Optimal Toll Design from Reliability Perspective

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1. Introduction

Road pricing has been advocated as a potentially powerful travel demand management (TDM) strategy capable of significantly influencing travel demand characteristics ((*1*), (*2*)). Generally, the policy objectives of road pricing can be summarized as: managing demand, optimizing congestion level, reducing environmental impacts, maximizing social welfare gains, raising revenues to recoup maintenance cost and construction cost, etc. The optimal charges obtained with different policy objectives are different.

Nowadays, reliability is becoming an increasingly important performance characteristic of road networks and transport services. Reliability is generally defined as the ability of a system or process to perform a required function under given environmental and operational conditions and for a stated period of time. Reliability, however defined, provides a measure of the stability of the quality of service, which the transport system can offer to its users. Whereas road pricing has been mainly considered to relieve congestion, it will be argued in this paper that it can also help provide more reliable transport services to the users.

To our best knowledge, no studies have advocated improving network travel time reliability as an objective of road pricing. Some researchers (e.g., (*3*)) suggested to evaluate road pricing from a network reliability perspective. A new or additional objective of road pricing from a road authority's point of view, aiming at optimizing network travel time reliability, is suggested in this paper and is used to determine the optimal charge or optimal links for tolling of a charging system.

2. Problem definition of the reliability-based toll design problem

In an optimal toll design problem, optimal toll schemes will result from different policy objectives. Designing toll levels from a reliability perspective is a new proposition of the road pricing objective. Conceptually, in the short term, road pricing influences travelers' trip choices, departure time choices, mode choices, and route choices. Trip choices and mode shifts together may result in a drop of travel demand during the tolling period. Departure time choices lead to temporal shifts of travel demand, thus a decrease of travel demand as well within the tolling period. Consequently, decreased travel demand and route flow shifts in the charged period are the prevailing determinants of network reliability.

In this paper, we assume that the road authority aims to improve road network reliability and to optimize system performance (i.e., improve reliability and minimize travel time on a network level). The tolls set by the road authority are assumed to influence travelers in their route and trip decisions where the decision to make a trip or not during the tolled period represents temporal and modal shifts as well. In addition, the travelers are assumed to take only travel time and tolls into account when making these decisions, however not trip reliability. Therefore, in this paper reliability is predominantly viewed as a network objective from a planning point of view, however it could be extended towards a user's objective as well. Network performance reliability will be analyzed considering stochastic network characteristics (i.e., stochastic link capacities), elastic demand and fluctuated travel demand from day-to-day. A reliability-based optimal toll design model, by choosing the optimal toll levels for a subset of links, subject to dynamic equilibrium traffic assignment is proposed in this paper. A dynamic travel analysis is followed because it will give more accurate estimates of congestion patterns and derived trip characteristics such as travel times. Due to the integrated stochastic properties, dynamic assignment, elastic demands, and various toll schemes, this reliability-based toll design problem quickly becomes very complex reason why application is only done to small networks so far.

The optimal toll design problem is a bi-level problem. On the upper level, the road authority determines the tolls on a set of links aiming to improve network travel time reliability, while on the lower level, travelers react on these tolls and change their route choices or decide to change mode or not travel at all. These latter two decisions are assumed to be simultaneously captured in an elastic demand framework. Since we will only analyze road pricing schemes with uniform tolls over the day, departure time choice is not expected to play a large role.

Special attention is needed for the time domain in this problem. Reliability refers to the day-to-day variations in quality of service of the network such as caused by day-to-day fluctuations in demand and capacity. Reliability is defined by a long term series of day-to-day trip travel times of a particular period of the day, e.g. the morning peak. To that period of the day a demand pattern is attached encapsulated by an analysis period, mostly of longer duration, for which the traffic flows including their trip times are determined that follow from the demand pattern. In a dynamic analysis this analysis period is further subdivided into smaller intervals.

3. Mathematical formulation of reliability-based toll design problem

The proposed design problem is a mathematical program with equilibrium constraints (MPEC) where the upper level is an optimization problem and the lower level is an equilibrium problem ((*4*), (*5*)). MPEC is a special case of a bi-level programming problem. This section formulates the reliability-based toll design problem as an MPEC in which the design variables are the toll levels on each of the links in the network.

