# THE GREENLAND AIR TRANSPORT MODEL SYSTEM

## - A COMBINED TRANSPORT MODELLING AND OPTIMISATION PROBLEM

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**SHORT ABSTRACT**

The paper describes a combined demand and optimisation model for air transport in Greenland. Demand and route choice for passenger and freight is addressed at the lower level problem, whereas the service network, flight schedules (time-table optimization), decision on airplane types (considering cost and capacity), and finally creation of airplane schedules are dealt with in the upper level problem. Due to special capacity restrictions combined with the need for a schedule based assignment model, this bi-level problem cannot be solved analytically. The lower level model is a non-analytical non-linear non-continuous mapping of the solution of the upper level problem.

The model is now used by the Greenland Home Rule to decide upon a new airport structure in Greenland (change of main airports and creation of new airports), policies to obtain more competition and to liberalise the market, and to design the main air transport system. In the paper, we present the model as well as a study in which the location of a new Atlantic Airport is analysed. The cost-benefit analysis indicates that the present airport structure is highly in-optimal and that significant benefits will result if the location of the new airport is moved to Nuuk, the Capital of Greenland.

## 1. INTRODUCTION

Being the worlds largest island, but with only 56,000 inhabitants in a mountainous arctic area with no possibilities for interurban road transport, air transport plays a central role for the society. However, the present transport network needs large public subsidies yet having very high user costs and often inconvenient transfers for the users.

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## 1.1 Background

The main airport of Greenland is not located at the Capital (Nuuk), but at the location "Sdr.Strømfjord" at the bottom of the fjord "Sønderstrømfjord" 150 km. from the cost-line and only 10 km. from the ice cap. This location was originally chosen due to historical reasons, since the US built the runway and airfield during the Second World War. At that time, the main consideration was to "minimize" accessibility for potential German submarines. Clearly, today this objective has changed and with increasing air transport in Greenland, the efficiency of the system has been questioned. Three major issues have been discussed;

- There is a feeder-traffic structure between Nuuk and Sdr.Strømfjord. The link has to be serviced by DASH 7 turbo-prop airplanes (50 seat max.), which is the largest airplane that can land and take-off at the 899 meter runway in Nuuk. To compare, the 4½ hour flights from Sdr.Strømfjord to Copenhagen are presently served by a 245 seats Airbuses-200 with additional cargo facilities.
- There have to be an "Artificial community" in Sdr.Strømfjord to support the operation of the airport and there is large operation costs associated with this. A second problem is that, as a result of global warming, the permafrost on which the Sdr.Strømfjord runway is based has started to thaw, making it costly to maintain.
- For domestic flights, a Dash7 air network has Nuuk as hub. Therefore, the Island operates with two hubs, even though the population is only 56,854 inhabitants.

Even though it might seem very obvious to build an "Atlantic" airport in Nuuk, the decision is not trivial due to the construction costs related to this. Realising a need for better decision tools, the Greenland Home Rule asked the Technical University of Denmark (DTU) to develop an integrated system of a transport model, an air network optimization model, and a socio-economic decision support model.

## 1.2 Overview of the model system

While the whole model system includes demand models, and socio-economic impact models, the present paper focus on the interaction between the assignment models and the optimisation models.

The route choice model assigns passengers onto the (air) network. The model operates with exact timetables, and takes into account capacity constraints (seat as well as weight restrictions). The model is formulated as a stochastic (mixed probit) multi-class user equilibrium model. A special consideration is that post (letters) have the highest priority, and that postal sacks literally can replace passengers. Passengers then have higher priority than freight.

The choice of the passengers depends on the network, i.e. the leg-structure, the departure times (and frequencies) and the air crafts being used (fast jet-plains may make a difference compared to less comfortable slow propelled airplanes).

The optimization model design the overall air transportation network, leg structure (departure time), chooses plain types, and calculates the costs of operations including the necessary number of aircrafts.

## 1.3 Decision context

The model context may be considered to represent at “three player game” consisting of – and finding an equilibrium between;

1. Home rule decisions (which are feed into the system manually as exogenous assumptions)
    - Airport structure (location, length of runways)
    - Subsidies and/or packages of routes
    - Minimum service frequencies defined by the Home rule
2. Air company decisions
    - Legs (and general structure of network)
    - Schedule
    - Airplane allocation
    - Non-Greenland flights
    - Fare
3. Passenger decisions
    - Number of trips, destination, mode choice (domestic to some limited extend), route choice, time of travel (partly part of route choice), fare level, company.

## 1.4 Object functions in the optimisation model

The object function of the air model includes the following elements;

$$\beta_1 \cdot passenger(dis)utility + \beta_2 \cdot costs\_of\_operation + \beta_3 \cdot fare\_revenue$$

The function is maximised, whereby the model creates the air network.

The passenger dis(utility) is a function of frequency (dis-utility of few departures), waiting time, travel time, transfer time, number of transfers (the extra disutility of transfering), and fare.

The costs of operations of take-off (including handling at the airport), a distance dependent cost and a turn-arround cost (the costs of not using the airplain between arrival and departure, i.e. lost return of investment). These costs depend on the type of aircraft. The total costs usually increases with the size of the aircraft, however, the unit cost per PAX (passenger) is usually lower for larger aircrafts. In Greenland, helicopters have to be used to locations without runways, however, helicopters have a much higher unit cost than airplanes.

