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### Modeling Residential and Spatial Search Behaviour: Evidence from the Greater Toronto Area

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#### Abstract

This papers attempts to conceptualize the residential mobility and spatial search process as a part of comprehensive Residential Mobility and Location Choice (REMLOC) model to be implemented within a microsimulation-based integrated modeling framework and presents empirical results of household mobility-decision models using Greater Toronto Area (GTA) retrospective survey data. In implementing mobility models, both discrete choice and hazard-based duration models are tested in order to aid time- and event-driven microsimulation respectively. Within discrete choice framework, binomial choice panel logit models are examined that include fixed effects, random intercept and random parameter models. On the other hand, parametric hazard duration models are investigated with various assumptions concerning the baseline distribution. In addition, the paper investigates frailty models that account for unobserved heterogeneity for both univariate and multivariate hazard models. While the Random Parameter (RP) model performs better in identifying residential stressors that lead to mobility, the log-logistic Gaussian shared frailty model shows promising results in explaining termination of passive-state duration. The study reveals that most stressors that relate to life cycle events such as job change, birth of a child, increase/decrease in number of jobs etc. are significant in the RP Model. On the other hand, dwelling and neighbourhood characteristics are dominant in the continuous-time shared frailty model. The estimation results of both techniques give important behavioural insights and better understanding of residential mobility processes, which can be incorporated in the microsimulation-based integrated urban models.

Keywords: mobility and search, binomial logit, random parameter, hazard model, frailty

#### 1. Introduction

Since residential mobility and location choice is an important part of integrated land use and transportation models to understand relationships between transportation and land uses, it is necessary to explicitly model the "decision-making process" of relocation incorporating its underlying behaviour. This paper presents a comprehensive conceptual model of residential mobility and location choice (REMLOC) and identifies various decision components starting from the decision to become active in the housing market through spatial search of dwellings that ends up with bidding process to secure a new location. It also implements households' mobility decision component by using two different techniques: discrete choice and hazard-based duration modeling. Understanding the mobility process is very important to identify behavioural responses due to changes within the households and their surroundings, which will provide tools to test various public policies and link with the residential location choice processes that has significant influence on the composition of neighbourhoods, travel patterns and commuting. One of the significant contributions of this paper is that it extends standard binomial choice models of residential mobility explicitly recognizing panel effects. With the retrospective survey data three different specifications, fixed effects, random intercept and random parameter models are investigated to incorporate unobserved heterogeneity due to panel structure. To our knowledge, the empirical application of Random Parameter (RP) model in analyzing residential mobility in this paper is unique in literature. On the other hand, hazard-based duration models are examined to recognize dynamics of duration in taking mobility decisions. In other words, these models attempt to test likelihood of termination of passive-state duration depending upon the length of time spent from the beginning of the event (being active in the housing market) as well as other related covariates.

The paper is organized as follows: the next section discusses a comprehensive residential mobility and spatial search process to be implemented within REMLOC. Section 3 provides concepts and estimation procedures of the techniques applied to model residential mobility. Section 4 briefly describes the data used for the empirical application. Section 5 discusses results of both binary choice panel logit and hazard-based duration models. Finally, Section 6 concludes with a summary of contribution of this study and future research directions.

#### 2. Conceptual framework: Mobility and Spatial Search

Assuming a sequential decision making process (see discussions in Habib and Miller, 2005, 2007) the Residential Mobility and Location Choice (REMLOC) Model consists of three interrelated components: Mobility decisions, Spatial search and Bidding process to secure alternative locations. In this section the conceptual framework of each of these modeling components are briefly discussed.

The residential mobility component determines for each decision making unit (DMU) the decision to search for dwellings and the timing of mobility decisions. It is hypothesized that DMUs take these decisions for adjustments with housing needs based on stressors arising from changes in household composition and different life cycle events as well as surrounding environments (for recent literature reviews see Habib and Miller, 2007; Clark and Ledwith, 2005). Since DMUs could act differently in response to stressors (for example in case of job change one could either buy a new car to reduce burden of commuting or change dwelling), a stress management component is introduced to deal

with stress-release mechanism. Once a DMU take a decision to search, it enters into the spatial search process, which is a very complex process involving an array of decisions to make before the bidding for selected alternative dwellings to relocate. Figure 1 details a conceptual framework to model spatial search.

At first, it is assumed that every DMU has a mental perception how to search. It has certain objectives. Either they are a "forced mover" due to occurrence of any major triggering event (such as in-migration, formation of new household through marriage or separation) or "choice movers" under a certain level of stress (for example, room-stress due to birth of a new child, commuting stress due to change in job etc). As such, DMUs know what they need (at least a crude estimate). This mental model would be tried to capture by "(Selection of) search strategies" component. That means the outputs of stress and stress-release mechanism will guide the formulation of strategies. In addition, socio-economic characteristics might have impact on selecting strategies.

Of course, there are arguments that DMUs might have some advance knowledge of the market through day-to-day activities like reading newspapers, driving through a neighborhood and seeing rental advertisements. These phenomena arguably also have impacts on selecting strategies. But they are not sufficient condition for considering a move and deliberately searching dwellings to improve existing utility of residential location. So this model would use this "past knowledge and experience" as one of the factors in shaping strategies, but the process will be dealt with a separate component ("passive information") and placed within the "information channel" component.

In reality, search strategies could be of numerous kinds. For simplicity, this conceptual model would preliminary focused on following strategies:

- Anchor-based search (anchor at current job and/or residence),
- Supply-driven search, area-based sequential search,
- Area-based simultaneous search and
- Learning-based search (see Huff, 1986; Pushkar, 1998).

