# SEQUENTIAL MODELS FOR THE MOBILITY DECISIONS: EXPERIMENTATION FOR THE HOLDING VEHICLES CHOICES 

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## Extended abstract:

The demand models used in literature, in the field of a behavioural approach, generally simulate the user choices through a discrete choice models.
The consolidated approach isn't explicitly able to simulate the variation of choice probability, in consequence of the different events, that characterize the evolution of the transportation system. So, we define static demand models the models that give the choice probability of the single alternative, independently of the actual choice of the decision-maker, relative to the actual and the previous system condition, and dynamic demand models the models that give the choice probability according to evolution system and earlier decisions.
The need to introduce dynamic models, considering the state of the decision-maker, in regards to different main decisions, among which it's possible to recall: the path choice, for a private transportation system user, and the run choice, for a transit system user, considering the travel choices; the holding choice and particularly the holding decision of vehicles, considering the mobility choices.
Predominantly, the models used in this area of research are members of the family of discrete choice models, derived from the random utility theory. This theory is based on the hypothesis that every individual is a rational decision maker, maximizing utility relative to his own choices.
Different random utility models can be derived by assuming different joint probability distribution functions for the random residuals. The relevant models used in literature are: the Generalized Extreme Value (GEV) model, proposed by McFadden (1978); the Probit model (Daganzo, 1979); the Multinomial Logit model (Ben Akiva and Lerman, 1985); the Mixed Multinomial Logit model (McFadden and Train, 1996) and the ordered GEV (Bhat, 1998) model.
These mathematical models have been proposed and applied in a way which doesn't give the possibility to represent adequately the dynamic of the process choice. In particular, they don't represent the influences exercised by the passed decisions on the actual choice.

In literature, the known models that consider some of the effects of the previous choices on the present one are the holding vehicle models.
Many of these studies (Manski and Sherman, 1980; Train, 1986) simulate the holding vehicle choices using the Multinomial Logit model and introduce the parameter
transaction search cost, in order to consider the influence exercised by the previous choices. The transaction search cost is a dummy variable which takes the value zero for vehicles currently owned by the household and one for all vehicles obtainable on the market, representing the search and transactions costs associated with changes in vehicle holding.
Hensher and Le Plastrier (1985) develop a series of linked choice models to explain household vehicle holdings and adjustments in the holdings over time. The effects of the experience are considered introducing the brand loyalty and the parameter experience index. The brand loyalty supports the hypothesis that a vehicle of a make previously chosen in the household's fleet is preferred to one of makes not experienced by the household. The experience index is a summary of retrospective utility, which links models related to several periods. It is introduced for the first time from Hensher and Johnson (1982) and is defined as the natural $\log$ of the denominator of the last estimated model in the previous period, representing the experience effects on the decision maker.
Train and Winston (2004) simulate the type of vehicle choice using a Mixed Multinomial Logit model, which:

- goes over the independence of irrelevant alternatives assumption maintained by the Multinomial Logit model;
- considers that the vehicle price is endogenous because it is related to unobserved vehicle features;
- considers heterogeneity among vehicle consumers;
- treats appropriately the effects of the experience through the brand loyalty.

However, these models are based on static structures, that can be defined pseudodynamic.

The transition matrix (Gottman and Roy, 1990), that represent the variation of user decisions over time, and the sequential model, that represents the time - dependencies of path choice (Russo, 1999), suggested the vehicle transition choice model. According to the sequential approach, the model simulates the permanence or the transition of the actual system state. It is different from the pseudo-dynamic models, because it simulates explicitly the choice of an alternative, in a given period of time, conserving or modifying the choice set relative to the previous period.
One of the literature model limits derives from the choice alternatives definition, which are represented as numbers, if it is necessary to simulate the holding choice, as vehicle classes related to determinate periods, if it is necessary to simulate the vehicle choice.
In comparison with previously considered models, the transition model can simulate the vehicle choice, in a given period of time, as maintenance or variation of the holdings level and composition, that is, in terms of sequential approach, as permanence or transition from the current system state.
At first, the proposed model has been applied by the authors to the holding vehicle, in order to compare the results obtained with the experimentation of the pseudo-dynamic models, used in literature.
Moreover, vehicle ownership plays an important role in determination of travel behaviour, because the availability in the household encourage its utilization, increasing network flows, traffic congestion, air pollution. On the other hand, understanding the behavioural responses of consumers regarding vehicle holding should be of interest to government and business.

In the year $t+l$, the alternative of choice for the decision maker derives from the possibility to conserve unchanged the vehicles set relative to the year $t$ or to add or deduct a generic number of vehicles.
We assume that the current decisions are directly influenced by the most recent previous decision and only indirectly influenced by the decisions earlier than the previous one via their influence on the subsequent decision. In other words, we assume that there is no connection between the transition from the period $t$ to a period $t+1$ and the transition from the period $t-1$ to a period $t$ (Markov Process), because the choice variations only derive from attribute variations on which the same choice it depends.
According to the Gottman and Roy (1990) approach, we can construct the frequency transition matrix (Fig.1). We assume that, in a given period of time $t$, a decision maker owns a number of vehicles as $n(t-1)+q, n(t-1)+1, n(t-1), n(t-1)-1$ or $n(t-1)-q, q$ generic positive integer, in comparison with the vehicle number $n(t-1)$ owned in the period $t-1$. Then, in the year $t$, the choice alternatives for the decision maker derives from the possibility to conserve unchanged the vehicle set relative to the year $t-1$ or to acquire or deduct a generic number $q$ of vehicles. In formal terms, the alternatives are: $n(t)+q$, $n(t)+1, n(t), n(t)-1$ or $n(t)-q$ (Fig.1).

