A Column Generation Approach for the Door Assignment in LTL-Terminals

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1 Introduction

The main task of less-than-truckload (LTL) terminals is to guarantee an errorless and on-time transshipment of all units from arriving trucks to waiting trucks within an operation period of several hours. Decisions which have to be taken in this regard, can be divided into assignment decisions and inner transport decisions. Arriving trucks, so-called tours, have to be assigned to inbound doors and to suitable time slots for unloading. Simultaneously, waiting trucks - each representing an offered relation in the underlying transportation network - have to be allocated to outbound doors. To guarantee a coordinated material flow between all trucks, decisions also have to influence the inner flow of shipments and the utilization of resources (e.g. buffer areas, forklift trucks). Two aims shall be achieved by the optimization. The first aim is to minimize the total inner distances. In addition, the waiting time for each truck should be minimized as long waiting times result in too high times of unproductiveness for the trucks and in too congested yards.
Bermúdez and Cole (2001) were one of the first tackling this kind of logistical problem. They used a genetic algorithm to minimize transport volumes inside of breakbulk terminals. They assumed that a single door only serves a single truck, time constraints do not exist. An enlargement was developed by Bartz-Beielstein et. al. (2006) who introduced a 1+1 evolution strategy to the problem by integrating the aspect of time.
A major drawback of the approaches is that they solely work on the optimization of the door utilization. Inner processes and resources are not integrated. A first approach for optimizing the processes inside of transshipment terminals can be found in Li and Rodrigues (2004) who solved the task by using a hybrid evolutionary algorithm. Yet, they did not combine the inner view with an optimization of the door utilization. Stickel and Furmans (2005) in contrast concentrated on the optimization of the door utilization in in combination with the creation of vehicle routing tours for the inbound tours - the view outside. The associated mathematical model is very complex, only small problem instances could be solved with CPlex.
Another common characteristic of most work that has been done on this field of research is that - as the underlying problem is related to quadratic assignment problems - solution approaches from discrete optimization are randomly pursued. Most researchers develop solution methods from the field of computational intelligence. Chmielewski and Clausen (2005) were one of the first developing an enhanced mathematical model for optimizing the door assignment in LTL-Terminals. The model is based on a time-discrete multi-commodity flow network with several side constraints. The resulting MILP was programmed with GAMS. Afterwards, the standard algorithm for MILP (Branch-and-Cut algorithm, CPlex 10.0) has been applied to different scenarios. The reinterpretation of the results to the world of logistics showed that the mathematical model is suitable for the optimization task but due to slow solution times the Branch-and-Cut is not.
Therefore in this work, a decomposition and column generation approach based on the applicable model is presented. The new approach has also been programmed with GAMS. When applying to the same test scenarios as the Branch-and-Cut algorithm, the new approach showed faster solution times as well as better solution results in terms of lower objective function values.
The logistical background of the optimization task "Door Assignment in LTL-Terminals" as well as the mathematical model are briefly presented in section 2. The new approach is introduced in section 3. Experiments with test scenarios and a comparison with the solution process of the Branch-and-Cut algorithm is presented in 4. An outlook on future research work on this problem class is given in section 5.

2 Problem description and mathematical model

The transport of LTL goods within a country is organized via a transportation network which consists of several terminals interlinked by line-haul. The core element of a terminal is the transshipment building with

several doors. The doors are separated in inbound doors $I = \{1, ..., I'\}$ for unloading and outbound doors $J = \{1, ..., J'\}$ for loading. A terminal has two main operation periods: the inbound arrival of local collection tours with subsequent outbound departure of trucks for the long distance relations (12am - 8pm) and the inbound arrival of long distance tours with the outbound departure of trucks for the local delivery relations (12pm - 9am). In both periods, arriving trucks are called tours and waiting trucks are set equate with relations. Therefore, the following model is designed for both operation periods.
K' tours arrive according to a certain timetable with earliest arrival times a_k and latest departure times b_k $(K = \{1, ..., K'\})$. The set of all relations is given by $L = \{1, ..., L'\}$. Each relation truck arrives at a_l and leaves at b_l. Tour k has w^k shipments on board. The number of shipments from k that go to relation l is given by d_{kl}. Relation l gets v^l shipments in total. The inner material flow of a shipment is pictured in the left part of figure 1. The right part shows the appropriate time-discrete multi-commodity network structure that represents the main decisions (here: just two time slices exist for higher transparency):

