Inventory Routing Problem for the LNG Business

Roar Grønhaug[1] Marielle Christiansen[1] Guy Desaulniers[2,3]
Jacques Desrosiers[2,4]

[1]Norwegian University of Science and Technology, [2]GERAD, [3]Ecole Polytechnique de Montreal

[4]HEC Montreal

e-mail: roar.gronhaug@iot.ntnu.no

1 Introduction

Natural gas has traditionally been transported in pipelines, but transportation by ships is more efficient for transportation over long distances. One way of transporting natural gas by ships is to cool the gas down to liquid state at atmospheric pressure before loading it into special designed tank ships. In 2003 there were 151 liquefied natural gas (LNG) tankers in operation and 55 LNG tankers were under construction (EIA, 2003).

The purpose of this paper is to introduce a real ship routing and inventory management problem for the LNG supply chain. The paper will also present the characteristics of this problem and how it is distinguished from other maritime inventory problems. The problem is solved by use of a column generation algorithm. An introduction to column generation is given by Lübbecke and Desrosiers (2005a), while Lübbecke and Desrosiers (2005b) give an updated survey of column generation with special emphasis on solving integer problems.

The rest of the paper is organized as follows: Section 2 gives the problem description, and a short literature review is covered in Section 3. Section 4 presents the mathematical model and its decomposition, and finally, computational results will be discussed in Section 5.

2 Problem Description

Suez Energy International (SEI) is a global energy actor, and is involved within most of the LNG supply chain except exploration and extraction. Thus, the company is involved in liquefaction, transportation, storage, and regasification of LNG. SEI currently purchases and distributes approximately 8 million tons of LNG per year, using a number of liquefaction plants and regasification terminals throughout the world. The LNG operations are continuously increasing.

The transportation at sea is carried out with SEI's own fleet of LNG tankers. Currently, this fleet consists of 7 LNG tankers, but will increase as SEI's LNG operations increase. There are several important issues regarding the LNG tankers. First, LNG tankers use its cargo as fuel because some of its content vaporizes each day and will otherwise be of no use, this vaporizing is called boil-off. The second issue is due to sloshing induced forces in the cargo tanks. In order to maximize safety, the volume in the tanks should be as close to maximum or minimum as possible. The third issue is that if a tank on board an LNG tanker is completely empty, the tank's temperature will begin to rise. Before any LNG is reloaded into the LNG tanker, the tank must be recooled, which is expensive and time consuming. In order to cope with these issues an LNG tanker loads its tanks completely at a liquefaction plant, and unloads discrete number of tanks at the regasification terminals. The LNG tanker must have just enough LNG left in the tanks to cover the boil-off until it reaches a liquefaction plant. Only a limited number of ships can (un)load at the same time at a port since it requires special equipment.

Corporate planners at SEI oversee the routing and scheduling of the LNG tankers in order to keep the inventories between their upper and lower limits. Their objective is to maximize the total profit from selling LNG to end-customers. The sales contracts include fixed contracts where the agreed volume cannot be violated, contracts with upper and lower limits on the delivering volume, and short term contracts where SEI can decide to sell LNG if profitable. If advantageous, SEI can buy spot cargoes of LNG delivered directly to SEI's regasification terminals. The typical planning horizon is $3 - 4$ months. Problems where one actor is responsible for both the transportation and the inventory at the sources and the destinations are called inventory routing problems. Thus, SEI's problem can be viewed as a *maritime inventory routing problem*. A review of inventory routing problem literature can be found in Kleywegt et al. (2004).

The problem can then be defined as maximizing the total supply chain profit from the LNG operations within a tactical planning horizon. The profit consists of sales revenues less purchase and transportation costs. In order to maximize the profit the ships must be routed in the most efficient way. Furthermore, the inventories must be kept between their upper and lower limits; these limits might differ from day to day. Spot cargo purchase of LNG is possible. In addition, the number of ships simultaneously in a port is limited by an upper bound.

