Delays, variation and anticipation in walker models

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A large variety of pedestrian flow models have been proposed in the last two decades, ranging from macroscopic models [1], Cellular Automata [2], and microscopic simulation models that are continuous in time and space [3], [4]. Especially the latter models are able to correctly describe collective phenomena in empirical pedestrian flow, such as the emerging relation between flow and speed, spatiotemporal patterns, and self-organisation of lanes, diagonal stripes, etc. Recently, a parameter estimation approach was developed that enabled identification of the behavioural parameters for individual pedestrians based on trajectory data [5]. This study yielded a couple of important new insights into the microscopic properties of a pedestrian flow, in particular with respect to the inter-pedestrian differences. Judging from the large variance in the estimated parameters, it turns out that the inter-pedestrian behavioural variability is considerable. Another aspect that turned out to be of importance, is the presence of a finite reaction time in the walking decision process: using the new estimation technique, it was shown that the response to stimuli is in fact delayed by (on average) 0.3 seconds.

The main contributions of the presented paper are an empirically based theory of walking behaviour featuring finite reaction times, anticipation and estimation and control errors, and finally inter-pedestrian differences. Based on this new theory, a novel walker model is proposed. This model is applied to a simple test-case example in order to show effects of the new behavioural theory on aggregate flow characteristics such as capacity and total congestion time.

Pedestrian behavioural theory

The new empirically based pedestrian behavioural theory is based on the theory presented by [3,6], where pedestrians are described as optimal controllers with a limited prediction horizon. The behavioural hypotheses in this theory are [5,6]:

1. Pedestrians are:
   a. anisotropic particles, i.e. they react mainly to stimuli in front of them,
   b. compressible, at least to a certain extent.
2. Pedestrians minimise predicted discounted costs resulting from:
   a. straying from the planned trajectory,
   b. vicinity of other pedestrians and obstacles,
   c. applying control (differentiating between longitudinal acceleration and lateral acceleration (side stepping)).
3. Walkers will be more evasive when encountering a group of pedestrians than when encountering a single pedestrian (described by assuming proximity costs additively).
4. Pedestrians continuously reconsider their walking choices by using current observations and the resulting predictions into the subjective utility optimization (rolling horizon). Pedestrians are thus feedback-oriented controllers.

5. Walkers anticipate on the behaviour of other pedestrians by predicting their walking behaviour according to non-cooperative or cooperative strategies.
6. Pedestrians have limited predicting possibilities, reflected by discounting utility of their actions over time and space, implying that they mainly consider pedestrians in their direct environment.

The main contributions with respect to the theory presented in this paper originating from parameter estimations on empirical data are:

1. Pedestrians have a delayed response to the stimuli offered (finite reaction times).
2. Pedestrians (subconsciously) take into account these finite reaction times and anticipate on changes in the traffic conditions by means of temporal anticipation.
3. Pedestrians make both estimation and control errors.
4. Behaviour is different from one pedestrian to the other.

Mathematical walker model

Starting point of the model development is the model derived by [4] and improved in [6]. This model distinguishes two components: 1) a physical (close-range) model and 2) a control model. The physical model describes the forces acting upon the pedestrians, for instance when pedestrians collide. Pedestrians are described as compressible (circular) particles upon which both normal forces and tangential forces (friction) act [3,4,6]. The control model describes the control decisions made by the pedestrians and is derived by application of the theory of differential games.

Clearly, finite reaction times, anticipation and errors will only affect pedestrian description at the control level. This to a certain extent also holds for the inter-pedestrian differences, although differences in size and weights amongst pedestrians are bound to affect the physical model as well.

Without going into details, the basic mathematical walker model has the following form:

$$\begin{aligned} \frac{d}{dt}\vec{r}_p(t) &= \vec{v}_p(t) \\ \frac{d}{dt}\vec{v}_p(t) &= \vec{a}_p(t) + \vec{b}_p(t) \end{aligned} \tag{1}$$

where $\vec{a}_p(t)$ denotes the acceleration due to physical interactions, and $\vec{b}_p(t)$ denotes the controlled accelerations; $\vec{r}_p(t)$ and $\vec{v}_p(t)$ respectively denote the position and the velocity of pedestrian *p* at time *t*.