The two overall optimization criteria for the service network design are partly conflicting;

- To minimize the operations costs and maximize revenue (company objectives)
- To maximize the utility for passengers and society

The object function can be configured as pure company profit maximization, i.e.

*Max[care revenue -  costs of operations]*

Normally it is reasonable to assume that airline companies tries to maximize this objective and that the route network would reflect such function in a completely free market[2].

The function can also be configured as a pure socio economic optimization, i.e.

*Min[ passenger(dis)utility + cost of operation]*

In other words, the passengers should obtain minimum generalised travel cost taking the operation costs into account.

This socio-economic optimization results in a different route network than the operational optimization. In case of substantial differences between the two solutions it could favour regulations. I.e. a change in the operational conditions through subsidizes or taxes and duties

A combined function even in a socio-economic valuation could also be a solution, i.e.

$$Min\ [\ \beta_1 \cdot passenger(dis)utility + \beta_2 \cdot operation\ cost + \beta_3 \cdot ticket\ revenue\ ]$$

It can be argued, that since a share of the ticket revenues is submitted to the Home rules, this objective might be reasonable. This could as well be the case, especially if the revenue comes from foreigners. Also, it might be less distorting than the tax on income (less tax distortion). $\beta_3$ is between 0 and -1. Isolated from Greenland's point of view it could be argued that the operation costs has a partly increasing affect on employment and that $\beta_2$ should be set below 1.

Hence, in political analysis it is possible to implement sensitivity calculations and evaluate the optimized structure of the route network given different assumptions and political priorities. Within the model tests so far there are however used a purely operational configuration as it is presumed that Air Greenland primary designs the route network from profitable considerations.

## 1.5 Passenger behaviour prediction

Each passenger will typically maximize his/her own utility. However, in addition, it can be assumed that passengers have incomplete knowledge of alternatives and as well different preferences. As a result, the model operates with two different objective functions in the traffic model and the optimisation model respectively;

- *The objective function for the traffic model describes the preferences of the passengers (e.g. the value-of-time) and includes a random term and stochastic coefficients.*
- *The objective function of the optimisation model is weighted function of system optimal criteria's and operational criteria's, which both is deterministic.*

The different models operates by trip purpose (business, tourists, and natives), each one

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[2] $\beta_3$ may, however, be less than 1, if the ticket price includes taxes.

with a separate utility function

## 2. MAIN STRUCTURE OF THE MODEL

The model framework consists of two separate modules;

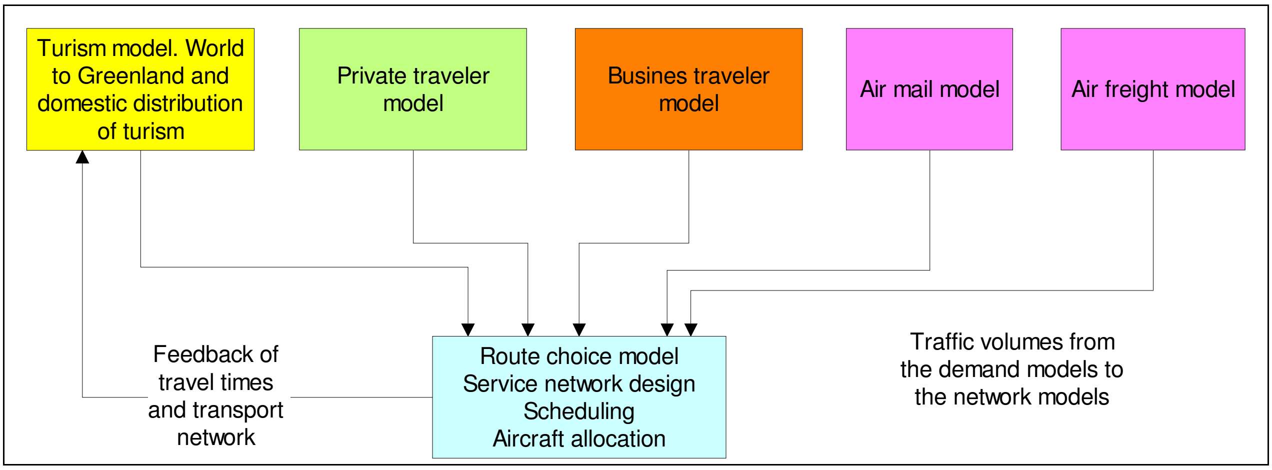
1. Demand models, which describe number of trips and destination choice. The models have been stratified according to trip purpose (business, tourists, natives, post, and freight). In addition, most of the demand models have been formulated with separate modules for national and international trips.

2. Supply models, which define the leg-structure (plane connections), stock of planes (type and number of planes), and the passengers choice of route as a function of traffic demand. The route choice models distinguish between modes, but evaluate the assignment simultaneously due to capacity constraints, which is affected by all purposes.

OD-matrices are feed from demand models to supply models, describing the number of travelers between the different airports. The supply model calculates travel time and costs at the OD level, which is used to re-calculate demand. As a result, the two models will iterate until equilibrium is reached.

Because there are great seasonal variation in demand and supply in Greenland, the model is split into 4 models, one for each quarter. The figure below illustrates the overall model structure (note the feedback from all 5 demand models).