3 Related Literature

Inventory routing is rarely discussed in maritime context, and more research has been sought after Christiansen et al. (2004, 2006). However, Christiansen (1999) studies such a problem where she considers the ship routing and inventory management for an ammonia producer. In her case, the supply chain for ammonia consists of several locations that either produce or consume ammonia at a constant rate, and the transportation network between those locations. The shipper has a fleet of ships available for transporting ammonia between these ports, and he is responsible for keeping the inventory levels between predetermined upper and lower bounds. Although the problem from Christiansen (1999) share some similarities with the LNG supply chain problem, there are some important distinctions between them. One major difference is that Christiansen (1999) assumes constant production and consumption rates at the terminals, while SEI's demand and production of LNG might differ from day to day during the planning horizon. Furthermore, Christiansen (1999) does not have to cope with boil-off or sloshing.

The overall problem in Christiansen (1999) is solved by column generation (Christiansen and Nygreen, 1998a) with two types of subproblems which are solved by dynamic programming algorithms (Christiansen and Nygreen, 1998b). Another solution approach to the problem described in Christiansen (1999) is developed by Flatberg et al. (2000). They have solved the problem with an iterative improvement heuristic combined with a linear programming solver. Furthermore, Christiansen and Nygreen (2005) have extended the model described in Christiansen and Nygreen (1998a) in order to deal explicitly with the stochastic nature of maritime transportation. They introduce a set of soft inventory limits within the hard inventory limits. Thus violating the soft inventory constraints yields a penalty, but it is not possible to violate the hard inventory limits.

4 Mathematical Formulation

The problem has been formulated as a path flow model, where the paths represent possible routes for the ships. The problem is solved by column generation, and branch-and-price is used to obtain integer solutions. The path flow model represents the (restricted) master problem and has a more extensive form than the more usual set partitioning or set covering formulations. It handles the production and sales of LNG in addition to the port capacity and spot cargo purchases. Furthermore, there is one subproblem for each ship that generates new columns for the master problem.

4.1 Master Problem

In the mathematical description of the problem each port is represented by an index i, and the set of liquefaction plants (pickup ports) is given by N_P, while the set of regasification terminals (delivery ports) is given by N_D. The set of all ports is given by N. Furthermore, let V, indexed by v, be the set of available ships. The set of possible arcs for a ship in the network is given by A_v. The LNG tankers have several cargo tanks, thus there exist several (un)loading combinations for each ship, denoted by the set W_v and the index w. The set of time periods is given by the T, and is indexed by t. The sales at regasification plant $i \in N_D$ and the production of LNG at liquefaction plant $i \in N_P$ is calculated by the variable y_{it} during the different time periods $t \in T$. y_{it} is bounded by the interval $\{Y_{MNit}, Y_{MXit}\}$, and the unit sales revenues and production costs are given by R_{EVit} and C_{OSTit}, respectively. Furthermore, the variable s_{it}, which is bounded by the interval $\{S_{MNi}, S_{MXi}\}$, represents the inventory level at the regasification terminals and liquefaction plants in the different time periods. s_{i0} is a parameter representing the initial inventory.

For each ship $v \in V$ we need to find feasible routes with respect to the tanks loading capacities and the boil-off. The variable λ_{vr} equals one if ship v chooses to sail on route $r \in R_v$, where R_v is the set of generated routes or columns. The parameter Z_{ivtr} equals 1 if ship v visits port i in time period t on route r and 0 otherwise, while the the chosen (un)loading volume is given by the parameter Q_{ivtr}. The cost of sailing on route r is given by the parameter C_{vr}.

The spot orders are indexed by k, and the set of spot orders for port i are given by K_i. The binary variable $z_{SPOTikt}$ decides whether a spot cargo k at port i in time period t should be purchased or not. The spot order has a unit cost of $C_{SPOTikt}$, and the variable $q_{SPOTikt}$ decides the volume which is bounded by the interval $\{U_{MNik}, U_{MXik}\}$. U_{NUMX} is the maximum number of spot cargoes purchased during the planning horizon, while N_{CAPi} is the maximum number of ships that can (un)load simultaneously at port i.