The key outcome of the "strategy of search" component is that it will give a big sample of dwelling units that are available in the market in a given period of time for a particular DMU. However, a searcher, in reality, does not know this choice set. But, from the modeling perspective, inference on strategic choice has a great impact. Having subsample of alternatives (even if it is big) from the citywide set (which will be exogenously determined) would be very helpful. However, the actor who is searching will not be given this information (only the model would know it). Rather, a filtered set of choice will be passed to the actor, which will be produced from the level of information it might gain through active or passive search (and also from social networks).

As such, the "information channel" is very important part of search stage. In reality, information search activities can take a variety of forms: first, relying on Internet search (predominantly, for renters, thanks to IT technologies), attending open houses, searching commercial advertisements, contacting real estate agents, reviewing advertisement, calling developers etc. It would also include physically visiting a neighborhood to see "For Rent" or "For Sale" signs on poles or front yards. In addition, a DMU could be informed by friends, relatives, neighbors, colleagues and other "alters" of his/her social network.

The information a searcher could gain from these channels would actually filter the "big choice set" (obtained from previous component) and would help creating "awareness space" (i.e. a set of dwelling units that an actor knows for certain they physically exist). For this set of dwellings a searcher can somehow assign "place utilities". However, it is assumed that the DMU will not visit each of these dwellings, even may not extensively search more information on all of those that requires defining "search space" which is a sub-set of "awareness space". The argument is that the filtered information might produce some useless options that do not match the "mental aspiration region" of a person seeking a dwelling. Mental aspiration regions reflect an upper-limit and lower-limit that the DMU has in mind about the attributes of the dwelling (in terms of structural, neighborhood, even transport options). For example, some people seek to rent

only one-bedroom apartment, and some "captive transit riders" have a minimum and maximum walking distance from the residence in mind. To capture these limits, this "mental aspiration region" is proposed, which will further screen out the "choice set" while taking actual decisions during search.

Until now, the searchers actually do not take any real decision other than collecting information and processing in response to mental simulations. The main decision components during search are: (a) Final screening of dwelling units from the "choice set" obtained from "search space" and matching with "mental aspiration region", (b) Defining prospect set (a set of dwelling that would be physically visited (i.e. "vacancy visit") to see and understand the value and utility of owning/renting), (c) Evaluation of selected dwellings in the prospect set (d) Defining bidding set after vacancy visit and evaluation. Once a DMU decides on alternative dwellings on which to bid, a bid formation, negotiation and acceptance/rejection process would take place within which both buyers and sellers interact.

Although the entire REMLOC model as outlined in the Figure 1 considers a sequential process involving mobility, search, and bid, it is expected that once each component is individually modeled potential feedbacks could be introduced to make the model more realistic and behaviourally sound. It is expected that it will be possible to operationalize this conceptual model of relocation with two retrospective surveys, the Residential Mobility Survey (RMS II 1998) and Residential Search Survey (RSS 1998) conducted for the Greater Toronto Area, and some additional survey needed in the near future. However, this paper attempts only to model the first component of decision to search using RMS II 1998 data for a sample of 270 GTA households that contains housing career, employment history and household composition changes for a period of 1971-1998. In the following section model structures of the mobility component are briefly discussed.

#### 3. Model structures for empirical application

For mobility, a DMU first decides whether to become active in the housing market at a given point of time or not. Two different modeling techniques are applied for the purpose: discrete choice methods and hazard-based duration models.

#### 3.1 Discrete choice models

Within discrete choice framework, binomial logit panel data models are investigated with different assumptions on the heterogeneity structures that include Fixed Effects (FE), Random Intercept (RI) and Random Parameter (RP) models.

The FE model specification is used to accommodate individual heterogeneity in the panel models by examining group specific effects:

 $P(y_{it} = 1) = g(\alpha_i + \beta' x_{it})$  (1)

where  $P(y_{ii} = 1)$  represents probability of binary choice of being active in the housing market and  $\alpha_i$  denotes group specific effects. In this particular application, a group means a DMU that has sequence of choice occasions over time. For a given set of DMUs (i = 1, 2, ..., n) at different unit time periods (i.e. choice occasions,  $(t = 1, 2, ..., T_i)$  the unconditional log-likelihood function is given by:

$$LogL = \sum_{i=1}^{n} \log \left[ \prod_{t=1}^{T_i} \Lambda[(2y_{it} - 1)(\alpha_i + \beta' x_{it})] \right]$$
(2)

where  $\Lambda$  is the CDF of logistic distribution. The conditional log-likelihood can be obtained by conditioning the contribution of each group on the sum of the observed outcome, which can then be maximized with respect to the slope parameters without estimating fixed effects parameters (see Chamberlain, 1980). However, direct maximization of the log-likelihood function with all parameters is possible by brute force maximization taking advantage of the properties of the sparse second derivatives matrix (see Greene, 2001 for details). While the parameter vector  $\beta_i$  has a fixed component and a sub-vector that varies across groups in the FE models, it is constant across groups and periods for the Random Intercept (RI) models. But the later type has a time invariant component, which is the latent heterogeneity that enters into the model in the form of random effects:

$$P(y_{it} = 1) = g(\beta' x_{it} + \psi_i)$$
(3)

where  $\psi_i$  is the unobserved heterogeneity same in every period.