Figure 1: Overall model structure.




### 2.1 Main flow

The main flow in the model is;

1. Run the demand models. These models calculate the transport demand on a weekly basis, i.e. number of trips to and from each airport and the distribution of trips between the airports. Thereby this model step produces 5 OD trip matrices (Origin-Destination matrices)

2. The trip matrices are at first divided into the 7 weekdays which is then subdivided into 7 time intervals. The split-functions vary between the 5 trip purposes.

3. The next step is the run of a scenario generator that generates the possible airline connections between all the airports (incl. the heliports) given the available plane types, the length of the runways, and the distance between the airports.

4. The plane optimization model optimizes the leg structure (airline connections between the airports), number of departures, and the used plane types with iterations with the assignment model as an iterative process with an assignment model

5. The supply data – travel times, costs etc. called LoS (Level of Service) is aggregated to weekly basis for each trip purpose as a weighted average of days and time periods.

6. The demand models (tourism, business, visiting, mail and cargo) are repeated with the new supply data

7. The matrices are subdivided into weekdays and time periods as in step 2.

8. The calculations with the plane optimization model is repeated

9. A plane balancing model is started, which calculates and balances the use of aircraft material. During the process each leg is upgraded and new legs could be added.

10. The final assignment model

It could be argued that the demand models and the supply models should iterate several times but there is an upgrade function built in the assignment model which upgrades the material after the demand calculation. This issue is therefore less critical.

## 3. ROUTE CHOICE MODEL

The passenger route choice model finds the user-equilibrium, e.g. the situation where 'no passengers perceived utility can be improved by he/her unitarily changing route at the desired time of departure'.

Route choice is dependent on trip purpose, preferences of passengers, and the use of time. The utility is strongly flow dependent, since most airports in Greenland can only be served by small air plains, the main type being the DASH7 with maximum 50 seats. Only three international civil airports exist[3];

- Sdr.Strømfjord, which is served by a 245 seat Airbus-200
- Nasarsuaq, which is served by a 180 seat Boing-757
- Kulusuk, which is served by a another turbo-prop airplane (Fokker 50 – which is quite similar to DASH7)

[3] The Military airport located at Thule is not open for civil air traffic.

The strict capacity restrictions combined with the low frequency led to the decision to implement a Stochastic User Equilibrium timetable-based assignment model (Nielsen & Frederiksen, 2006) that is run on a weekly schedule (since some leg are only served once or twice a week).

This model was then modified to reflect a number of special issues in Greenland;

- Passenger capacity is a strongly limiting factor; passengers are simply rejected, not only delayed as e.g. in classical route choice models. This causes some difficulties in the model formulation and solution algorithm.
- Air mail has higher priority than passengers, i.e. a need for a sequential approach with regard to this compared to traditional multi-class equilibrium methods.
- Payload restrictions may restrict the number of passengers and the amount of cargo.
- Airplane (schedules) has to be part of the model.
- Trips may even have to wait to the next week or they may origin from the week before. First a cyclic graph approach was explored for this, whilst a before and after demand and network period was finally decided.

### 3.1 Utility functions

The model has the following explanatory variables that are optimized in a linear-in-parameter utility function for each class of passengers (and by simulating statistical distributions of each passenger as well);

Suppose that there are k passenger classes and $i \in I$ route alternatives, then the utility functions is given as;

$$U_{k,i} = \beta_{k,ed} \bullet ED_{k,i} + \beta_{k,ld} \bullet LD_{k,i} + \beta_{k,tp} \bullet TP_{k,i} + \beta_{k,tt} \bullet TT_{k,i} + \beta_{k,op} \bullet OP_{k,i} + \beta_{k,it} \bullet IT_{k,i} + \beta_c \bullet C_i + \varepsilon_{k,i}$$

Where;

$ED_{k,i}$: Early departure penalty (sort of “hidden waiting time)

$LD_{k,i}$: Late departure penalty (traditional “hidden waiting time)

$IT_{k,i}$: Travel time (traditional In vehicle time spend in the airplane)

$TT_{k,i}$: Transfer time.

- Strongly non-linear, as long transfer times can be used constructively, e.g. on musk ox trips, or visiting the inland ice at Sdr.Strømfjord using long transfer times for e.g. tourism

$TP_{k,i}$: Transfer penalty (disutility by non-direct travels)

$OP_{k,i}$: Overnight penalty (disutility and cost when need for overnight transfers/stays)

$C_i$: Cost (Ticket costs)

$\varepsilon_{k,i}$: Independent identical Gumbel error.

In the model, the behavioural parameters $\beta_{k,ed}$ , $\beta_{k,ld}$, $\beta_{k,tp}$, $\beta_{k,tt}$, $\beta_{k,op}$, $\beta_{k,it}$, $\beta_c$ all followed log-normal distributions (over the population), with the relative size between passenger classes and time components based on experience from Copenhagen.

## 3.2 Chosen parameters

Since it is difficult and computational hard to implement non-linear utility functions the assumed non linearity in the total travel times was approximated by using a high transfer penalty (as proxy for non-linearity) and a low value of transfer time. Different values were tested where 40 % of the free travel time gave the best result.