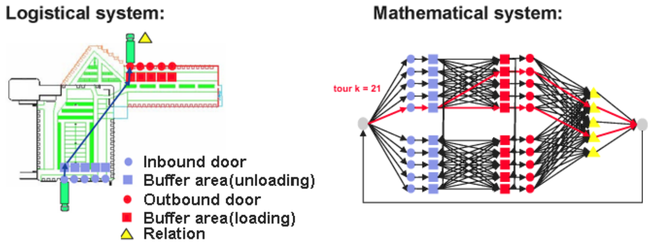

Figure 1: Transformation of the logistical system into a network structure

Within its attendance time, a tour k is allocated to an inbound door (first node layer). All shipments are unloaded and buffered on a defined area (second node layer) where they are consolidated according to their relation. Next, the shipments are transported by forklift trucks to those outbound doors (fourth node layer) where the trucks for the different relations (fifth node layer) have been assigned to. As the loading process usually starts much later, the shipments are stored on a defined buffer area (third node layer) in front of the assigned door. Consequently, the inner material flow can be split into three processes: unloading (second arc layer), inner transport (third arc layer) and loading (fourth arc layer). For each process, the number of available resources r per time slice with speed capacity g in [km/h] is given. Those nodes representing the doors and buffer areas are duplicated for each time slice $t \in T = \{1, ..., T'\}$ within the optimization period (node layers 2 - 5). The length of the arcs in arc layers 2-4 are equivalent to real distances in the terminal building. Different tours are integrated as flow variables of different "commodities" on the set of arcs A: $x^k_{arc_i} \in \mathbb{R}_+,\ arc_i \in A$.
The first arc layer represents the assignment of tours to inbound doors. A binary variable $\delta_k^{i'}$ is introduced that checks if there is a positive flow of tour k through inbound door i' in any of the time slices t:

$$w^k * \delta_k^{i'} \geq \sum_{j \in \bigcup_{t=1}^{T'} {A_1}_t^{i'}} x^k_{arc_j} \quad \text{and} \quad w^k * (1 - \delta_k^{i'}) \geq \sum_{i,\, i \neq i'} \sum_{j \in \bigcup_{t=1}^{T'} {A_1}_t^{i}} x^k_{arc_j},\ \forall\, k \in K,\ \forall\, i' \in I. \qquad (1)$$

Set $\bigcup_{t=1}^{T'} {A_1}_t^{i'}$ consists of all arcs starting in the common sink node and ending in one of the nodes representing inbound door i' in different time slices. As it is not allowed to unload a tour at different doors, the following restriction is necessary: $\sum_{i=1}^{I'} \delta_k^i = 1,\ \forall\, k \in K$.
Similar unsplittable flow restrictions for certain sets of nodes hold for the assignment of relations to outbound doors. The fifth arc layer represents these assignment decisions. A binary variable $\theta^l_{j'}$ is introduced which checks if there is a positive flow of any tour k over an arc that connects one of the nodes representing outbound door j' in any time slice and the adequate node for relation l in node layer 5:

$$v^l * \theta_l^{j'} \geq \sum_{k \in K} \sum_{i \in {A_5}_l^{j'}} x^k_{arc_i} \quad \text{and} \quad v^l * (1 - \theta_l^{j'}) \ \geq\ \sum_{k \in K} \sum_{j,\, j \neq j'} \sum_{i \in {A_5}_l^{j}} x^k_{arc_i},\ \forall\, l \in L,\ \forall\, j' \in J \qquad (2)$$

Each relation has to be loaded at the same door. Furthermore - in contrast to the inbound arrival - a one-to-one assignment is necessary: $\sum_{j=1}^{J'} \theta_l^j = 1, \forall\, l \in L$ and $\sum_{l=1}^{L'} \theta_l^j \leq 1, \forall\, j \in J$.
Arc layers 1 and 5 represent assignment decisions and no real transport, their length is set to zero.
As a timetable exists, relation l might not be available in time slice t. In this case, no arcs will be implemented between nodes representing outbound doors in earlier or later time slices and the node for relation l. For arriving tours, additional binary variables γ_t^k have to be introduced to check if there is a positive flow over an arc to an inbound door in time slice t:

$$\sum_{i \in A_{1t}} x_{arc_i}^k \leq w^k * \gamma_t^k \quad \text{and} \quad \sum_{i \in A_{1t}} x_{arc_i}^k \geq \gamma_t^k, \forall\, k \in K, \forall\, t \in T \tag{3}$$