Then, the master problem can be modeled as follows:

$$\max \sum_{t \in T} \sum_{i \in N_D} R_{EVi} y_{it} - \sum_{t \in T} \sum_{i \in N_P} C_{OSTi} y_{it} - \sum_{t \in T} \sum_{i \in N_D} \sum_{k \in K_i} C_{SPOTikt} q_{SPOTikt} - \sum_{v \in V} \sum_{r \in R_v} C_{vr} \lambda_{vr} \tag{1}$$

$$s_{it} - s_{i(t-1)} - \sum_{v \in V} \sum_{r \in R_v} Q_{ivtr} \lambda_{vr} - \sum_{k \in K_i | i \in N_D} q_{SPOTikt} + I_i y_{it} = 0, \forall i \in N, t \in T \tag{2}$$

$$\sum_{v \in V} \sum_{r \in R_v} Z_{ivtr} \lambda_{vr} + \sum_{k \in K_i} z_{SPOTikt} \leq N_{CAPi}, \forall i \in N, t \in T \tag{3}$$

$$\sum_{t \in T} \sum_{i \in N_D} \sum_{k \in K_i} z_{SPOTikt} \leq U_{NUMX} \tag{4}$$

$$\sum_{r \in R_v} \lambda_{vr} = 1, \forall v \in V \tag{5}$$

$$U_{MNik} z_{SPOTikt} \leq q_{SPOTikt} \leq U_{MXik} z_{SPOTikt}, \forall k \in K_i, i \in N_D, t \in T \tag{6}$$

$$S_{MNi} \leq s_{it} \leq S_{MXi}, \forall i \in N, t \in T \tag{7}$$

$$Y_{MNit} \leq y_{it} \leq Y_{MXit}, \forall i \in N, t \in T \tag{8}$$

$$z_{SPOTikt} \in \{0,1\}, \forall k \in K_i, i \in N_D, t \in T \tag{9}$$

$$\lambda_{vr} \geq 0, \forall r \in R, v \in V \tag{10}$$

$$\sum_{r \in R_v} X_{ijwvtr} \lambda_{vr} \in \{0,1\} \, \forall (i,j) \in A_v, w \in W_v, v \in V, t \in T \tag{11}$$

The objective function (1) maximizes total profit from the LNG supply chain, which consists of sales revenues minus the production, transportation, and spot cargo costs. Constraints (2) monitor the inventory levels at the ports. The parameter I_i is equal to 1 for regasification terminals and -1 for liquefaction plants. Furthermore, the (un)loading volume parameter, Q_{ivtr}, is positive for regasification terminals and negative for liquefaction plants. Constraints (3) and (4) limit the number of simultaneous ships in a port and the total number of spot cargoes purchased, respectively. Constraints (5) are the convexity constraints, limiting the ships to sail no more than one route, while constraints (6) restrict the spot cargo volume between upper and lower limits, if the spot cargo is purchased. The bounds on the different variables can be found in constraints (7)-(10). Finally, constraints (11) give the binary requirements for the routing variable λ_{vr}. The constraints also give the connection between the master problem and the subproblems, the constant X_{ijwvtr} represents the geographical route for a ship as it equals 1 if ship v starts sailing on arc (i,j) in time period t after (un)loading with tank combination w on route r, and 0 otherwise.

4.2 The Subproblems

The subproblems are shortest path problems for each ship and solved by specific dynamic programming (DP) algorithms. Although the networks are topological sorted, the boil-off from the LNG in the ships' tanks and the sloshing complicate the network design and the DP.

Two different network structures have been designed for the problem. In the first network structure, there is a node for each port and time combination. An arc in the network represents the sailing from node (i,t) to node (j,τ) combined with the volume LNG (un)loaded at node (i,t). With this network design, the DP needs to calculate the cumulative boil-off for what we call a duty, which is a journey between two liquefaction plants where the ship visits one or more regasification terminal. The reason for this is to be able to calculate the correct unloading volume at the regasification terminals.

In the other network structure, there are nodes only for the liquefaction plants. Thus, an arc covers an entire duty. When the duty is known a priori the cumulative boil-off is also known, and is not a complicating factor for the DP. On the other side, we have to generate arcs for all possible duties which lead to a significant larger network in terms of arcs than the first network structure.

5 Computational Results

Computational results from experiments with real data from SEI will be presented.

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