The scaling of the different time values for each passenger trip purposes is in general similar. However the hidden waiting time for visiting trips is assumed to have a smaller impact on the utility than business trips and an even smaller impact for tourism trip purposes.

For airmail and cargo it is assumed that travel time from O to D and the price of the transportation is the only parameters influencing the utility. Which time components the trip can be subdivided into is considered without influence for customers. The access time is however given a higher time value as this is regarded as a discomfort for customers and the freight companies.

The ticket prices are in general priced in DKr. Because the rest of the variables are scaled into DKr using time values the coefficients for ticket prices is normally set to 1. For theoretical reasons this coefficient is fixed.

In the model it is possible to define a maximum wait time and a maximum early departure time to implement how long time the passengers are willing to wait and leave before the planed departure.

Table 1 below shows the final parameters in the route choice model.

Concerning the random variation (the error term) it is assumed to be additive non negative distributed (gamma distributed) and that the passengers are familiar with the route network why the variance is rated low (5 %).

The last definitions describe the average weight per passenger inclusive luggage and weight units for mail and cargo. For some plane types the passenger seats can be exchanged with mail sacks.

Finally there is a specified ranging of the travellers, mails and cargo in the model. This results from interviews in Greenland where mails have higher priority than passengers and passengers have higher priority than cargo.

***Table 1 Parameters in the route choice model***

| Parameter name | Business | Tourism | Visiting | Mail | Cargo |
|---|---|---|---|---|---|
| BASIS Value of time | 2,29 | 1,62 | 1,62 | 1 | 0,5 |
| Access time (WalkTimeW) | 3,435 | 2,430 | 2,430 | 1,500 | 0,750 |
| Variance (WalkTWVar) | 0,3435 | 0,2430 | 0,2430 | 0,1500 | 0,0750 |
| Change time (WaitTimeW) | 0,916 | 0,648 | 0,648 | 1 | 0,5 |
| Variance (WaitTWVar) | 0,0916 | 0,0648 | 0,0648 | 0,1000 | 0,0500 |
| Time to airport (ConTimeW) | 2,748 | 1,944 | 1,944 | 1 | 0,5 |
| Variance (ConTWVar) | 0,2748 | 0,1944 | 0,1944 | 0,1000 | 0,0500 |
| Hidden wait time in zone (WaitIZoneW) | 0,687 | 0,162 | 0,486 | 1 | 0,5 |
| Variance (WaitZWVar) | 0,0687 | 0,0162 | 0,0486 | 0,1000 | 0,0500 |
| Change penalty (ChangePen) | 137,4 | 97,2 | 97,2 | 0 | 0 |
| Variance (ChPenVar) | 6,87 | 4,86 | 4,86 | 0 | 0 |
| Early departure (EarlyDW) | 0,687 | 0,162 | 0,486 | 1,000 | 0,500 |
| Variance (EarlyDVar) | 0,069 | 0,016 | 0,049 | 0,100 | 0,050 |
| Ticket price (CostW) | 1 | 1 | 1 | 1 | 1 |
| Variance (CostVar) | 0 | 0 | 0 | 0 | 0 |
| Distribution type (DistAll) | 7 | 7 | 7 | 7 | 7 |
| Variance (StocCofAll) | 0,05 | 0,05 | 0,05 | 0,05 | 0,05 |
| Weight per seat (SeatUnitWeight) | 100 | 100 | 100 | 100 | 100 |
| Seats in plane (Seat) | 1 | 1 | 1 | 1 | 0 |
| Cargo room (Cargo) | 0 | 0 | 0 | 1 | 1 |
| Ranking Load (PreLoad) | 1 | 1 | 1 | 0 | 2 |

## 3.3 Capacity restrictions

In each of the three assignment steps described above the capacity restraints are handled. And for each passenger class loaded in the iteration the following procedure are followed:

- For each departure in the route network :
    - If the passenger class is freight which can not be transported on passenger seats and the plane type does not have a cargo room, the departure is closed for travellers
    - If the passenger class only can travel on passenger seats and the plane type only has a cargo room the departure is closed for travellers

- Otherwise the departure is examined for exceeded capacity (see below). If capacity is exceeded the departure is closed for more travellers
- If departure is not closed for new passengers or freight it is possible for the specific passenger class to use this departure in this specific iteration.

### 3.3.1 Examination of exceeded capacity

For each departure examined for exceeded capacity the following procedure are followed.

In case of prior iteration steps the number of occupied passenger seats and the total pre-loaded weight is:

$Seats_{preload}$ , $Weight_{preload}$

The sums of occupied seats and weight in the current iteration are calculated by working through each passenger classes loaded on the specific departure in the current assignment calculation. In this procedure the three running totals is calculated:

$PassengerSeats_{total}$, $Weight_{total}$ , $PostSeats_{total}$

This is done as follows:

For each passenger class *i* the traffic in seat units is $t_i$:

- If $t_i$ only can use passenger seats (passengers):
  - $PassengerSeats_{total} = PassengerSeats_{total} + t_i$
  - $Weight_{total} = Weight_{total} + t_i * seat\ unit\ weight_i$
- If $t_i$ only can use the cargo room (cargo)
  - $Weight_{total} = Weight_{total} + t_i * seat\ unit\ weight_i$
- If $t_i$ can use both cargo room and seats (airmail):
  - If the plane type has cargo room:
    - $Weight_{total} = Weight_{tota} + t_i * seat\ unit\ weight_i$
  - If the plane type has no cargo room:
    - $PostSeats_{total} = PostSeat_{total} + t_i$
    - $Weight_{total} = Weight_{total} + t_i * seat\ unit\ weight_i$

For some plane types groups of seats are removed to make room for mail e.g. groups of 4, 8 or 12 seats. If this is the case, the $PostSeats_{total}$ is rounded up to the nearest multiple.