On the other hand, in the Random Parameters (RP) model, all the parameters can vary randomly over individuals. The structure of the model is based on the conditional probability:

$$\operatorname{Prob}\left[y_{it} = 1 \middle| x_{it}, \beta_i\right] = g(\beta'_i x_{it}), i = 1, \dots, N, t = 1, \dots, T_i$$
(4)

The model is operationalized by writing:  $\beta_i = \beta + \Delta z_i + \Gamma v_i$  (5)

where  $z_i$  is a set of M observed variables which do not vary over time and enter into the means of the random parameters.  $\Delta$  represents a coefficient matrix that forms the observation-specific term in the mean. And  $\Sigma$  is the diagonal matrix of scale parameters (i.e. standard deviations). The random vector  $v_i$  induces the random variations in the reduced form parameters of the model. The unconditional likelihood function is given by

$$L_{i}(\Phi \mid y_{i}, x_{i}, z_{i}) = \int_{v_{i}} \prod_{t=1}^{T_{i}} g(y_{it}, \beta'_{i,t} x_{it}) f(v_{i}) dv_{i}$$
(6)

Since this likelihood function is a multivariate integral that cannot be evaluated in closed form, the parameters are estimated by simulation (See Train, 2003). The simulated log-likelihood function is

$$LogL_{s} = \sum_{i=1}^{N} \log \frac{1}{R} \sum_{r=1}^{R} \prod_{t=1}^{T_{i}} g(y_{it}, \beta'_{i,r} x_{it})$$
(7)

The simulation is carried out over R draws on  $v_{i,r}$  through  $\beta_{i,r}$ . The maximum simulated likelihood estimator is obtained by maximizing equation (7) over the full set of structural parameters  $\Phi$ . Eventually estimates of structural parameters and their asymptotic standard errors are generated from this simulated maximization procedure (see Greene, 2004, and Train, 2003 for details).

In the RP Model it is needed to assume specific distributions for random parameters. In most applications such as Revelt and Train (1998), Mehndiratta (1996), and Ben-Akiva and Bolduc (1996) it has been specified to be normal or lognormal. On the other hand, Revelt and Train (2000), Hensher and Greene (2001), and Train (2001) have used triangular and uniform distributions. This paper assumes all random parameters to be normally distributed with zero mean and unknown variance. In practice, it is often found that some of the parameters are random while others are nonrandom. In such cases, nonrandom parameters in the model are implemented by constraining corresponding rows in  $\Delta$  and lower triangular matrix to be zero. Note that in the Random Intercept (RI) model only the constant term is assumed to be random while all other parameters are nonrandom. In other words, the RI model can be seen as the RP model in which only the constant term is random.

#### 3.2 Hazard-based duration models

In addition to discrete choice models, this paper investigates continuous time hazardbased duration models to analyze mobility decisions. Hazard-based duration model recognizes dynamics of duration since likelihood of termination of duration depends on the length of time spent from the beginning of an event. It has wide applications in the fields of engineering, medical sciences and labour force analysis and the basic principles are well discussed in Kalbfleisch and Prentice (2002), Lancaster (1990) and Hougaard (2000). For a housing market application, let *T* be a continuous, non-negative valued random variable representing time until active in the housing market of a Decision Making Unit (DMU). If the probability of a DMU leaving the passive-state within a short interval  $\Delta t$  at or after *t* is  $P(t \le T < t + \Delta t | T \ge t)$  while the DMU is still passive in the market at *t*, then the hazard rate can be obtained simply dividing the probability by  $\Delta t$ that represents average probability of leaving the state per unit of time. Considering this average over very short intervals the hazard function  $\lambda(t)$ , which is the instantaneous rate of failure at t, is given by

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{P(t \le T < t + \Delta t \mid T \ge t)}{\Delta t}$$
(8)

This basic formulation of hazard function allows relating it with probability density function f(t), cumulative distribution function  $F(t) = P(T \le t)$  and survival function  $S(t) = P(T \ge t) = 1 - F(t)$ . Since the probability density function of *T* is

$$f(t) = \lim_{\Delta t \to 0} \frac{P(t \le T < t + \Delta t)}{\Delta t} = \frac{dF(t)}{dt} = \frac{-dS(t)}{dt}, \text{ the hazard function can be written as}$$
$$\lambda(t) = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{S(t)} = \frac{-d\log S(t)}{dt}$$
(9)

Since we are interested in investigating factors affecting termination of duration this nonparametric model is extended to incorporate explanatory variables (in the form of covariates) leading to semi-parametric and parametric models. Semi-parametric models assume hazard rate to be proportional and baseline hazard to be parametrically unspecified. The most popular proportional model exploits partial likelihood estimation techniques put forwarded by Cox (1972) and takes the following form:

$$\lambda(t, x) = \lambda_0(t) \exp(x(t))$$
(10)

where x(t) is the vector of observed covariates and  $\lambda_0(t)$  is the baseline hazard which is not parametrically specified. But parametric models assume a distribution for the baseline hazard and hence for the survival function. In many cases knowledge of the baseline hazard is unnecessary, for example, in comparing control and treatment groups for mortality due to a specific disease while applying a new drug. However, like many other instances tackled by event history researchers, in microsimulating urban systems, it is useful to have clear inference about  $\lambda_0(t)$  since such a modeling framework requires identification of timing of entry and exit in the housing market, labour force and school etc. Although it is possible to retrieve baseline hazards in semi-parametric models (for details see Box-Steffensmeier and Jones, 2004, Kalbfleisch and Prentice, 2002 and Collett, 1994 among others) this study prefers parametric models since they provide direct inference on the duration dependence. In addition, this paper makes an accelerated failure time (AFT) assumption (i.e. the covariates directly rescale time), which can be expressed as a log-linear model:  $\log(T) = \beta'_i x + \sigma \varepsilon \ (11)$ 

where  $\beta_j$  are the coefficients of the time independent covariates x and  $\varepsilon$  is a stochastic error term with type-I extreme value distribution scaled by  $\sigma$ . The hazard function of the AFT models is given by

$$\lambda(t, x) = \lambda_0(t \exp^{(-\beta x)}) \exp^{(-\beta x)} (12)$$

Here the effect of covariates is to alter the rate at which a person proceeds through time by either accelerating or decelerating the termination of duration.