3.1 Formulation of Lower Level Equilibrium Problem

Consider a network $G = (N, A),$ where *N* is the set of nodes, and *A* is the set of directed links. Let $R \subseteq N$ denote the set of origins and $S \subseteq N$ denote the set of destinations in the network, where each combination $(r,s),\ r \in R,\ s \in S,$ denotes a certain origin-destination (OD) pair in the network. Furthermore, let P^{rs} denote the set of feasible routes from *r* to *s*. Time is considered to be discretized into small time periods $t \in T$. Travel demand is assumed for the period $K \subseteq T,$ also subdivided into small periods $k \in K$.

For each link $a \in A$ the travel time $\tau_a(t)$ on *a* when entering the link at time *t* is expressed as:

$$\tau_a(t) = f_a\left(\mathbf{q}, C_a\right), \tag{1}$$

where $f_a(\cdot)$ is a link specific function. $\mathbf{q} \equiv [q_a(t)]$. $q_a(t)$ is the link inflow rate (veh/h) at time *t*. The vector $\mathbf{q}$ denote the inflow rates on all links in all time periods (previous and current time periods). C_a is the capacity (veh/h) on link *a*. Reliability of the network is the main focus in our optimal toll design problem. The sources of network unreliability within the transport network can be classified into two main categories: (a) travel demand variation, and (b) supply variation (e.g. link capacities). Both the link flow rates as well as the capacities are considered stochastic variables, such that also $\tau_a(t)$ is also stochastic.

In reality, capacity is not a continuous variable, dependent on the number of lanes. However in order to capture the stochastic properties of link capacity caused by weather, etc, we assume link capacity as a continuous variable, following a class of distribution. Current research applies different assumptions of the distribution of stochastic link capacity, such as uniform distribution, normal distribution, exponential distribution, gamma distribution, weibull distribution, etc. Depending on different characteristics of road facilities in different area, different link capacity distributions can be chosen. Brilon et al. (*6*) have tested different types of distribution function based on the value of the likelihood function and found that Weibull distribution turned out to be the function that best fitted the observations. Since in our analysis, the distribution type of capacity is not of importance and doesn't influence the outcomes, each link capacity C_a is assumed independently distributed, following a normal distribution with $\omega\%$ coefficient of variation (value to be derived from empirical data, see (*7*)), expressed as:

$$C_a \sim N\left(\bar{C}_a, \left(\omega \bar{C}_a\right)^2\right) \tag{2}$$

where $\bar{C}_a$ is the mean capacity. In reality, the correlation of link capacities do exist, however it is not important in our analyses, thus ignored.

The travel demand $Q^{rs}(k)$ (veh/h) for each OD pair (r,s) and departure period $k \in K$ is assumed to follow a normal distribution as well with $\beta\%$ coefficient of variation (derived from empirical data as well, see (*8*)), expressed as:

$$Q^{rs}(k) \sim N\left(\bar{Q}^{rs}(k), \left(\beta \bar{Q}^{rs}(k)\right)^2\right) \tag{3}$$

where $\bar{Q}^{rs}(k)$ is the mean elastic demand for time period *k*. This elastic demand is derived from some base travel demand $Q_o^{rs}(k)$ and depends on the generalized OD costs $c^{rs}(k)$ (€) which include the tolls denoted by the vector of (uniform) link tolls $\theta \equiv [\theta_a]$ (€). The mean elastic demand is calculated using a price elasticity of demand. The generalized OD cost is calculated by the route flow weighted average route costs $c_p^{rs}(k)$ for each route $p \in P^{rs}$. In reality, the OD demands may have large positive correlations or negative correlations. We don't take into account the demand correlations for our current preliminary analyses.

The stochastic dynamic user-equilibrium assignment problem can be formulated as the following equivalent variational inequality (VI) problem (see e.g. (*9*)) where we would like to find equilibrium route flow rates $\tilde{q}_p^{rs}(k)$ such that:

$$\sum_{(r,s)} \sum_{p \in P^{rs}} \sum_{k} \tilde{F}_p^{rs}(k)\left(q_p^{rs}(k) - \tilde{q}_p^{rs}(k)\right) \geq 0, \quad \forall q_p^{rs}(k) \in \Omega, \tag{4}$$

where

$$\tilde{F}_p^{rs}(k) = \left(\tilde{q}_p^{rs}(k) - P_p^{rs}(k) Q^{rs}(k)\right) \frac{\partial \tilde{c}_p^{rs}(k)}{\partial q_p^{rs}(k)}, \tag{5}$$