Afterwards it is tested whether the capacity restraints are exceeded which is described by:

*PlaneMaxSeats, PlaneMaxWeight, PlaneMaxPostSeats*

The the following is tested:

If

- $(PassagerSeats_{total} + PostSeats_{total}) > (\text{Plane}MaxSeats + Seats_{preload})$

or

- $PostSeats_{total} > PlaneMaxPostSeats$

or

- $Weight_{total}$ > (Plane$MaxWeight$ - $Weight_{preload}$)

If one of these criteria is true the capacity is exceeded and the flight is closed for the present iteration in the assignment calculation.

## 4. NETWORK DESIGN MODEL

The network design model design/forecast the

- Air network (service network), i.e. between which airports there is a direct leg.
- Schedule, in the model formulated as discrete departure times in a 30.min. segmentation
- Use of airplanes (types and numbers)

This is calculated conditional on the expected passenger flows, which are the results of the route choice model on the output of the network design (i.e. a bi-level optimisation problem[4]).

In the solution algorithm, the air model (the set of optimisation models) and the route choice model is run iteratively in a tabu-search algorithm, which turned out to be the most efficient solution algorithm. The model takes the following decisions;

- Passengers choice of routes, and possible upgrading of airplane type due to this
- Downgrading of airplane type due to small passenger volumes
- Airplane allocation model
- Removing or adding direct legs (conditional to the economics of operation and some minimum service restriction)
- Time-schedule design

### 4.1 Main solution algorithm

The main solution approach was decided as follows;

- Step 1; A gross network is generated
  - 30 minutes departures between all relevant destinations
  - Feasible airplanes for each leg is set
    - Depends primarily of the lengths of the runway
    - Secondarily from this by how long it is possible to go with the most far reaching airplane that can operate

[4] Where the sub-problems are non-linear, discrete and with a non-closed form formulation, and is solved internally by iterative procedures.

- Step 2; Calculations are made based on weekly schedules with 3 days additionally before and after (some destinations may have down to 1 connection each week)
  - Each leg (flight between airports) can be operated by a list of possible airplane types with different sizes and operations costs
  - Based on modelled passenger potentials, all legs are downgraded to the optimal type of airplane
- Step 3; Route choice with no capacity constraints, i.e. the "optimal" schedule is estimated in the passengers viewpoint
- Step 4; Links with very few passengers are deleted (However, only to the extent this does not violate minimum restraints). Since the graph was enormously, this heuristic was added in order to reduce the optimisation problem to a size that could be solved by more rigid methods in the later phases of the overall optimisation algorithm.
- Step 5; Downgrading of airplanes to fit demand based on pure business economic decision.
- Step 6; Iterative network improvement
  - 6.1 Closing of legs with least utility (given a set of minimal criteria for operations and whether the leg has not been investigated before)
  - 6.2 A new assignment is made. After 5 iterations, airplanes may be upgraded during assignment. The network is redesigned then due to possible increased speed
  - 6.3 Airplane disposition / scheduling (estimation of turn-arround costs and balancing of legs and airplanes on airports) using a heuristic method.
  - 6.4 New stochastic capacity dependent route choice (legs can be closed, upgraded and using bigger and faster plains), which influence route choices.
  - 6.5 The overall utility function is calculated. If this is improved, the changes takes place, otherwise, they are regretted and marked as tabu
  - 6.6 Stop criterion
- Step 4-6 is repeated nine times with gradually increased restrictions.
- Step 7; detailed optimization of the airplane scheduling using a linear program formulated in the MOSEL software.
- Step 7 a separate module calculates the operation cost including turn around cost.
- Step 8; Final route choice model

Basically, this heuristic first use a very simple passenger-based approach to reduce the possible solution space (step 4). In practice this approach reduces the problem size with respect to number of legs from 58,000 to 5,000.

Then a heuristic is used for the calculation of the optimal airplane disposition and scheduling. (Step 6.3). The method for this problem, which consist of two core elements, calculation of turn-around time as a lower level problem and balancing of airplanes as the upper level problem, where the cost of balancing is given by the lower level calculation. This heuristic provides a much more precise estimate of the object function value, than just the passenger flows. However, it was first found feasible for network sizes below 5000 legs.

Finally a more rigid optimisation was attempted for the airplane disposition as this was formulated as a mathematical problem and solved by MOSEL. This, however, was only able to solve problems up to 600 legs, and was too slow to be run iteratively with the time-table and leg optimisation models. It was also necessary with simplifications to solve it as a pure mathematical. In the final model, it was therefore decided not to use step 7, and to use the heuristic (refer to section 4.2) for the last step as well.

All-in-all using the above combination of heuristics made it possible to optimise the air network in Greenland conditional to the passengers' choice of route and departure time. The overall calculation time is about 84 hours, which may be considered high, but which however indeed was able to pin-point possibilities of restructuring the air transport network with considerably benefits for the society.