If a tour k is not available in a certain time slice, the appropriate binary variable will be set to zero. Furthermore, a special rule for the γ-vector is necessary to guarantee that the unloading process is not interrupted. Patterns like 1|0|1 or 1|0|0|1 ... should be prevented. This means, for each part of the γ-vector, the sum of the two outer elements minus the sum of all inner elements must be below 1:

$$\gamma_t^k + \gamma_{t+i}^k - \sum_{j=t+1}^{t+i-1} \gamma_j^k \quad \leq 1, \forall\, k \in K, \forall\, i \in \{2, ..., T'-1\}, \forall\, t \in \{1, ..., T'-i\} \tag{4}$$

For each of the three main processes a certain resource capacity level per time slice should not be exceeded. The corresponding restriction is shown exemplarily for the unloading process (= ent):

$$\sum_{i \in A_{2t}} \sum_{k=1}^{K'} x_{arc_i}^k * \frac{arc_i^{length} * 60}{g_{ent}} \leq q * r_{ent}, \forall\, t \in T \tag{5}$$

Set A_{2t} represents the unloading process in time slice t. The length of an arc is multiplied by the number of shipments flowing over it and divided by the transshipment resources' velocity g_{ent}. The resulting number of minutes needed for all unloading activities in time slice t must be less than the number of resources available r_{ent}, multiplied by the length of a time slice q in minutes.
To guarantee that each relation gets the requested number of shipments of tour k, the following restriction is set for all arcs in the sixth arc layer: $x_{arc_l}^k = d_{kl}, \forall\, l \in L, \forall\, k \in K$.
Finally, additional restrictions are implemented to control the areas' capacity or the level of traffic at a door in a time slice and for controlling the flows over arcs (Kirchhoff, maximum capacity) are implemented.

One objective is to find an allocation that leads to minimal total distances and consequently to a minimal number of resources needed in operations. A second objective is the minimization of waiting times. The following objective function has been developed to achieve this:

$$\begin{aligned} min \quad & \sum_{k=1}^{K'} \sum_{t=1}^{T'} \left(\sum_{i \in A_{2t}} B^t * x_{arc_i}^k * \left(\frac{arc_i{}^{length} * c_{ent}}{g_{ent}} \right) + \right. \\ & \left. \sum_{i \in A_{3t}} x_{arc_i}^k * \left(\frac{arc_i{}^{length} * c_{ver}}{g_{ver}} \right) + \sum_{i \in A_{4t}} x_{arc_i}^k * \left(\frac{arc_i{}^{length} * c_{bel}}{g_{bel}} \right) \right) \end{aligned} \tag{6}$$

The three inner summands represent the costs for unloading (ent), transport (ver) and loading (bel). The coefficient B^t is set equal to 1 for the first time slice and will be increased slightly for each ascending time slices to punish late actions. This also influences an early assignment of trucks to doors as can be seen in figure 3.
The resulting model which is based on a time-discrete multi-commodity flow network and supplemented by necessary constraints belongs to the class of mixed integer linear models. In the next section, a decomposition and column generation approach will be presented that works on the presented mathematical model.

3 A Decomposition and Column Generation Approach

First, a decomposition approach has been applied to the problem. The basic idea was to avoid the parallel assignment of tours and relations to doors since this raises the problem's complexity:

Definition: Feasible Routing for a tour

An non-empty set $R_k = \{(arc_1, x_{arc_1}, c_{arc_1}), ..., (arc_m, x_{arc_m}, c_{arc_m})\}$ which consists of m arcs ($m \geq 5$) with flows x_{arc_i} and costs c_{arc_i} represents a feasible routing of tour k through the time-discrete network if the arcs are organized in n paths ($n \geq 1$) and hold the following conditions:

1. *For each node t_l in the fifth layer there has to be at least one arc in the routing that is connected to it. The sum of flows over all arcs being connected to t_l has to be equal to d_{kl}.*
2. *All arcs of the routing that belong to the first arc layer have to be connected to nodes in the first node layer representing the same inbound door.*
3. *All arcs of the routing that belong to the first arc layer have to be connected to nodes in the first node layer that keep the time-restrictions (timetable, continuous unloading process).*
4. *For any two arcs $arc_i = (i_1, i_2)$ and $arc_k = (k_1, k_2)$ in the routing belonging to the fifth arc layer the following rule applies: $(i_1 = k_1) \Leftrightarrow (i_2 = k_2)$.*
5. *Each flow x_{arc_i} in the routing has to keep the maximum and minimum capacity levels of arc_i.*
6. *The sum of flows over all arcs belonging to the second arc layer has to keep the resource capacity level for unloading in a certain time slice (same applies for the transport and loading).*

All rules formulated in the definition as well as the underlying path structure can be found in the mathematical model of section 2. A feasible routing is pictured exemplarily by the red line in figure 1 for tour $k = 21$. The decomposition idea was adapted from Savelsbergh and Sol (1994) who worked on the vehicle routing problem with time windows and Huisman (2005).