There are wide varieties of distributions that can be employed in the parametric models including exponential, Weibull, log-logistic, log-normal, gamma, generalized F, Gompertz, Makeham etc. (for detail discussion on each distribution see Lancaster, 1990; Kalbfleisch and Prentice, 2002; Deshpande and Purohit, 2006). This paper tested Weibull, log-logistic and exponential distributions. While the exponential distribution has the no-ageing phenomenon due to lack-of-memory property, the Weibull distribution is a generalization of exponential distribution that provides constant, strictly increasing (or decreasing) hazard functions. Hence the hazard rate for Weibull distribution can be expresses as

$$\lambda(t) = \varphi p(\varphi t)^{p-1}$$
(13)

where  $\varphi$  is a positive scale parameter and p is known as the shape parameter. When p > 1, the hazard rate increases as t increases from 0 to  $\propto$ . In other words, there is a positive duration dependence (i.e. positive ageing, also called "snowballing effect") which means that the longer the elapsed duration the unit is more likely to exit soon. On the other hand, if 0 then the hazard decreases with time (i.e. negative ageing, also called "inertia effect"). In case of <math>p = 1 it becomes exponential with mean  $1/\varphi$  and if p = 2 it becomes Rayleigh distribution for which hazard rate is a straight line passing through the origin with slope  $2\varphi$ . Hence, Weibull models can accommodate both increasing failure rate (IFR) and decreasing failure rate (DFR) probability distributions depending upon the free shape parameter p.

The paper also considers log-logistic distribution that permits non-monotonic hazard form in contrast to monotonic Weibull models. The hazard function of the log-logistic distribution can be written as

$$\lambda(t) = \varphi p(\varphi t)^{p-1} / (1 + (\varphi t)^p) \quad (14)$$

In this case, the hazard decreases monotonically from  $\propto$  at the origin to zero provided that p < 1 and decreases monotonically from p provided that p = 1 as t approaches to  $\propto$ . On the other hand, if p > 1, the hazard gets a non-monotonic shape increasing from 0 to a maximum of  $t = (((p-1)^{1/p})/\varphi)$ , and decreasing thereafter as t approaches to  $\propto$ .

The parameters of the parametric hazard models are estimated using full information maximum likelihood estimation method (see Kalbfleisch and Prentice, 2002). Since our data is right Type-I censored, denoting  $\delta_i$  as the censoring indicator (taking the value 0 if case *i* is censored and the value 1 if case *i* experienced the event), the likelihood function is given by

$$L = \prod_{i}^{n} [f(t_{i}, x_{i})]^{\delta_{i}} [S(t_{i}, x_{i})]^{1-\delta_{i}} = \prod_{i}^{n} [\lambda(t_{i}, x_{i})]^{\delta_{i}} [S(t_{i}, x_{i})]$$
(15)

This univariate formulation is adequate for analyzing single-spell residential mobility (see an application in Vlist *et al.*, 2001). However, this study is using retrospective data that has repeated events recorded for each DMU. Failure to account for this repeatability might violate independence assumption on the occurrence of events taken in single-spell models. Two general approaches are applied in such cases: variance-corrected approach and random effects/frailty approaches (for extensive review see Box-Steffensmeir, 2004 and Hougaard, 2000). While variance-corrected approach estimate a model and then fix up the variance to account for the fact that the observations are not independent (rather repeated and therefore correlated), the shared frailty models assumes a stochastic variation across the parameters that is shared (common) among individuals. This paper investigates shared frailty models with different assumptions on frailty distributions. If decision making unit (DMU) i has multiple episodes j, the hazard rate for the j th episode of the i th DMU can be expressed as

 $\lambda(t_{ii}) = \lambda_0(t) \exp(\beta' x_{ii} + \psi' \omega_i) = \lambda_0(t) \exp(\beta' x_{ii}) v_i$ (16)

where  $v_i = \exp(\psi'\omega_i)$  represents group-specific heterogeneity (i.e. shared frailty) that is distributed across groups (in this case DMUs with repeated episodes) according to some distribution function  $G(v_i)$ . Consequently, the likelihood function is given by

$$L = \prod_{i=1}^{g} \int_{0}^{\infty} \left( \prod_{j=1}^{n_{i}} \left[ \lambda(t_{ij}, x_{ij}) \right]^{\delta_{ij}} \left[ S(t_{ij}, x_{ij}) \right] \right) dG(v_{i})$$
(17)

This likelihood function is maximized in order to obtain parameter estimates. The likelihood ratio test is used to assess the need for inclusion of the frailty component  $v_i$ .

For both single-episode and repeated events models goodness-of-fit statistics are obtained by estimating Rho-square, which is one minus the ratio of log-likelihood of the full model and null model (i.e. constant only model).

#### 4. Data preparation

As discussed earlier this paper uses the RMS II retrospective survey data for a random sample of 270 GTA households. Detail description of the data can be found in Habib and Miller (2007). For the discrete choice models, 28 years of longitudinal data (for a period of 1971-1998) has been extracted from the database that contains yearly observations of whether decision making units are active in the market or not. It also contains information on employment history, household composition changes as well as socioeconomic characteristics of the households. Key residential stressors are identified for each observation year by comparing states between the consecutive years (for example, job increase/decrease, DMU size increase/decrease etc.). Indicators of lags and leads of these stressor events are also identified to test in the models. Additional explanatory variables are generated from census tabulations of the corresponding years. In total, 4097 observations are used for modeling in which DMUs were active in the housing market at 408 choice occasions in different years between 1971 and 1998. Note that the event "active" in the housing market includes actual moves as well as instances in which the DMU became 'active' in the market but did not end up moving. This 'active but did not move' information is unique in the literature and provides an unbiased database for mobility model development. That is, most data sets only include successful moves and so underestimate mobility participation rates.