## 4.2 Leg balancing

The general principle in the leg-balancing model is to ensure balance for each airport one by one. By starting with the airport with the smallest number of legs, the balancing problem with the fewest degrees of freedom is solved first. In this way, the algorithm always finish in the largest hup's (Nuuk, Sdr. Strømfjord), where it should be easier to ensure a balance. Even if there is a lack of balance here (e.g. from 45 to 44 legs) it is in relative terms less problematic than if a small airport are assigned 1 from leg and 2 to-legs.

The balance is ensured by successively removing legs if balance is not meet initially. Subsequently balance is ensured by plane types by upgrading legs to larger plane types.

### 4.2.1 Solution algorithm

$SumBal_{OK}$ := 0; $SumBal_{EjOK}$ := 0 (The two balancing variables are initialised)

The simplified turnaround cost model is run in order to generate an initial turnaround cost (refer to section 3).

Airports are sorted according to the total number of trips in and out of the airport. Lufthavnene A sorteres efter hvor mange legs der samlet går til og fra lufthavnen

For each airport A in the sorted sequence (the smallest airport first), legs are balanced as follows[5] {

> Legs, which is equal to the minimum service criteria is marked as "tabu". The number of legs $L_f$ (legs, which depart from the airport) and number of legs $L_t$ (legs, which arrive to the airport) are counted.
>
> If $L_f > L_t$ {
>
> > If at least one from-leg $L_f$, which is not marked as "tabu" {
> >
> > > The from-leg $L_f$, which is not "tabu" and have the lowest

[5] The trick is that the larger the airport the more chance of being able to balance. If there is balance in the small airports, the risk of larger airports not being balanced is smaller.

object value (e.g., revenue – operation costs – turn-around costs)[6] are removed.

$Bal_A$ := OK (the airport is balanced)

$SumBal_{OK}$:= $SumBal_{OK}$ + 1 (the status variables is updated)

} Or (the airport could not be balanced)[7]

$Bal_A$ := NOTOK (The airport could not be balanced)

$SumBal_{EjOK}$:= $SumBal_{EjOK}$ + 1 (the status variables is updated)

}

If $L_t > L_f$ the same procedure as for from-legs are carried out, but with to-legs removed.

It should be noted, that even though balance is not ensured for the airport as a whole, the balancing will continue in that it might be possible to ensure a better balance on plane types.

If there are at least two different plane types in an airport, legs for each plane types are investigated. The largest plane type is investigated first and the procedure continues until the second smallest airplane type.[8] {

The number of legs from $L(X)_f$ (legs, which depart from the airport of plane type X) and the number of legs $L(X)_t$ (legs, which arrive to the airport) are counted.

If $L(X)_f > L(X)_t$ {

All to-legs of plane type $L(X\text{-}1)_t$, and which arrive from an airport that allows plane type X exists[9]

If there is at least on leg in this set[10]

Cturnarround := null, Lopt:=null

For each of these legs {

The turnarroundcost is calculated for the set {all L(X) + the actual L(X-1)} by same principle as for the simplified turnaround model[11].

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[6] This is the reason why the simple turn-around model needs to be run before the solution algorithm. If not, there would not be an estimate for the turn-around costs.

[7] The purpose of this updating is mainly internal validation.

[8] The smallest airplane is a residual of the other upgrades.

[9] All to-legs, which can be upgraded is detected, however, only within the present plane type.

[10] If there are not relevant legs of type X-1, the algorithm proceed to check X-2 and, etc.

[11] It will give a biased estimate of the turn-around cost if only the calculation is carried out for the

If Cturnarround:=null or the calculated turnarround cost for L(X-1); C(L(X-1)) < Cturnarround, then define Lopt := L(X-1) and LCturnarround :=C(L(X-1))[12]

}

The optimal leg LCturnarround is upgraded from X-1 to X

}

Or it is not possible to upgrade legs, e.g. the plane optimization model must form empty legs. This is done, only when the models have converged and no more can delete legs.

}

If $L(X)_t > L(X)_f$ the same procedure is carried out, but where from legs is upgraded.

}

}

The simplified turnarround cost model is run[13]

The following information is loaded to the plane optimization model {

Deleted legs

Upgraded legs

New turn around costs

}

### 4.2.2 Discussion

The general plane optimisation model delete links one by one (or several links in one process, but without considering balancing). Initially, up to 50% of all airports are unbalanced. However, because many airports meet the minimum service constraints, it is expected to be less.

After running the heuristic for the leg-balancing, most if not all airports will be balanced. Only in very special cases balancing may not be met.

After the balancing on legs the balancing on plane types are carried out. It must be assumed that, in most cases, it is possible to upgrade plane types.

All-in-all the heuristic will ensure a much better balancing of the final solution. This

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actual L(X-1). However, there is a limited list of alternatives, which is checked according to the FIFO principle (pure linear operation). As a result, the complete is investigated.

[12] The actual leg is the best candidate so far for upgrading of plane type from X-1 to X.

[13] The leg-structure is changes, which is why we need to calculate new turnaround costs.

ensures that the input to the optimisation model is better. However, it should be underlined that even though balance is ensured, the final plane optimization model may still upgrade legs in order to reduce turn-around costs. Also, it may in rare occasions be beneficial to introduce empty legs.