and where $P_p^{rs}(k)$ is the proportion of travelers choosing route *p* among all route alternatives for an OD pair (*r*,*s*). It can be calculated using any choice models such as path-size logit model, C-logit model, which account for the route overlapping. Ω is the set of feasible route flow rates defined by flow conservation and nonnegativity constraints, respectively:

$$\sum_{p\in P^{rs}} q_p^{rs}(k) = Q^{rs}(k), \quad \forall (r,s)\in RS, \forall k\in K, \tag{6}$$

$$q_p^{rs}(k) \geq 0, \quad \forall (r,s)\in RS, \forall p\in P^{rs}, \forall k\in K. \tag{7}$$

It is important to note that $c_p^{rs}(k)$, $P_p^{rs}(k)$, and $Q^{rs}(k)$ depend on the link toll levels θ, such that also the equilibrium route flow rates $\tilde{q}_p^{rs}(k)$ depend on the toll levels.

3.2 Formulation of Network Travel Time Reliability

We define network reliability based on route and trip travel time reliability. The travel times constitute a distribution of a longer series of day-to-day fluctuations in travel time of trips made during a particular period of the day (e.g. morning peak). Route-based network travel time reliability is defined to express the quality of service that a transport network provides to its users over a longer stretch of time. On the route level, travel time is a stochastic variable following a class of distributions, due to stochastic capacities and stochastic travel demand. Travel time reliability is useful to evaluate network performance under normal daily operations. Different measures have been proposed as a travel time reliability indicator: probabilistic measures ((*10*); (*11*); (*12*)), travel time standard deviation ((*13*)), travel time skew (*14*), coefficient of variance, and the like. Standard deviation is chosen as an indicator of route travel time reliability, since it is a good measure to provide the range of travel time variations. Although the day-to-day travel time distribution is typically asymmetric, the effects of negative and positive deviations of travel time can be formulated as a function of travel time standard deviation ((*13*)). This provides some justification for the widespread use of standard deviation as an indicator of travel time reliability. The smaller the standard deviation is, the more reliable the route travel time is.

The unreliability $\hat{\rho}_p^{rs}$ of travel time valid for time interval $\hat{K}\subseteq K$ on route *p* from origin *r* to destination *s* is defined as:

$$\hat{\rho}_p^{rs} = \sqrt{\mathrm{Var}\left(\hat{\tau}_p^{rs}\right)}, \tag{8}$$

where $\hat{\tau}_p^{rs}$ is the average route travel time for route *p* from *r* to *s* over period $\hat{K}$. Thus, given link tolls θ, stochastic day-to-day link travel times $\tau_a(t)$ follow, from which the stochastic day-to-day route travel times $\tau_p^{rs}(k)$ and the stochastic average route travel times $\hat{\tau}_p^{rs}$ can be determined, yielding route travel time unreliability $\hat{\rho}_p^{rs}$. Travel time variability is caused by day-to-day capacity variations and travel demand fluctuations, and denotes the variability of average travel times at the same time period $\hat{K}$ over the days. Within-day dynamics during the time period of each day are not considered.

Trip travel time unreliability is an unreliability indicator for a specific OD pair which is important for users when making a trip from one place to another. Trip travel time unreliability $\hat{\rho}^{rs}$ for a specific OD pair during time interval $\hat{K}$ for a traveler is defined on the basis of the corresponding route travel time unreliabilities of all used routes $p\in P^{rs}$ and using average route flow rates as weights:

$$\hat{\rho}^{rs} = \frac{1}{\hat{\bar{Q}}^{rs}} \sum_{p\in P^{rs}} E(\hat{q}_p^{rs})\hat{\rho}_p^{rs}, \quad \text{with } \hat{\bar{Q}}^{rs} = \sum_{k\in\hat{K}} \bar{Q}^{rs}(k), \tag{9}$$

where $E(\hat{q}_p^{rs})$ is the expected average route flow rate during time period $\hat{K}$ on route *p* from *r* to *s* over a series of days.