On the other hand, for continuous time hazard-based duration models the same dataset is selected where 270 households have 623 episodes (that represents passive-state duration of DMU) including censoring spells. The data is right-censored in the year of 1998. Note that these duration spells are the continuous time-period within which DMUs are in passive-state. The state terminates by the occurrence of the event decision-to-move that triggers actively searching the housing market for potential alternative dwellings. The average observed duration is 2.3 years with a minimum one year and maximum seven years. Covariates used for hazard models are also taken from the same sources, but only those that are constant or assumed to be constant for the entire duration of an episode. Time varying covariates are not tested in this paper. As such, year of birth of the DMU head, number of bedrooms and other structural attributes of the dwellings, and neighbourhood attributes are employed in the models. It also tests whether the first spell since household formation as well as immigration affects termination of passive-state duration or not.

#### 5. Discussion of results

#### 5.1 Panel logit models

Table 1 reports results of binomial panel logit models described in Section 3.1. Fixed effects (FE) model estimated through brute force maximization gives similar results as of the conditional FE model put forwarded by Chamberlain (1980) where fixed effects parameters are not estimated. In fact, conditional maximum likelihood estimation is carried out due to presence of incidental problem (i.e. if there is same choice throughout the observation period). The results shows that incidental problem is not acute for the sample used for this study and fixed effects models can be estimated using direct maximization (proposed by Greene, 2001) where all the group-specific effects are estimated along with coefficients of explanatory variables at the same time. However, Random Intercept (RI) model shows greater value of good-ness-of fit statistics (i.e. Rho-

square) than that of FE models. In the RI model we have a time invariant constant term, which captures unobserved heterogeneity across decision making units (DMU). But the highest goodness-of-fit statistics is achieved in the Random Parameter (RP) Model that captures heterogeneity across the parameters. Hence this paper selected the RP model as the final model. Description of parameter estimates of the final RP model is given below.

It is found that most of the hypothesized residential stressors (such as increase/decrease in number of jobs, birth of a child, job change, decrease in DMU size, retirements etc.) are found to be significant in explaining residential mobility. The fifth column of Table 1 reports mean and standard deviation of the parameters estimated for the RP model. It can be seen that mostly dynamic variables are found to be random parameters whereas all of the static variables are nonrandom.

Age of the household head (often used as a proxy variable to mark stages of life cycle, see Clark *et al.*, 1986, Mulder, 1993, Vlist *et al.*, 2001 among others) have significant effect on the decision to move. It shows that younger-head DMUs are more frequent movers than older heads. However, there is some variability of the effect as seen in the significant standard deviations. Another significant life-cycle event, birth of children also induces mobility. Similarly, job change increases the probability of moving where the mean of the parameter is 0.296 with a standard deviation of 0.76. This means that although change of job location on average significantly encourages a relocation decision in order to relieve commuting stress, this effect varies considerably across households, with certain DMUs preferring other stress-release mechanisms (possibly, such as buying a new car) and do not become active in the housing market in response to this stressor.

While decrease in number of jobs in the DMU increases the probability of moving, a dummy variable reflecting retirements doubles this effect. Interestingly, these stressors are found to be non-random across the sample households. However, increase in jobs shows a very interesting behaviour having negative sign in the parameter value. Our prior hypothesis was that such an increase would increase the probability of becoming active in

the housing market. But the model result suggests an opposite effect. This could partly be explained by the results of the static variable number of jobs in the household, which is found to be nonrandom with a coefficient value of -0.086. That means if there are more workers in the household, the probability of becoming active is lower, all else being equal. This presumably reflects inertia effects associated with having more job locations within which they have already been settled in terms of mode choice, commuting patterns and other short-term activity agendas. Therefore, an increase in jobs in a DMU actually brings a similar stationary effect that prevents considering a residential relocation. However, the parameter of this variable also exhibits a very high variability among the decision makers having a standard deviation of 1.254 compared to the mean of -0.198. This suggests that in some cases an increase in jobs actually does increase the probability of moving.

Duration at the current home is also found to be one of the significant determinants of mobility decision. The higher the duration in the current location, the lower the probability of moving. It proves the hypothesis of inertia that impedes relocation due to strong community linkages created by longer durations of living in a neighborhood. It is very much consistent with many other previous findings (such as McHugh *et al.*, 1990). This study also tried to capture duration effects at different time scales using dummy variables (such as the first three years in the GTA, three to five years, more than five years, etc. as well as other alternative combinations), none of which provides expected impacts on mobility. Hence, in the final model, only total duration is retained, which is also found to be a random parameter.

In many cross-sectional studies (as well as in some longitudinal mobility research), tenure was considered to be an important factor in explaining residential mobility. Often it was found that renters are more mobile than owners. Again, it was found that highly educated persons tend to have higher mobility rates compared to less educated workers. Similarly, household size, dwelling type, number of rooms, number of bedrooms, number of people per room etc. were found to be contributing factors for mobility (see Vlist *et* 

*al.*, 2001 for a recent review). But this research indicates that most of these static attributes of the dwelling as well as decision makers are not significant when dynamic variables are taken into account in the mobility model.

It is also found that neither job-residence distance nor distance to CBD is significant in explaining mobility decisions. Rejection of distance to CBD could be explainable due to the GTA being a multi-centric metropolitan area (although the Toronto CBD is still a very important employment, shopping and cultural centre within the region). However, the study was expecting to see effects of job-residence distance or changes in average job-residence distance within the mobility decision, but these hypotheses were not confirmed. One possible reason for this unexpected result might be that the stressor "job change" already captures some of the effects of change in commuting distances.