|  | $t+1$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{n}(\mathrm{t})+\mathrm{q}$ | $\ldots$ | $\mathrm{n}(\mathrm{t})+1$ | $\mathrm{n}(\mathrm{t})$ | $\mathrm{n}(\mathrm{t})-1$ | $\ldots$ | $\mathrm{n}(\mathrm{t})-\mathrm{q}$ |  |
| $\mathrm{n}(\mathrm{t}-1)+\mathrm{q}$ | $\mathrm{a}_{11}$ | $\ldots$ | $\mathrm{a}_{1, \mathrm{j}-1}$ | $\mathrm{a}_{1 \mathrm{j}}$ | $\mathrm{a}_{1, \mathrm{j}+1}$ | $\ldots$ | $\mathrm{a}_{1 \mathrm{~m}}$ | $\mathrm{a}_{(\mathrm{n}+\mathrm{q})+}$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $\mathrm{n}(\mathrm{t}-1)+1$ | $\mathrm{a}_{\mathrm{h} 1}$ | $\ldots$ | $\mathrm{a}_{\mathrm{h}, \mathrm{j}-1}$ | $\mathrm{a}_{\mathrm{hj}}$ | $\mathrm{a}_{\mathrm{h}, \mathrm{j}+1}$ | $\ldots$ | $\mathrm{a}_{\mathrm{hm}}$ | $\mathrm{a}_{(\mathrm{n}+1)+}$ |
| $\mathrm{n}(\mathrm{t}-1)$ | $\mathrm{a}_{\mathrm{i} 1}$ | $\ldots$ | $\mathrm{a}_{\mathrm{i}, \mathrm{j}-1}$ | $\mathrm{a}_{\mathrm{ij}}$ | $\mathrm{a}_{\mathrm{i}, \mathrm{j}+1}$ | $\ldots$ | $\mathrm{a}_{\mathrm{im}}$ | $\mathrm{a}_{\mathrm{n}+}$ |
| $\mathrm{n}(\mathrm{t}-1)-1$ | $\mathrm{a}_{\mathrm{k} 1}$ | $\ldots$ | $\mathrm{a}_{\mathrm{k}, \mathrm{j}-1}$ | $\mathrm{a}_{\mathrm{kj}}$ | $\mathrm{a}_{\mathrm{k}, \mathrm{j}+1}$ | $\ldots$ | $\mathrm{a}_{\mathrm{km}}$ | $\mathrm{a}_{(\mathrm{n}-1)+}$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $\mathrm{n}(\mathrm{t}-1)-\mathrm{q}$ | $\mathrm{a}_{\mathrm{m} 1}$ | $\ldots$ | $\mathrm{a}_{\mathrm{m}, \mathrm{j}-1}$ | $\mathrm{a}_{\mathrm{mj}}$ | $\mathrm{a}_{\mathrm{m}, \mathrm{j}+1}$ | $\ldots$ | $\mathrm{a}_{\mathrm{mm}}$ | $\mathrm{a}_{(\mathrm{n}-\mathrm{q})+}$ |
|  | $\mathrm{a}_{+(\mathrm{n}+\mathrm{q})}$ | $\ldots$ | $\mathrm{a}_{+(\mathrm{n}+1)}$ | $\mathrm{a}_{+\mathrm{n}}$ | $\mathrm{a}_{+(\mathrm{n}-1)}$ | $\ldots$ | $a_{+(\mathrm{n}-\mathrm{q})}$ |  |

Fig. 1 - Frequency transition matrix
The generic element $\mathrm{a}_{\mathrm{ij}}$ of the transition matrix represents the family number which passes from the state $i$ to the state $j$ of the system, passing from period $t$ to period $t+1$.
We can extract the probability transition matrix from the frequency transition matrix, dividing the generic $\mathrm{a}_{\mathrm{ij}}$ by the total line $\mathrm{a}_{\mathrm{i}(+)}$ (Fig. 2). The probability transition matrix represents the family percentage which passes from the state $i$ to the state $j$ of the system, passing from period $t$ to period $t+1$.


Fig. 2 - Probability transition matrix

The tested transition model gives the probability $\mathrm{p}_{+(\mathrm{n}(\mathrm{t})+1)}, \mathrm{p}_{+(\mathrm{n}(\mathrm{t})}, \mathrm{p}_{+(\mathrm{n}(\mathrm{t})-1)}$ that the family owns, in the period $t+1$, a vehicle number like $n(t)+q, \ldots, n(t), \ldots, n(t)-q$ in comparison with the number vehicle owned in the period $t$.
In this work, we assumed that the number of transactions for each household is limited to only one per year, limiting the transaction choice set.
In other words, we assumed that the stochastic process model has a state space comprising: to acquire a new vehicle; to maintain the number vehicle owned earlier; to sell a vehicle. Subsequently we subdivide the second state considering that each family can trade one of its vehicles for another vehicle or to maintain the number and the type (family, compact,...) and vintage (new, old, usage...) of vehicle owned earlier.
Therefore, it is important to capture the transaction behaviour in dynamic context, as a process of adjustment to the households' vehicle fleet (sequential approach).
The time-dependencies are considered introducing some attributes that are function of the passed state.
The transition model has been specified, calibrated and validated using:

- a database relative to the socio-economic evolution of a sample family, which captures dynamic longitudinal effects;
- a database relative to the technical classification of vehicles, defined by an Italian company of car hire;
- a database relative to the technical-performances characteristics of vehicles, obtained by a specialized car review published in Italy.
The results obtained by the experimentation of the model confirms the need of a dynamic sequential approach for the holding vehicles choices. They are presented in the paper and compared with the results obtained through the application of the models present in literature to the same sample.
The comparison gives a favourable index for the sequential model in relation to the others experimented models.


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