The basic mathematical walker model predicts the two-dimensional acceleration vector $\vec{a}_p(t)$ due to physical interaction as a function of the free velocity $\vec{v}_p^0$, the current speed $\vec{v}_p(t)$, and distance $d_{pq}(t)$ between pedestrians *p* and *q* as follows:

$$\vec{a}_p(t) = \vec{f}_p(\vec{v}_p(t), \vec{r}_p - \vec{r}_q, ...) = \frac{\vec{v}_p^0 - \vec{v}_p(t)}{T_p} - A_p \sum_{q \in Q_p} \vec{u}_{pq}(t) e^{-\frac{d_{pq}(t)}{R_p}} \tag{2}$$

where Q_p denotes the set of pedestrians that influence pedestrian *p*, and where

$$d_{pq}(t) = \| \vec{r}_q(t) - \vec{r}_p(t) \| \quad \text{and} \quad \vec{u}_{pq}(t) = \frac{\vec{r}_q(t) - \vec{r}_p(t)}{d_{pq}(t)} \tag{3}$$

The basic interaction model has four *pedestrian specific parameters*, namely the free speed $V_p^0 = \| \vec{v}_p^0 \|$, the acceleration time T_p, the interaction constant A_p and the interaction distance R_p that are to be estimated from data. Note that the desired walking direction $e_p^0 = \vec{v}_p^0 / V_p^0$ determined by pedestrian route choice is assumed known.

The dynamics in the controlled acceleration $\vec{b}_p(t)$ are described by a function $\vec{f}$, which is a function of the difference between the positions of pedestrian p and another pedestrian q $\vec{r}_q'(t) - \vec{r}_p'(t)$, the current velocity of pedestrian p $\vec{v}_p(t)$, and the difference in velocity $\vec{v}_q'(t) - \vec{v}_p'(t)$:

$$\vec{b}_p(t) = \sum_q \vec{f}_p\left(\vec{r}_q'(t) - \vec{r}_p'(t), \vec{v}_p(t), \vec{v}_q'(t) - \vec{v}_p'(t) \mid \theta_p\right) + \vec{\varepsilon}_p(t) \tag{4}$$

where θ_p is a set of pedestrian specific parameters. In Eq. (4), the prime indicates that we are considering anticipated locations and velocities rather than the actual ones. This reflects the fact that a pedestrian reaction will be delayed by τ_p, and the fact that the pedestrian will to a certain extent be aware of this delay. We will use a constant velocity heuristic to describe this anticipation [7]:

$$\begin{aligned} \vec{r}_p'(t) &= \vec{r}_p(t-\tau_p) + \tau_p \vec{v}_p(t-\tau_p) \\ \vec{r}_q'(t) &= \vec{r}_q(t-\tau_p) + \tau_p \vec{v}_q(t-\tau_q) \end{aligned} \tag{5}$$

That is, the location of the pedestrian p and the other pedestrians q are predicted based on the delayed observations of the locations and velocities. The term $\vec{\varepsilon}_p(t)$ in Eq. 4 reflects the estimation and control errors made by the pedestrian.

Tab. 1 shows the parameter estimates for the free speed V_p^0, acceleration time T_p, interaction factor A_p, interaction distance R_p and the reaction time τ_p.

Tab. 1 Parameter estimates.

Parameter	Mean	Standard deviation
V_p^0 (m/s)	1.34	0.23
T_p (s)	0.74	0.23
A_p (m/s^2)	11.33	0.64
R_p (m)	0.35	0.11
τ_p (s)	0.28	0.07

Consequences for pedestrian flow modelling

The new parameters in the pedestrian behavioural theory presented in the previous section may have important implications for microscopic pedestrian flow modelling. In particular, these implications relate to:

1. Inter-pedestrian differences as expressed by the variability in the parameter estimates.
2. Correlation between parameter estimates.
3. Importance of delays in the correct description of microscopic behaviour.

Regarding the first point, the current models can in general be amended easily when sufficient insight has been gained into the distribution of the parameters for the pedestrian population to be simulated (see Tab. 1). The distribution that needs to be used will depend on amongst other things gender distribution, trip purpose and external conditions (such as weather). The variability in pedestrian behaviour will cause macroscopic properties of the pedestrian flow to become stochastic. This holds in particular for the bottleneck capacity and the jam-densities (and consequently the queue lengths). The second issue is of particular interest when generating pedestrians in a microscopic model. When the behavioural parameters are generated, it is important to take into account the correlations during the parameter generation process.

With respect to the third point, considerable changes in the properties of the pedestrian flow dynamics are expected. Amongst the phenomena that are likely to occur are instabilities in the flow, congestion probabilities and reduction in the queue discharge rate once congestion sets in.

To test whether this hypothesis is indeed correct, we have modified the NOMAD model [6] to include finite reaction times. We have applied the modified simulation model on a simple test-case example, namely a narrow bottleneck. The bottleneck has a width of 1.0 m. From experimental observations, we expect a bottleneck capacity of 1.6 P/s (see [8] for details). The pedestrian demand equals 1.8 P/s. The parameters used in the simulation correspond to the estimates shown in Tab. 1. Although NOMAD is able to model physical contacts and the resulting frictions between pedestrians, this and other advanced features of NOMAD were disabled in this study.