Network travel time unreliability should take into account all OD pairs' travel time unreliabilities. Based on trip unreliability, network travel time unreliability $\hat{\rho}$ for period $\hat{K}$ can be formulated as:

$$\hat{\rho} = \frac{\sum_{(r,s)} \hat{\bar{Q}}^{rs} \hat{\rho}^{rs}}{\sum_{(r,s)} \hat{\bar{Q}}^{rs}} = \frac{\sum_{(r,s)} \sum_{p \in P^{rs}} E(\hat{q}_p^{rs}) \hat{\rho}_p^{rs}}{\sum_{(r,s)} \hat{\bar{Q}}^{rs}}. \tag{10}$$

Since $\hat{\rho}_p^{rs}$ depends on the link tolls, as explained above, also $\hat{\rho}$ depends on the link tolls. This definition provides an indicator to assess network reliability with different toll schemes, showing the global average travel time deviations for a specific day time period for an arbitrary user of this road network. For a specific network, this network unreliability measure can be used to compare road network reliability performances under different conditions, such as with varying link capacities, with varying demand levels, with different tolling schemes, and so forth. For a comparison of network reliability between two different networks, the average network travel time needs to be taken into account in the definition of reliability.

The average network travel time indicator $\hat{\tau}$, which again implicitly depends on the link tolls, is also an important indicator of transport network services, which is calculated by

$$\hat{\tau} = \frac{\sum_{(r,s)} \sum_{p \in P^{rs}} E(\hat{q}_p^{rs}) \hat{\tau}_p^{rs}}{\sum_{(r,s)} \hat{\bar{Q}}^{rs}}. \tag{11}$$

These two measures, $\hat{\tau}$ and $\hat{\rho}$, will be used in combination in the objective function of the optimal toll design problem.

3.3 Formulation of Network Reliability-Based Optimal Toll Design Problem

Based on the formulated lower level equilibrium, the formulation of network travel time (un)reliability, and the formulation of network travel time, the reliability-based optimal toll design problem can be formulated as:

$$\min_{\theta} Z = \hat{\tau} + \tfrac{\gamma}{\alpha} \hat{\rho} = \frac{\sum_{(r,s)} \sum_{p \in P^{rs}} E\left(\hat{q}_p^{rs}(\theta)\right)\left(\hat{\tau}_p^{rs}(\theta) + \tfrac{\gamma}{\alpha} \hat{\rho}_p^{rs}(\theta)\right)}{\sum_{(r,s)} \hat{\bar{Q}}^{rs}(\theta)} \tag{12}$$

subject to

$$\theta_a(t) \geq 0, \quad \forall a \in A, \forall t \in T, \tag{13}$$

$$\sum_{(r,s)} \sum_{p \in P^{rs}} \sum_{k} \tilde{F}_p^{rs}(k)\left(q_p^{rs}(k) - \tilde{q}_p^{rs}(k)\right) \geq 0, \quad \forall q_p^{rs}(k) \in \Omega. \tag{14}$$

where Ω is the set of feasible route flow rates defined by constraints (6) and (7). In the objective function in expression (12), α and γ are value of time (VOT) and value of reliability (VOR), respectively. In (*10*) it was found the VOR increases or decreases accordingly to the increase or decrease of the VOT for different groups of people, while the reliability ratio γ/α is approximately constant.

Expression (12) defines the objective function from a road authority's point of view aiming to minimize network cost, which is a sum of network travel time and network unreliability. The network cost is a function of toll vector θ which we aim to derive, the stochastic capacity vector, and the elastic and stochastic OD demand. Thus, the reliability-based optimal toll design problem proposed in this paper is a complex problem. The formulated design problem belongs to the class of SN-SUE (Stochastic Network and Stochastic User Equilibrium, see (*15*)) problems, which models the stochastic network characteristics including stochastic link capacities, and also the travel demand fluctuations simultaneously. It should be mentioned that the model proposed in this paper can be extended to include different user classes.

Due to the stochastic capacity and the fluctuating travel demand, we use a Quasi-Monte Carlo approach (based on Halton draws, see (*16*)) to compute the unreliability measures in our design problem.

4. Experimental results

4.1 Test Network

As a preliminary analysis, the proposed reliability-based optimal toll design model is applied to a small hypothetical five-link network (see Figure 1) with a single OD pair (r,s). Since road pricing yields spare capacity on the charged link and the routes it belongs to, the charged routes can handle demand fluctuations more easily. However due to route flow shifts, the situation on other routes will become worse. Thus, on a network level, the network reliability is uncertain with tolls and needs to be analyzed. A single-link charging system is applied, in which only link 5 is tolled using a uniform toll for the whole time period. Links that will not be tolled receive a toll value of zero.