# 5. MODEL APPLICATION: LOCATION CHOICE AND SIZE OF ATLANTIC AIRPORT

During 2007, The Greenland Home Rule will take the most important transport decision ever, namely, the location and the size of a new Atlantic airport. To illustrate the importance of the decision, the construction cost alone may equate 10-20% of the annual GNP depending on the specific location. To help this decision, the model has been applied to three different scenarios, which varies with the length of the run-way in Nuuk as well as the location in the Nuuk area[14]. The alternatives are compared in a social cost-benefit analysis and benchmarked to a base situation, in which the location of the Atlantic airport remains in Sdr.Strømfjord. The three scenarios are outlined below;

- Nuuk 1799 meter: The runway in Nuuk is extended to 1799 meter enabling for a Boeing-757 to land.
- Nuuk 2200 meter: The runway in Nuuk is extended to 2200 meter enabling for Airbus-200 to land.
- Nuuk 3000 meter: The runway in Nuuk is extended to 3000 meter enabling for Airbus-200 to land.

Apart from the difference in the length of the runway in Nuuk there is considerable difference in construction and maintenance costs and also in the demand estimates, especially for tourists.

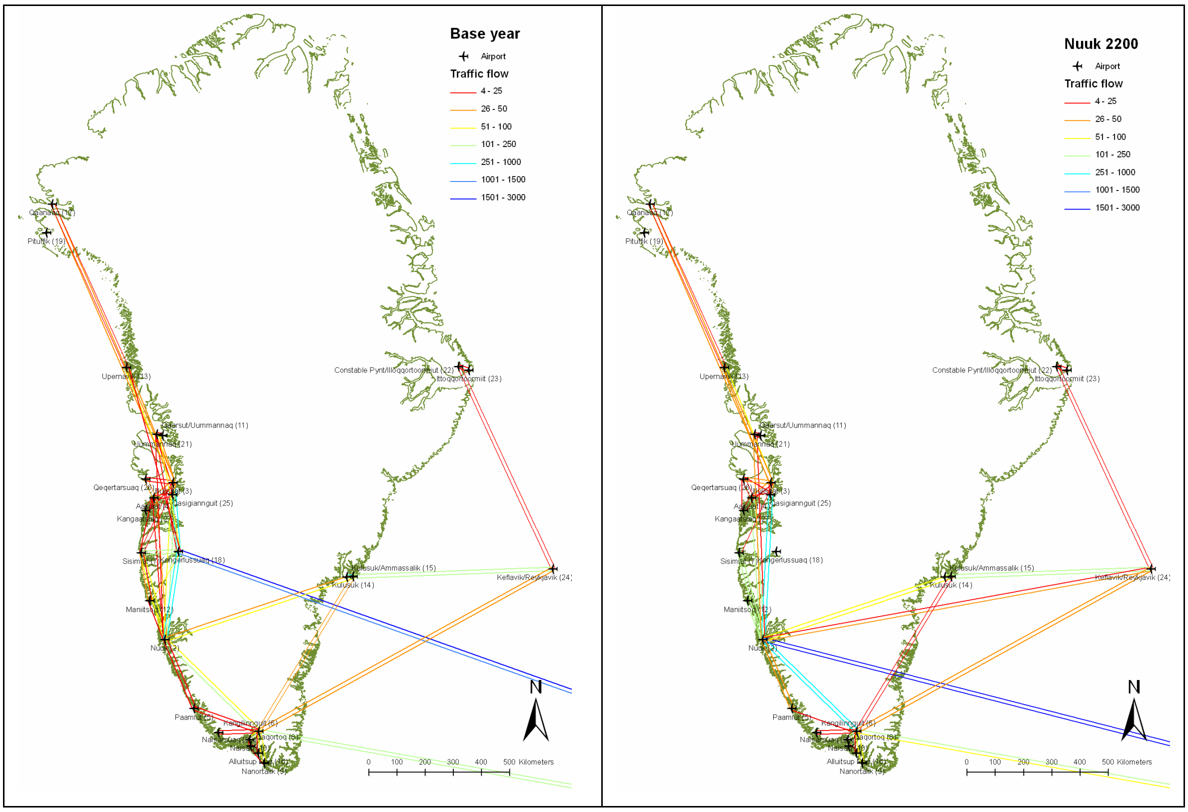
The service network and the traffic flow are modeled in the reference year with and without changes to the network. Afterwards, the service network and the traffic flow are modeled with the network changes and a new OD matrix is calculated on the basis of the change in the level of service.

The traffic flow of today has three main routes; from Copenhagen to Sdr.Strømfjord and from Sdr.Strømfjord to either Nuuk or Illulissat. With the extensions of the runway in Nuuk, these main routes change in that passenger will travel directly between Copenhagen to Nuuk and from Nuuk to either Illulissat or Narsarsuaq. The changes to the network results in direct routes from Denmark to Nuuk and direct route from Island. In general, the

[14] The present run-way can only be extended to 2200 meter and two alternative locations have been selected for the 3000 meter alternative, which at present is assumed to have the same construction costs.

number of routes is reduced considerably when Sdr.Strømfjord is closed because a great deal of the feeder-traffic is taken care of by direct routes.