Although this research examined three years of lag and lead effects for each of the life cycle stressors, the only significant lag/lead effect found is a two-year lagged response to a decrease in DMU size. That is, if the DMU size decreases it take two years to have an impact on the mobility decision. In other words, the probability of moving increases two years after a DMU size decrease. This is a plausible response, since it may well take a household some time to decide to adjust its dwelling size and/or location in response to a change in household size.

Regarding neighborhood dynamics, the model indicates that if a DMU lives in a stable community, represented by the fraction of non-movers (in the last five years) in the neighborhood, it is less likely to consider moving. On the other hand, the neighborhood labor force participation rate has a positive impact on household mobility. Both of these variables are found to be non-random. A large set of other neighbourhood attributes has also been examined (for example, average dwelling value, dwelling density, renter/owner ratio, percentage of immigrants etc.). None of those variables are found to be statistically significant.

Since housing supply data for a long period was not available for use in this study, it uses some housing market indicators such as change in mortgage rate, bank interest rate, etc. to include market dynamics in the model. Although this study finds both mortgage rate and bank interest rate to be significant, due to obvious correlation issues, only one of these two variables can be included in the final model. Since mortgage rates can vary for individual cases and RMS II does not provide any information of mortgage premiums, and equity or savings, the bank interest rate has retained in the final model. Note that this interest rate only differs across years, not for individual decision makers. Finally, it is found that changes in interest rates increase, decision making units are less likely to be active in the market and vice versa. This key market indicator is found to be a random parameter with a statistically significant standard deviation.

Most of the parameters included in the final model, including means and standard deviations, are statistically significant at the 95% confidence level or better. One or two estimates fall short of the corresponding t statistics value (1.64). However, they are retained in the model due to their importance as policy variables (such as change in interest rate) in an expectation that with larger sample the variables would be statistically significant.

#### 5.2 Hazard-based duration models

The first set of hazard-based duration models is estimated assuming single-spell for each termination. That means, each episode at the passive-state in the market for a given DMU is assumed to be independent and considered as separate observation in the analysis. But given the fact that we have collected retrospective data that contains information on the housing career of each household, it is more appropriate to consider repeated events models. As discussed earlier although there are different ways of dealing with this phenomena we estimated shared frailty models where the frailty component accounts for unobserved heterogeneity that is common for the households that have multiple sequential episodes. The study also tested frailty models for the single-spell assumption

in which frailty only considers individual-level heterogeneity (not group-level as in shared frailty). Since with the retrospective data we are more interested on the group-level unmeasured risk factors this paper only reports repeated events shared frailty models. Three different distribution assumptions are used for frailty in these models including gamma and inverse Gaussian (see Clayton, 1978 and Hougaard, 2000) and Gaussian distribution (see Greene, 2002). The goodness-of-fit measures for both basic single-spell models with different assumptions on baseline hazard and repeated events shared frailty models are tabulated in Table 2. As indicated by the goodness-of-fit measures, the log-logistic model with Gaussian frailty describes the termination probability best. While Table 3 presents coefficient estimates of all basic single-episode models, Table 4 shows some competitive shared frailty models. It can also be seen that most of the parameter estimates of the log-logistic Gaussian frailty model are statistically significant at the 95% confidence level in contrast to other candidate models. Since the log-logistic Gaussian frailty model is selected as the final model its results are detailed below.

It is found that age of the DMU head (represented by the year of birth) has significant impact on the passive-state duration. Older DMU head stays passive in the housing market longer than younger heads. Also, a DMU who has higher number of rooms in the dwelling unit it lives have longer duration. Similarly, DMUs living in the row houses have longer duration; possibly under income constraints they are less likely to become frequently active in the market. Tenure also affects duration; renters are more frequently active in the market than homeowners. Both higher dwelling density and single-detached housing density in the neighborhood increases duration of stay. However, impact of single-detached housing density is higher than that of all dwellings. This reflects that DMUs that secured housing in a single-detached dominated neighborhood are less likely to change their dwellings frequently.

While increased share of non-movers in the neighborhood increases duration, share of immigrants in the neighborhood does the opposite. This means that DMUs living in a

stable neighborhood are less likely to consider moving, but those who live in an immigrant-dominated area are more prone to move. On the other hand, labour force participation rate in a neighborhood increases duration of being passive in the housing market.

Distances to Toronto CBD also have impact on duration; people living close to the CBD are more frequently active in the market than people in suburban areas. It is intuitive that DMUs living in the suburbia have already obtained a stable location by purchasing dwellings. Both dummy variables indicating new household formation and new immigrants in GTA show positive impact on duration. That is, in the first spell since formation of household and immigration, DMUs tend to stay longer than other spells in general. However, DMUs forming new households have longer duration than that of immigrants.

Finally, the value of ancillary parameter (p) greater than one suggests that the hazard initially increases to a certain point and decreases thereafter. The model also shows considerable variance of the distribution of the random effects ( $\theta$ ), which is statistically significant when tested against a Chi-square distribution. That is, the null hypothesis ( $\theta = 0$ ) is rejected for the log-logistic shared frailty model.