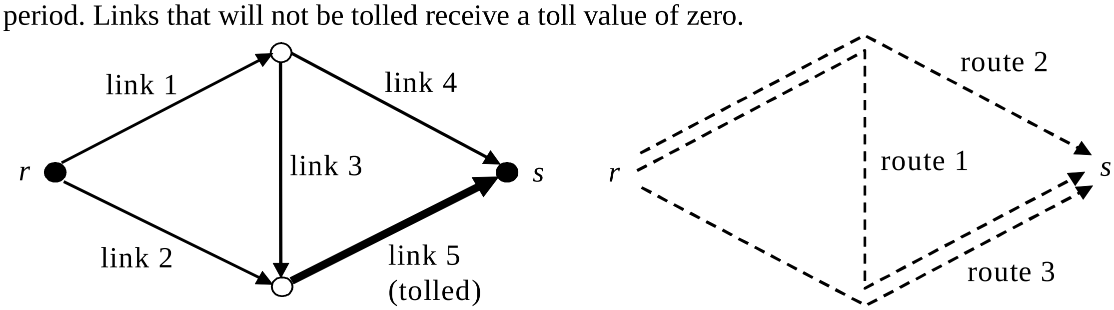


Figure 1 Five-link network

Table 1 lists free flow travel times and other characteristics of the five links. The INDY dynamic traffic assignment model (see (*17*)) is used for computing a stochastic dynamic user-equilibrium. Since in this study, we mainly focus on the analysis of the potential influence of road pricing on the network performance, to investigate how the impacts can be after implementing tolls in the network, firstly the multinomial logit (MNL) model is chosen for simplicity to model the choice behavior of travelers. All three routes are approximately equally attractive to the users when there are low flows through the routes, as the free flow route travel times are 3.5, 3.6, and 3.6, respectively for routes 1, 2, and 3.

A travel demand period K of an hour is considered in which the base OD travel demand is 5,000 veh/h for the whole hour. According to the capacities listed in Table 1, the network operates around capacity. Since the traffic situations is much more unstable when the network is operating around capacity, it is of more interests to look into the case when the network operates around capacity, than the case with demand far less than capacity with reliable travel times. The adopted coefficients of day-to-day variation for demand and capacity are $\beta = 0.2$ and $\omega = 0.1$, respectively, taken from (*8*) and (*7*). The demand elasticity to price is assumed to be 0.2. A reliability ratio of $\gamma/\alpha = 2.4$ (see Expression (12)) is used in this case study, taken from (*18*).

Table 1 Link characteristics

	Free-fow travel time (min.)	Capacity (veh/h)	Maximum speed (km/h)	Speed at capacity (km/h)
Link 1	1.7	2,500	120	80
Link 2	2.4	2,500	120	80
Link 3	0.6	2,800	120	80
Link 4	1.9	2,500	120	80
Link 5	1.2	2,500	120	80

4.2 Impacts of Tolling on Network Performance

Figure 2 shows total network costs (see dotted line) defined in objective function (12) as the sum of network travel time and weighted network unreliability. The dashed line indicates average travel time per vehicle in terms of varied toll levels on link 5, from which it can be seen that although the OD demand declines with increasing toll levels, the average travel time per vehicle (see Equation (11)) keeps nearly constant within the charge range of 0-2.5 euros on link 5, while network unreliability decreases greatly with increasing toll levels on link 5 within the charge range of 0-2.5 euros, as shown by the solid line. An optimal toll level of $\theta_5^* = 2.5$ is derived for this single-link charging system with which network reliability is optimized and network cost Z is minimized.