Assuming, the set of all feasible routings Ω_k of tour k is already known, the remaining task is to choose exactly one for each k so that the overall costs are minimized. The following binary represents this decision:

$$\psi_{R_k^i} := \begin{cases} 1, & \text{if routing } R_k^i \ (i \in \Omega_k) \text{ for tour } k \text{ is chosen} \\ 0, & \text{if not.} \end{cases}$$

Based on these ideas, a second mathematical **model for the optimal choice of routings** can be formulated:

$$\min \sum_{k \in K} \sum_{i \in \Omega_k} c_{R_k^i} * \psi_{R_k^i} \tag{7}$$

s.t.

$$\sum_{i \in \Omega_k} \psi_{R_k^i} = 1, \forall k \in K \tag{8}$$

$$\sum_{k \in K} \sum_{i \in \Omega_k} x_{arc_j}^{R_k^i} * \psi_{R_k^i} \leq arc_j^{cap_{max}}, \forall arc_j \in A'' \tag{9}$$

$$\sum_{s \in A_{2_t}} \sum_{k \in K} \sum_{i \in \Omega_k} ent_{arc_s} * x_{arc_s}^{R_k^i} * \psi_{R_k^i} \leq q * r_{ent}, \forall t \in T \tag{10}$$

$$(K' - 1) * (1 - \sum_{i \in \Omega_{k'}} m_{R_{k'}^i}^{j',l'} * \psi_{R_{k'}^i}) \geq \sum_{\substack{k \in K, \\ k \neq k'}} \sum_{\substack{j \in M \cup J, \\ j \neq j'}} \sum_{i \in \Omega_k} m_{R_k^i}^{j,l'} * \psi_{R_k^i} \quad \forall k' \in K, \forall l' \in L, \forall j' \in M \cup J \tag{11}$$

$$(K' - 1) * (1 - \sum_{i \in \Omega_{k'}} m_{R_{k'}^i}^{j',l'} * \psi_{R_{k'}^i}) \geq \sum_{\substack{k \in K, \\ k \neq k'}} \sum_{l \in L} \sum_{i \in \Omega_k} m_{R_k^i}^{j',l} * \psi_{R_k^i} \quad \forall k' \in K, \forall l' \in L, \forall j' \in M \cup J \tag{12}$$

The first restriction guarantees that for each tour exactly one routing is chosen. When developing new routings by using the time-discrete multi-commodity model, common used resource capacities as well as the assignment decisions are checked for each tour separately. In this new model, feasible routings of different tours are brought together. Therefore, in a first step, restrictions are needed to guarantee that the capacity levels are not violated. Restrictions 9 and 10 are shown as representatives of this class of restrictions. In the original model, there are far more restrictions with basically the same structure to be kept, but due to space limits, they are not listed here. In a second step, the technical correct assignment of relations to doors has to be kept when bringing the routings together. For example, if in a routing for tour k relation l is assigned to outbound door j ($m^{j,l}_{R^i_k} = 1$) and in a routing for tour k' relation l is assigned to outbound door j', these two routings can not be chosen. Restrictions 11 and 12 are implemented to guarantee that chosen routings have the same assignment of relations to outbound doors.

The decomposition approach allows to work in two models, one for the creation of good single routings and one for the optimal choice of routings for each incoming truck. After introducing a relaxation to the binary variables of the choice model, the concept of column generation can be applied to the problem. Figure 2 shows the interaction of decomposition and column generation approach:

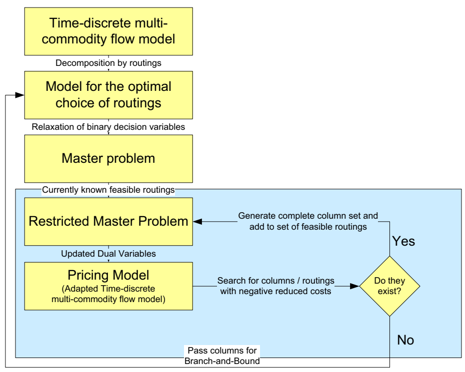

Figure 2: Interaction of Decomposition and Column Generation Approach

The relaxed choice model acts as master problem (MP). A column is equivalent to a routing for a certain tour through the time-discrete network. As the creation of all possible routings for one tour is a very complex task, an explicit column generation is not applicable, i.e. the pricing model has to identify routings with negative reduced costs and produce new routings / columns. Again, the proposition is to use the original time-discrete multi-commodity flow model with side constraints:

Definition: Pricing model for the Door Assignment in LTL-Terminals
The original time-discrete multi-commodity flow model with side constraints of section 2 acts as pricing model for the column generation approach if the model is defined just for one tour k and if the objective function of the model is enlarged by the dual variables from the restricted master problem.