#### 5.3 Comparison between panel logit and hazard models

It is not possible to directly compare these two modeling techniques examined to explain residential mobility. The focus of the discrete choice modeling was to see how households decide at each choice occasion to be active in the market. On the other hand, hazard-based duration models directly model termination of passive-state duration resulting the event of being active in the market. Although outcomes of the process modeled through these two techniques are the same, due to methodological differences and variable selection makes it hard to directly compare the results of both. In the panel data logit models, most of the variables selected mark dynamic changes at each unit time period (i.e. year). On the other hand, the variables that are constant (or assumed to be

constant) over the whole duration period are used for the continuous-time hazard models. Note that time-varying covariates are not tested in this paper. As such, mostly dwelling attributes and neighbourhood characteristics are found in the hazard models. However, common variables in both types of models show identical results in explaining mobility. For instance, younger-heads are more frequently active in the market than older-head households and higher labour force participation rates in the neighbourhood contribute larger duration meaning less prone to active in the market. On the other hand, the variables which are rejected in the panel logit models (such as tenure, dwelling density) are found significant in the hazard models. Perhaps, in the presence of dynamic variables these variables do not add explanatory power in the logit models. One interesting finding has been achieved through continuous-time hazard models is that the first spell of forming new households and also first spell after immigration lengthen duration of stay before considering a move to the next. This type of spell-specific characteristics presumably could only be incorporated in the continuous time hazard models.

#### 6. Conclusion

The paper portrays a comprehensive conceptual framework to model residential mobility and spatial search, which will provide an excellent guideline to carry forward modeling residential location choice processes. It also presents empirical results of residential mobility models that applied two different techniques: discrete choice and hazard-based duration modeling. In general, both modeling techniques perform well and provide useful results in understanding residential mobility behaviour. Tests and comparison of different models within each technique also provide important guidelines to cope with methodological challenges while working with retrospective data. While Random Parameter (RP) model shows better performance in incorporating latent heterogeneity for discrete-time data, shared frailty models are found quite satisfying for continuous-time settings. In general, both final models show reasonable explanatory powers. Also, their parameter estimates are mostly statistically significant. In the random parameter model, it is found that residential stressors mostly related to job and household composition dynamics are significant in explaining DMUs' desire to change location at each point of time. Notable stressors are increase and decrease in the number of jobs, change of job location, retirement, birth of children and a two-year lagged effect for decrease in DMU size. On the other hand, mostly dwelling and neighbourhood characteristics are found significant in the continuous-time shared frailty models in the absence of stressors that vary with time. Hence, the next step of the research should be testing these models by incorporating time-varying covariates and to see how the model responds.

The models presented in this paper significantly contribute to the residential mobility research, particularly in exploring methodologies to deal with panel effects when considering longitudinal data. In addition, it is expected that while the random parameter binomial logit model will support time-driven microsimulation of the mobility component in the Integrated Land Use, Transportation and Environment (ILUTE) project, the frailty model could be applied in case of event-driven microsimulation.

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	Fixed Effects	Fixed Effects	Random	Random
	Model	Model (Cond.)	Intercept	Parameter
Variables	(Uncond.)	· · · · ·	Model	Model
	Coefficient	Coefficient	Coefficient	Coefficient
	( <i>t</i> -statistics)	( <i>t</i> -statistics)	( <i>t</i> -statistics)	( <i>t</i> -statistics)
Age of head of decision	-0.18797847	-0.17533311	-0.02612479	-0.02918213*
making unit (DMU)	(-9.619)	(-9.398)	(-12.250)	(-12.637)
Birth of a child in the	0.37881621	0.34426617	0.31014448	0.32569854*
DMU	(1.574)	(1.510)	(3.546)	(3.549)
Lag of two years for	0.08649466	0.07705735	0.12778979	0.13347559
decrease in DMU size	(0.422)	(0.398)	(1.711)	(1.779)
Total duration in the	0.18803664	0.17236233	-0.05156842	-0.05392695*
dwelling	(7.325)	(7.058)	(-13.015)	(-11.367)
Change in job (any	0.51693548	0.46431397	0.43331847	0.29577452*
member in the DMU)	(3.090)	(2.959)	(8.691)	(4.290)
Increase in number of jobs	0.08196942	0.09177949	0.00124506	-0.19754624*
in the DMU	(0.311)	(0.369)	(0.016)	(-1.690)
Decrease in number of	0.69989859	0.61070743	0.42730730	0.47358793
jobs in the DMU	(2.466)	(2.291)	(4.141)	(4.506)
Dummy variable	0.57547997	0.53416598	0.46923746	0.44798726
representing retirement	(0.730)	(0.763)	(1.560)	(1.471)
Number of jobs in the	-0.10090589	-0.12056292	-0.07248508	-0.08618640
DMU	(-0.645)	(-0.799)	(-2.451)	(-2.875)
Ration of non-movers in	-0.45879471	-0.40384605	-0.08597773	-0.10976852
the neighbourhood	(-2.464)	(-2.336)	(-1.768)	(-2.189)
Labour force participation	-0.00013483	-0.00135888	0.00472632	0.00410920
rate in the neighbourhood	(-0.020)	(-0.210)	(2.288)	(1.935)
Change in bank interest	-0.00475829	-0.00031640	-0.01258810	-0.01312268*
rate	(-0.175)	(-0.012)	(-1.227)	(-1.233)
Constant			-0.27726666	-0.08359342
Constant			(-1.745)	(-0.503)
Scale parameter for random	m parameter			
Constant			0.00441068	
Constant			(0.114)	
Age of head of decision				0.00209713
making unit (DMU)				(2.119)
Birth of a child in the				0.21942446
DMU				(1.509)
Total duration in the				0.04460868
dwelling				(8.839)
Change in job (any				0.76225470
member in the DMU)				(8.609)
Increase in number of jobs				1.25401401
in the DMU				(7.942)
Change in bank interest				0.03465269
rate				(1.976)
Rho-square	0.047507	0.060672	0.05368	0.06396

