. The perturbed data point is regarded as an outlier and does not significantly affect the overall estimation of the L_1-norm model. As mentioned before, the L_1-norm model is insensitive to outliers. On the other hand, both L_∞-norm and L_2-norm models have to adjust the estimated flows on all observed links to accommodate the outlier to minimize the maximum error and RMSE, respectively. It should also be noted that even with a proper re-estimation, the indices of link-flow deviation (e.g., Max. error, MAE, or RMSE) become higher in all cases due to a higher degree of data inconsistency as shown in the figures.

5. CONCLUDING REMARKS

In this study, the PFE model was reformulated to internally resolve the inconsistency problem of traffic counts. Three different criteria for defining the estimation errors, namely the L_∞ norm, L_1 norm, and L_2 norm, were considered and incorporated into the PFE model. A partial linearization algorithm embedded with an iterative balancing scheme and a column generation procedure was developed to solve the modified L_p-PFE models. Numerical results indicate that the modified L_p-PFE models are capable of estimating trip tables that better reproduce the observed traffic volumes. The advantage of the modified L_p-PFE models is that they do not require a consistent (preprocessed) data or user-specified error bounds for each individual observation. However, it does require the users to specify penalty parameters to internally handle the inconsistent traffic counts in the estimation. Ideally, the cost penalty has to be set as high as possible to minimize the estimation error. However, due to numerical difficulty (i.e., ill-conditioned problem), this cost penalty could not always be set at the level at which the estimation error will be minimized especially when the L_2-norm criterion is considered. In spite of this difficulty and a higher computational cost, the proposed L_p-PFE models generally perform better in terms of replicating the observed link volumes and eliminating the bias of underestimating the total demand in the origin PFE model.

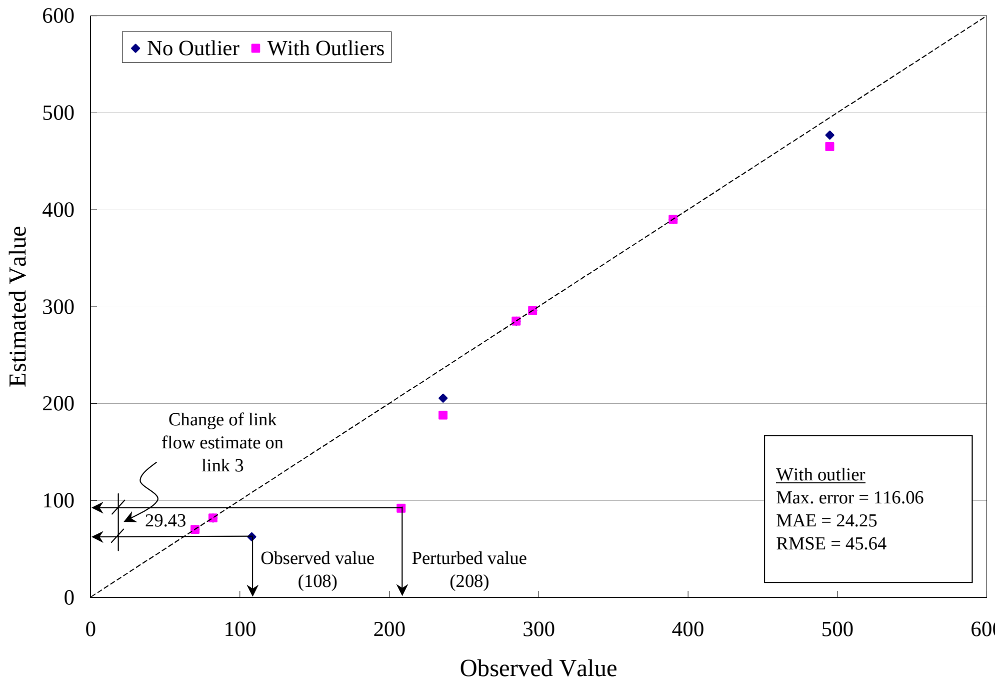


(a) Scatter plot of all observed links with and without an outlier

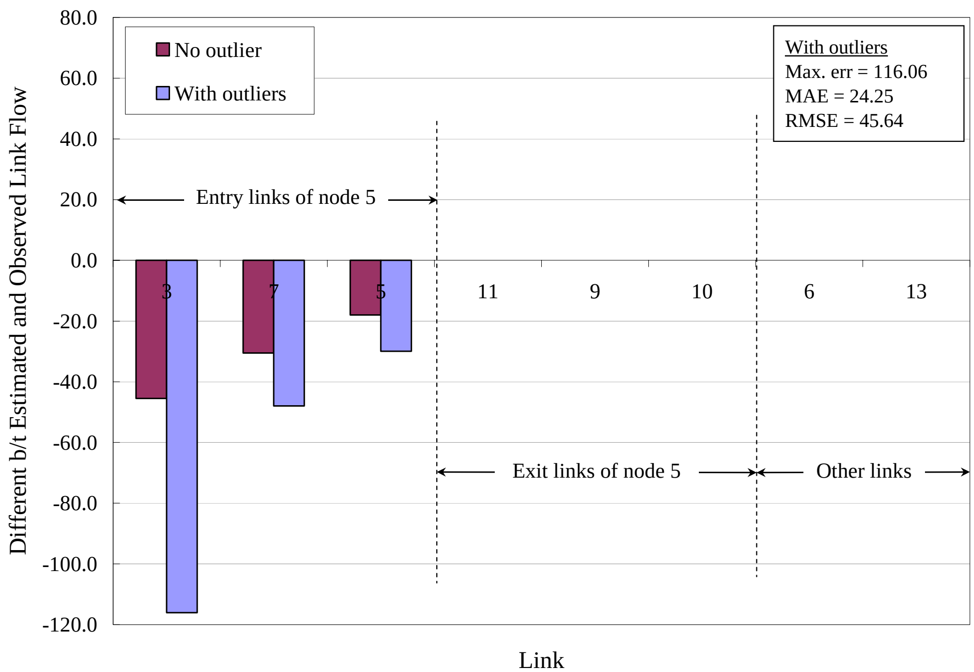


(b) Change of estimated and observed link flows with and without an outlier by link types