After having solved the RMP, the dual variables are passed to the pricing model to create new columns with negative reduced costs. The best performing column is chosen. It might happen that the assignment of relations to outbound doors in the chosen routing has been created for the first time. I.e. in the currently known column pool, not for all remaining tours routings do exist with this assignment. In this case, later the new column cannot be chosen. A procedure is implemented that creates complete routing sets according to a given assignment of relations to doors. All new columns are added to the column pool and the RMP is solved again - an iterative process has started. At the end of this process, the integrality property is restored for the choice model and the Branch-and-Cut algorithm is applied based on the current column pool.
As this approach is not integrated in a Branch-and-Price algorithm, it is not exact. But according to Huisman (2004) problems do exist, where similar approaches led to solutions with a MIP-gap between 1.5% and 12.1%.

4 Computational Experience

The decomposition and column generation approach has been implemented with GAMS and applied to several artificial test scenarios as well as to real world data from two freight forwarding companies.
The following figure shows the optimization results of a middle-sized scenario with 25 doors (rectangular layout), 40 arriving tours and 10 relations:

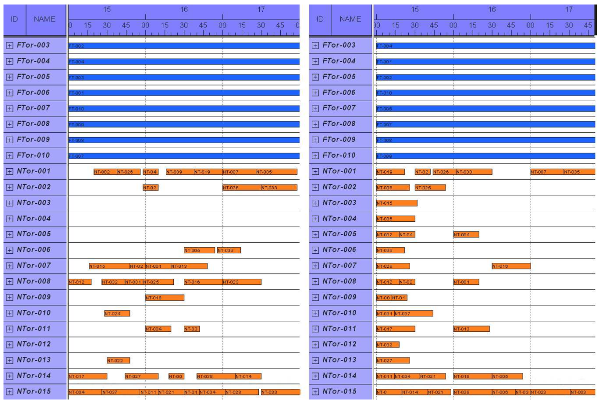

Figure 3: Optimization results of one test scenario as Gantt-Chart

The lines correspond to the doors. The blue areas represent relations which block the assigned door for the whole period. The orange areas stand for tours that utilize the assigned door for short periods. In the left part, the aim was a pure minimization of distances ($B^t = 1, \forall t \in T$). You can easily see, that inbound doors 1, 2, 14, 15 are frequently used. Doors 7 and 8 are lying in front of the outbound zone (doors 1 - 10), they also get many tours. In the right part the main optimization criterion was to minimize the waiting times ($B^t = 1 + (t-1) * 0,5$). Here, all tours get the earliest possible assignment.
The next figure shows the solution process of the column generation approach (blue line) and the Branch-and-Cut algorithm of CPlex 10.0 (pink line) when optimizing two other scenarios (42 doors, 20 relations, 40 (left part) and 50 (right part) tours):

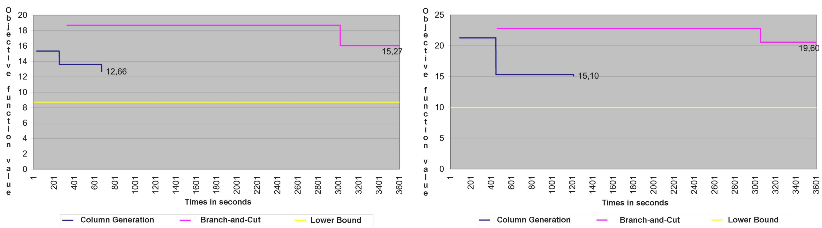

Figure 4: Comparison of tested algorithms for two scenarios

Although the column generation approach has been interrupted after a fixed number of iterations, in both cases the new approach outperforms the standard algorithm for MILP in terms of lower objective function values. The yellow line represents the lower bound which has been calculated by LP relaxation in the Branch-and-Cut. It it not clear how realistic this lower bound is.

5 Future Directions

In future research work, the decomposition and column generation approach should be integrated in a Branch-and-Price algorithm. Although, the test results have shown that this approach outperforms the Branch-and-Cut of CPlex 10.0 the quality of the solutions cannot be assessed. i.e. the MIP-gap which has been found by the Branch-and-Cut of CPlex is still not closed. It is not clear if this lower bound is a realistic guess of the theoretical optimum. So, another idea is to find better bounds for the mathematical model proposed in this paper.

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