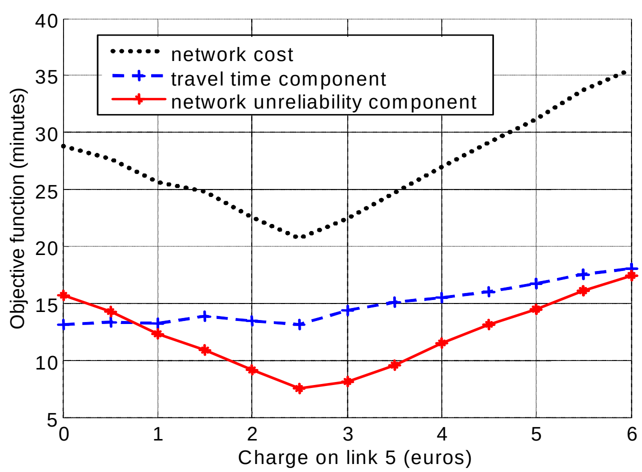


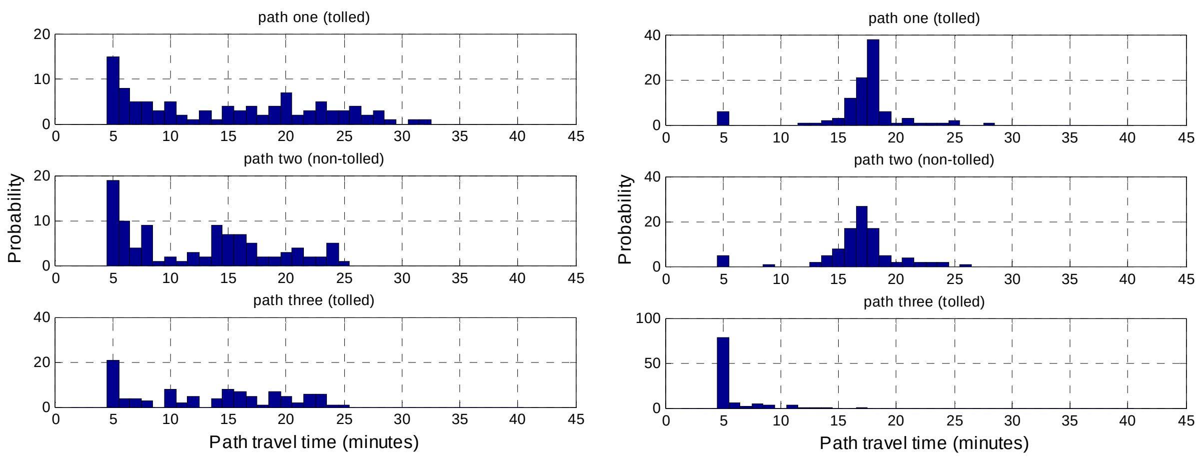
Figure 2 Objective network performances

The reason that network reliability can be improved by tolls is due to decreased OD demand and less frequent route flow switches with increasing charge levels on link 5. With low charge levels, decreased OD demand has a stronger positive influence on network reliability than route flow shifts do. With charge levels higher than 2.5 euros, most travelers shift to uncharged route 2, leading to traffic flows on route 2 reaching its capacity. The travel time on route 2 increases significantly. Small changes in capacities lead to great changes in travel time, thus leads to higher travel time unreliability. As seen in Figure 2, network unreliability increases with charge levels higher than 2.5 euros.

Since the objective is to find the optimal toll level θ^* with which the network reliability is optimized, it is important to investigate the impacts of the optimal toll scheme on network travel time reliability. Figure 3 shows for comparison the path travel time distributions without tolls and with the optimal toll level of $\theta^* = 2.5$. Without tolls, path travel times distribute widely with large standard deviations, as shown in Figure 3(a), while with the optimal toll vector θ^* the path travel time distributions are very centralized with much smaller variations (see Figure 3(b)). Travel time reliability has been improved greatly by charging a toll on link 5.

With respect to the expected route travel times, we notice an increase on route 1 and route 2, while a drop on route three after tolling. Travelers on charged route 3 save considerable travel time and get more reliable travel times by paying the tolls. Travelers on the other two routes take longer, but more reliable route travel times.

Based on our assumptions and above analyses, road pricing indeed may improve network travel time reliability. The potential impacts of road pricing on network reliability are analyzed. The proposed reliability-based optimal toll design model is promising at improving network reliability, which is a crucial factor involved in the transport network design problem nowadays.