Figure 4 Effect of an outlier on the link-flow estimates obtained by the L_∞-PFE model

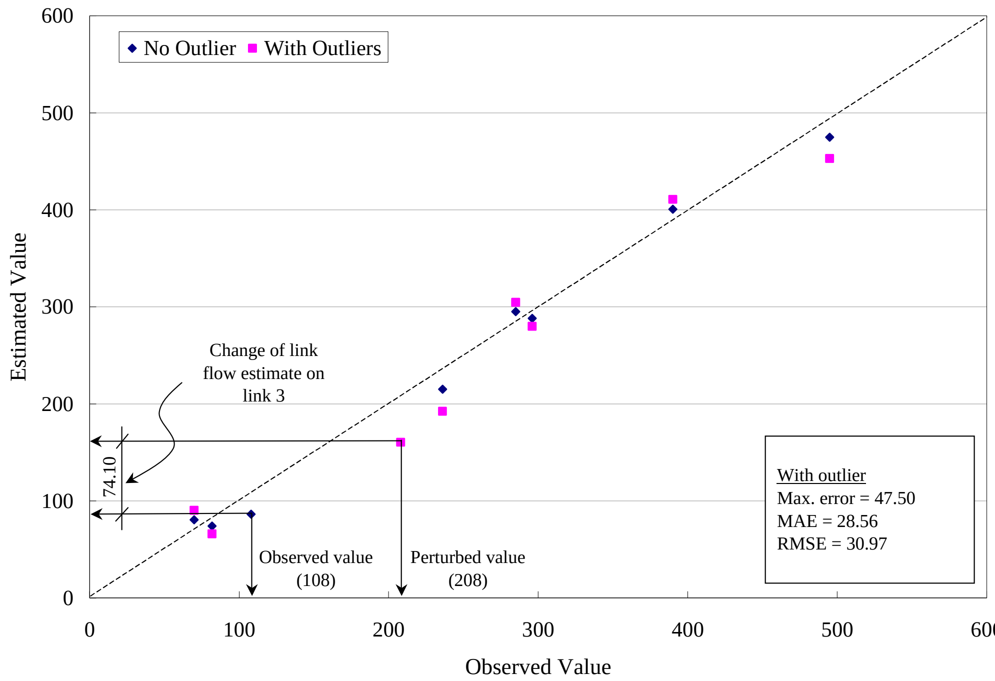


(a) Scatter plot of all observed links with and without an outlier

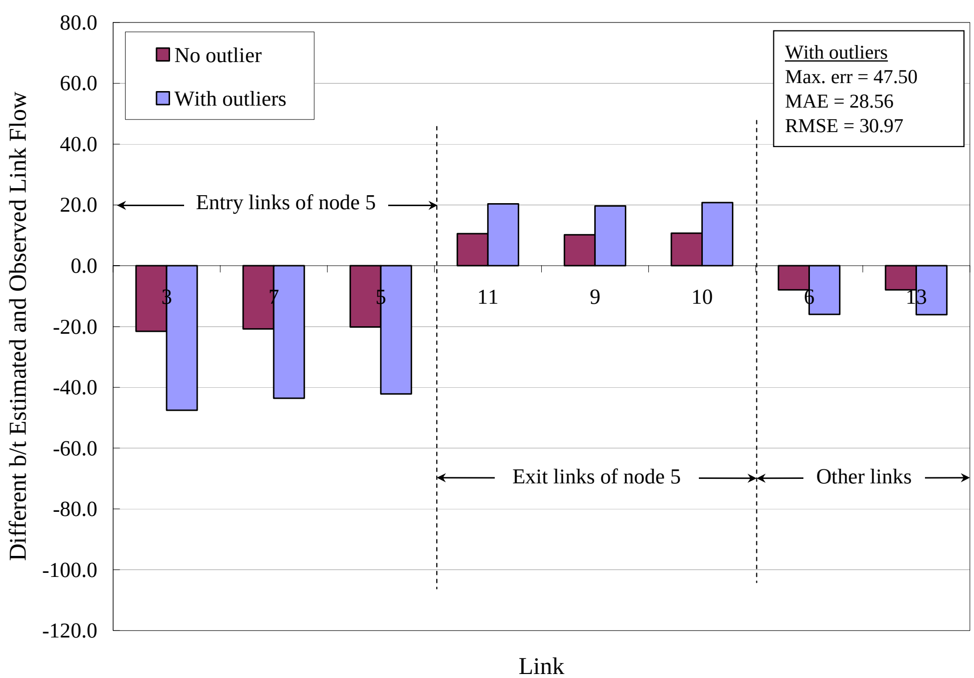


(b) Change of estimated and observed link flows with and without an outlier by link types

Figure 5 Effect of an outlier on the link-flow estimates obtained by the L_1-PFE model



(a) Scatter plot of all observed links with and without an outlier



(b) Change of estimated and observed link flows with and without an outlier by link types

Figure 6 Effect of an outlier on the link-flow estimates obtained by the L_2-PFE model

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