Fig. 1 shows a snapshot of a NOMAD simulation for this narrow bottleneck scenario. Pedestrians are generated at the right, and walk to the exit on the left. The figure shows the locations of the detectors that have been used to collect the synthetic data our analyses in the following are based on.

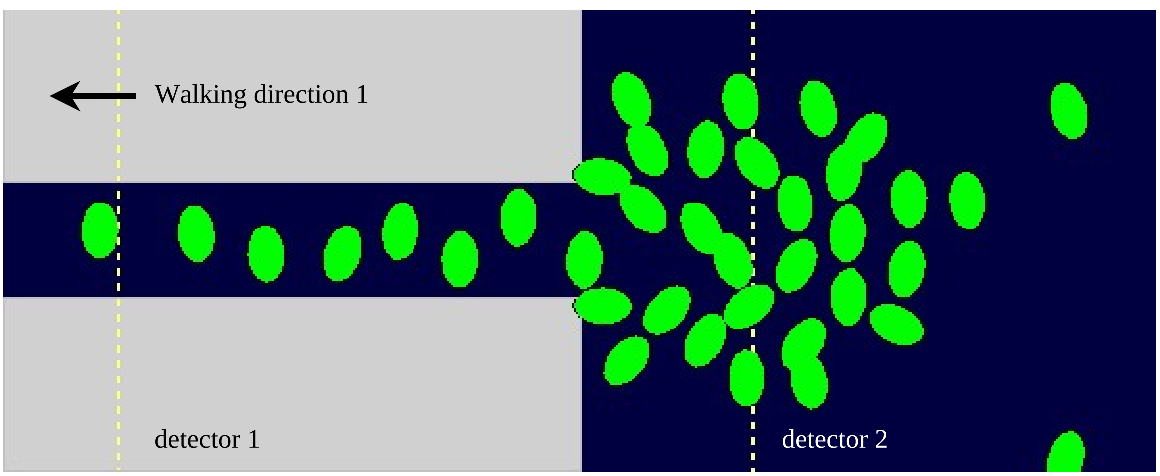

Fig. 1 Snapshot of situation at the narrow bottleneck as determined by the adapted NOMAD pedestrian simulation model. The figure shows the two detectors that have been used to compute the queue discharge rate. In the example shown, congestion is detected at detector 2. As a result, the flow observed at detector 1 is labelled as an observation of the queue discharge rate.

When comparing the instantaneous and the retarded models, several things become apparent. For one, the flow becomes more unstable, more erratic. The considered flow operations are near oversaturation (volume to capacity almost equal to 1). From our (preliminary) simulation results, it turns out that *before congestion occurs*, the bottleneck capacity is higher than once congestion has set in. This phenomenon is quite common in car traffic, but is also present in pedestrian flow operations. The reasons for this *capacity drop* is the fact that when congestion has set in, pedestrians are dispersed over part of the width of the walking area, i.e. they are not standing right in front of the bottleneck. When arriving at the bottleneck, they need to turn as well as to accelerate. This process is likely to be affected by the reaction time of the pedestrians:

when the response to prevailing traffic conditions is retarded, both the turning and the acceleration will be delayed and hence capacity of the bottleneck is likely to decrease.

To quantify the expected shifts in the bottleneck capacity, let us compare the distribution of the queue discharge rate for three situations: the instantaneous model (no reaction time), and two retarded models with reaction times of 0.3 s and 0.6 respectively. Fig. 5 shows the results of this comparison. Note that each data point in the figure corresponds to the outcomes of a 4-minute simulation in which the oversaturated bottleneck was considered. The capacity estimate for this simulation is determined by considering the period in which congestion occurred and subsequently considering the queue discharge rate during this period.