(a) *without tolls* (b) *with optimal tolls*

Figure 3 Route travel time distributions

4.3 Comparison of the Impacts of Capacity Variations and Demand Fluctuations

Both capacity variations and demand fluctuations have significant influences on network reliability. Now let us quantitatively analyze the impacts of simultaneous fluctuations in both capacity and traffic demand, compared with the impacts from capacity or demand fluctuations separately. Table 2 illustrates mean route travel times, standard deviations and coefficients of variation (CoV) in terms of different affecting factors, see also (*19*). The bold cells highlight the results from simultaneous fluctuations in capacity and travel demand as analyzed in this paper. As expected, simultaneous fluctuations lead to larger variations in route travel times than either capacity or demand fluctuations separately do. Only modeling capacity or demand fluctuations and ignoring the other always underestimates the travel time variability. Simultaneous variations lead to shorter expected travel times, which can be explained by the fact that the expectation of the minimum of a set of distributions is always smaller than or equal to the minimum of the expectation of each of the distributions (*20*), since there are always opportunities to find a shorter travel time. Without tolls, the coefficients of variation (CoV) of route travel times caused from simultaneous fluctuations in both capacity and OD demand are larger than 48%; with optimal tolls, variations of route travel times dramatically decline with much smaller CoV of route travel times. Tolls indeed can help to improve network travel time reliability due to decreased OD demand and less flow shifts after implementing road pricing, based on this case study.

Table 2: Comparison influence of capacity variations and demand fluctuations

Charge (€)	*Path*	*Affecting factors*	*Route travel time* σ (min.)	*Route travel time* μ (min.)	*CoV*
0	Path 1	capacity variation	3.6	16	23%
		demand fluctuation	7.6	16.9	45%
		simultaneous	**8**	**15**	**53%**
	Path 2	capacity variation	2.4	13	18%
		demand fluctuation	6	13.2	45%
		simultaneous	**6.3**	**12.4**	**51%**
	Path 3	capacity variation	2.15	14	15%
		demand fluctuation	6.2	14.9	42%
		simultaneous	**6.4**	**13.2**	**48%**
2.5	Path 1	capacity variation	0.25	17.6	1%
		demand fluctuation	2.9	17.6	16%
		simultaneous	**3.8**	**17**	**22%**
	Path 2	capacity variation	0.3	17	2%
		demand fluctuation	2.7	17	16%
		simultaneous	**3.6**	**16.8**	**21%**
	Path 3	capacity variation	0.1	5.2	2%
		demand fluctuation	1.8	5.6	32%
		simultaneous	**2.1**	**6**	**35%**

4.4 Reliability Paradox

Note that the route costs (sum of travel time and tolls) with a toll of 2.5 euros are not equal for all routes (see Table 3). This does not violate the equilibrium principle as we use a logit-based stochastic assignment instead of a deterministic equilibrium. Once DUE is applied, route 1 will definitely not be used by travelers, which means that a network with only two routes with the optimal toll is more reliable from road authority's point of view than the network with three routes. Adding one more alternative route may not improve network reliability, even may decrease network reliability.

Table 3: Route travel costs with the optimal toll

Charge (€)	Path	Affecting factors	Route travel costs (min.)
2.5	Path 1	capacity variation	32.6
		demand fluctuation	32.6
		simultaneous	**32**
	Path 2	capacity variation	17
		demand fluctuation	17
		simultaneous	**16.8**
	Path 3	capacity variation	20.2
		demand fluctuation	20.6
		simultaneous	**21**

6. Conclusions

In this paper, we propose network reliability as a promising policy objective of road pricing. A reliability-based optimal toll design model is proposed and formulated. Network performance reliability is analyzed with respect to stochastic link capacities and OD demand with varied toll levels, which integrates reliability and uncertainty analysis, network equilibrium models, and Monte Carlo methods, to evaluate the performance of a degradable road network with elastic and fluctuated demand.

The proposed reliability-based optimal toll design model is applied to a hypothetical network with a single-link charging system for a preliminary analysis. The optimal toll level can be determined with which network reliability is optimized and the network cost is minimized. Route travel times may increase, but will be more reliable. A reliability paradox is found with this network from road authority's point of view that adding one more alternative route may not improve network reliability, even may decrease network reliability.

The proposed model can be utilized to optimize toll schemes by searching for optimal toll levels for all links and a set of optimal links for charging, aiming to improve network travel time reliability. Improving network reliability is a promising policy objective of road pricing, especially in the context that reliability is becoming increasingly important in network design problems. The proposed model can be applied for a general optimal toll design problem aiming to improve network travel time reliability. Furthermore, it can be extended further by including departure time choice, and the design of time-varying toll systems.

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