Table 1: Results of binomial panel logit models

\*Means for random parameters

Single-episode models	Log-likelihood	
Log-logistic		-727.80549
Weibull		-748.56343
Exponential		-749.9647
Repeated event models		
Log-logistic Gamma shared frailty		-726.83519
Log-logistic inverse-Gaussian shared frailty		-726.8694
Log-logistic Gaussian shared frailty		-726.1688
Weibull Gamma shared frailty		-743.09058
Weibull inverse-Gaussian shared frailty		-744.05097
Weibull Gaussian shared frailty		-743.3892

# Table 2: Goodness-of-fit measures of both single-episode and repeated events models with various distributional assumptions

~	Log-logistic	Weibull	Exponential
Covariates	Coefficient	Coefficient	Coefficient
	( <i>t</i> -statistics)	( <i>t</i> -statistics)	( <i>t</i> -statistics)
Year of birth of the head of the	-0.04024524	-0.04593684	-0.04589226
DMU	(-10.913)	(-12.376)	(-11.678)
Number of rooms in the	0.01847907	-0.00076771	0.00031560
dwelling unit	(0.836)	(-0.035)	(0.013)
Dummy indicating row-house	0.09777811	0.09710156	0.10607144
(1 if row-house, 0 otherwise)	(0.712)	(0.742)	(0.729)
Dummy indicating renters (1 if	-0.03960180	0.01796712	0.01556447
renter, 0 otherwise)	(-0.391)	(0.192)	(0.150)
Dwelling density in the	0.03531795	0.04377769	0.04729649
neighbourhood	(1.611)	(1.782)	(1.773)
Single-detached dwelling	0.42565604	0.54647970	0.56450146
density in the neighbourhood	(3.001)	(3.789)	(3.551)
Ration of non-movers in the	0.38250610	0.42709794	0.43989081
neighbourhood	(2.901)	(3.249)	(3.047)
Ration of immigrants in the	-0.05329624	0.06092065	0.08448626
neighbourhood	(-0.255)	(0.301)	(0.376)
Labour force participation rate	0.00631783	0.00673219	0.00689517
in the neighbourhood	(1.091)	(1.274)	(1.174)
Euclidian Distance to Toronto	0.00468092	0.00394264	0.00431935
Central Business District (CBD)	(1.908)	(1.730)	(1.710)
Dummy indicating first spell	0.08583378	0.06701696	0.07180846
after formation of HH	(0.655)	(0.569)	(0.547)
Dummy indicating first spell	0.02770642	0.03098085	0.03452785
after immigration	(0.228)	(0.283)	(0.283)
Constant	2.36871246	3.07112720	3.01313741
	(4.817)	(6.964)	(6.148)
Ancillary parameter			
P	1.503843	1.072135	1.00 (fixed)
Log-likelihood	-727.8055	-748.5634	-749.9647

## Table 3: Parametric estimates of single-episode models

<b>A</b>	Log-logistic Gaussian	Weibull Gaussian	Log-logistic Gamma	Weibull Gamma
Coverietes	Frailty Model	Frailty	Shared	Shared
Covariates		Model	Frailty	Frailty
			Model	Model
	Coefficient	Coefficient	Coefficient	Coefficient
	( <i>t</i> -statistics)	( <i>t</i> -statistics)	( <i>t</i> -statistics)	( <i>t</i> -statistics)
Year of birth of the head of	-0.04048032	-0.0449556	-0.0386/33	-0.0446419
the DMU	(-19.273)	(-21.998)	(-8.63)	(-10.19)
Number of rooms in the	0.01747112	0.01005810	0.0225636	0.0121356
dwelling unit	(1.355)	(0.808)	(1.00)	(0.51)
Dummy indicating row-	0.13145958	0.12291582	0.1041151	0.1092205
house (1 if row-house)	(1.635)	(1.652)	(0.75)	(0.78)
Dummy indicating renters	-0.02673617	-0.00277430	-0.0426104	-0.0098313
(1 if renter)	(-0.480)	(-0.054)	(-0.42)	(-0.10)
Dwelling density in the	0.03273352	0.04209297	0.0329598	0.039585
neighbourhood	(2.502)	(2.981)	(1.23)	(1.28)
Single-detached dwelling	0.42109540	0.50733926	0.413634	0.4807128
density in neighbourhood	(4.891)	(6.012)	(2.49)	(2.79)
Ration of non-movers in	0.40935892	0.47885640	0.390927	0.4729032
the neighbourhood	(5.628)	(7.001)	(3.33)	(3.79)
Ration of immigrants in	-0.06555341	-0.00551386	-0.0702679	-0.0321404
the neighbourhood	(-0.592)	(-0.052)	(-0.36)	(-0.15)
Labour force participation	0.00707778	0.00792558	0.0067158	0.0076733
rate in the neighbourhood	(2.173)	(2.715)	(1.27)	(1.43)
Euclidian Distance to	0.00476633	0.00450069	0.0049392	0.0049255
Toronto CBD	(3.225)	(3.236)	(1.85)	(1.8)
Dummy indicating first	0.08646772	0.06384892	0.0959405	0.0744047
spell after formation of HH	(1.203)	(0.981)	(0.75)	(0.6)
Dummy indicating first	0.04147438	0.05021355	0.0326187	0.0558481
spell after immigration	(0.604)	(0.826)	(0.27)	(0.46)
Constant	2.33733191	2.86187217	2.212001	2.726221
	(8.233)	(11.433)	(4.33)	(5.49)
Ancillary parameters				
p	1.58519	1.194026	1.57096924	1.219213
heta	0.34463268	0.46280313	0.133638	0.3590112
Log-likelihood	-726.1688	-743.3892	-726.83519	-743.09058

 Table 4: Parametric results of selected repeated events models with various assumptions on shared frailty