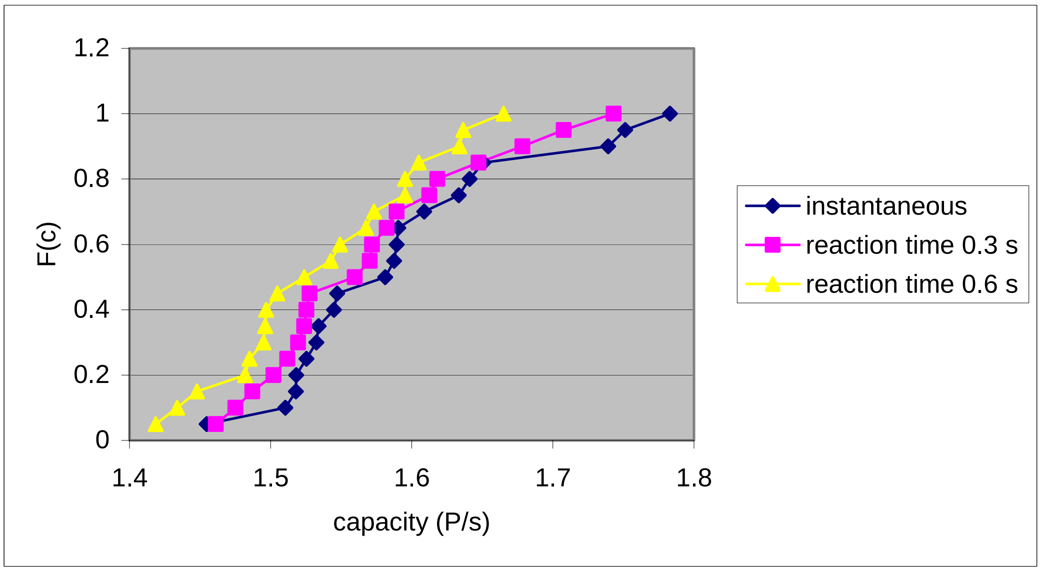

Fig. 2 Distribution of the capacity (queue discharge rate) of a narrow bottleneck and the changes resulting from including the finite reaction time in the model.

Fig. 2 clearly shows that the capacity of the bottleneck is not a constant value, but can best be described by a random variable. The figure also shows the expected changes caused by including the finite reaction time in the NOMAD model in relation to the queue discharge rate. Although the change in the average capacity is statistically significant, it is small (only a few percent).

Fig. 3 shows the distribution function of total congestion time (out of a simulation time of 240 s). In the figure, we see that the average congestion times for the retarded models are larger than for the instantaneous model, caused by the capacity reduction.

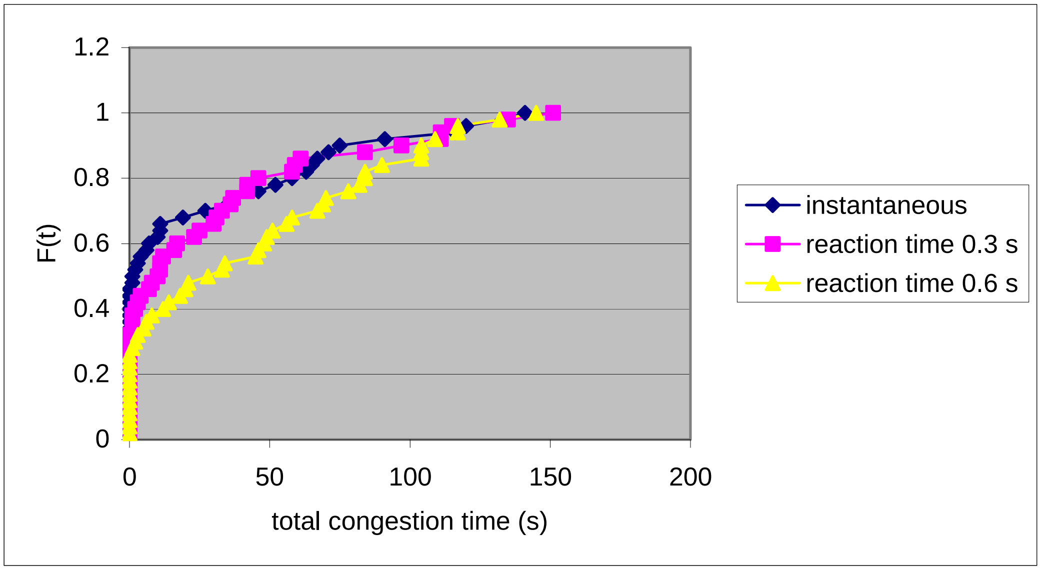

Fig. 3 Distribution of total congestion time (out of 240 s).

We found that the predicted probability of congestion occurring is larger when using one of the retarded models than when using the instantaneous model. More precisely, the instantaneous model predicts congestion occurring in 54% of all simulations. For the retarded model with a reaction time of 0.3 s, congestion occurs more frequently, namely in 68% of all situations. For the reaction time of 0.6 s, this number is even higher, namely 74%.

Although the increase in capacity is rather limited (few percents), the increase in congestion is significant (26% respectively 37%). In order to determine whether or not the additions to the new theory influence the pedestrian flows, additional statistics need to be performed.

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