***Figure 1:*** Illustration of how the network structure changes as a result of a new Atlantic airport in Nuuk.




***Traffic flow in reference year*** ***Traffic flow in scenario Nuuk 2200***

The results from the three scenarios points in the same direction. The service network has almost the same routes but the plane types differ between Copenhagen and Nuuk (refer to figure 4 below). This makes a slightly difference in level of service which give rise to different demand formations (OD matrices). Also, since the use of plane types differ between the scenarios, the production costs differ as well.

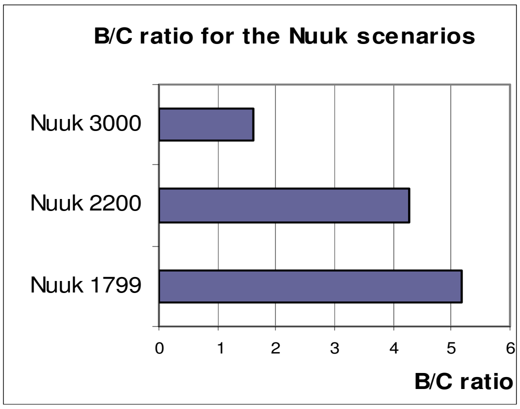



***Figure 2: B/C ratios for the Nuuk scenarios.***

In figure 2, the B/C ratios are shown for each scenario. The Nuuk 1799 scenario has turned out to be the most cost-effective of the three projects. The main reasons for the differences in the projects are the initial costs, production cost, and ticket revenue for operators. This difference resolves in an important difference in the calculated internal rate which is 26.3 % for Nuuk 1799, 20.9 % for Nuuk 2200 and only 8.9 % for Nuuk 3000.

On the figures below the different benefit and cost components is shown. Nuuk 2200 and Nuuk 3000 have equal benefits because the model settings allows the same type of planes to land, however, there is large difference when looking at cost components.

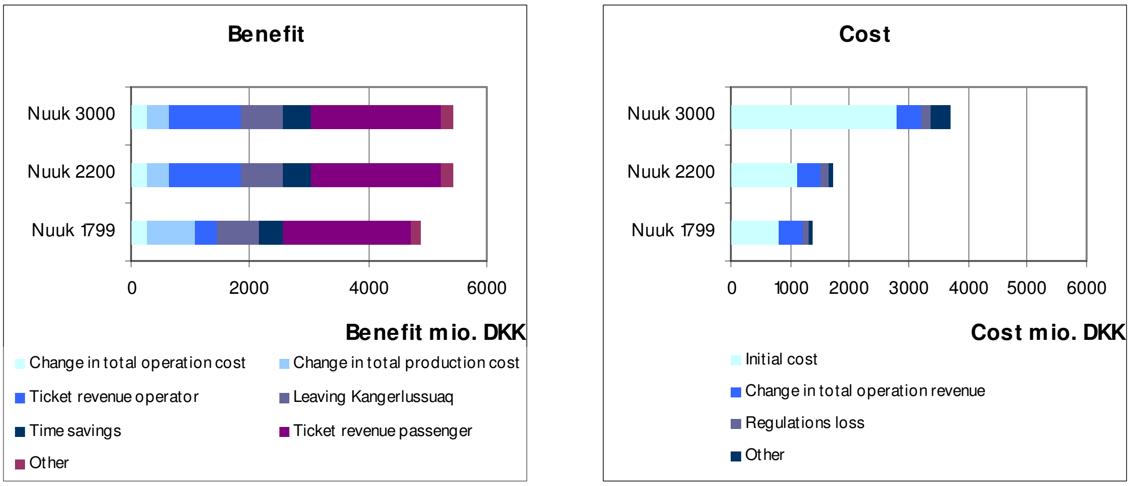
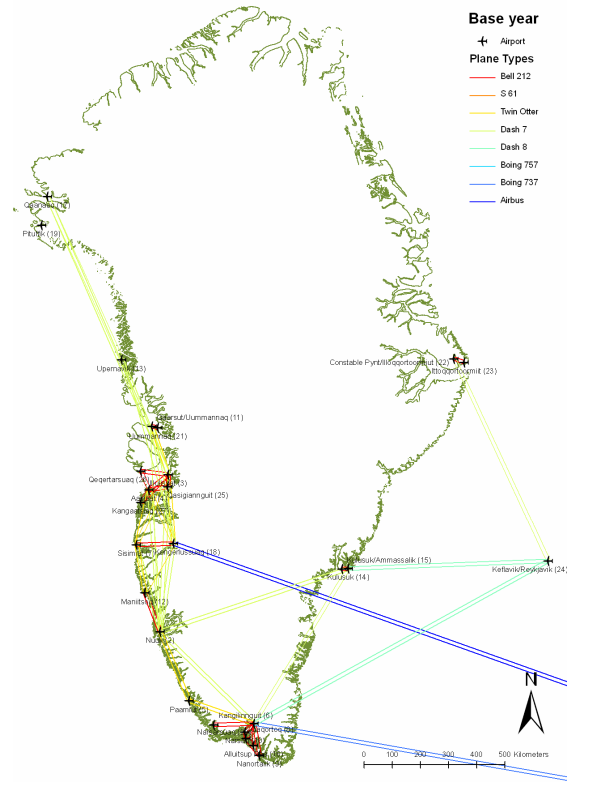


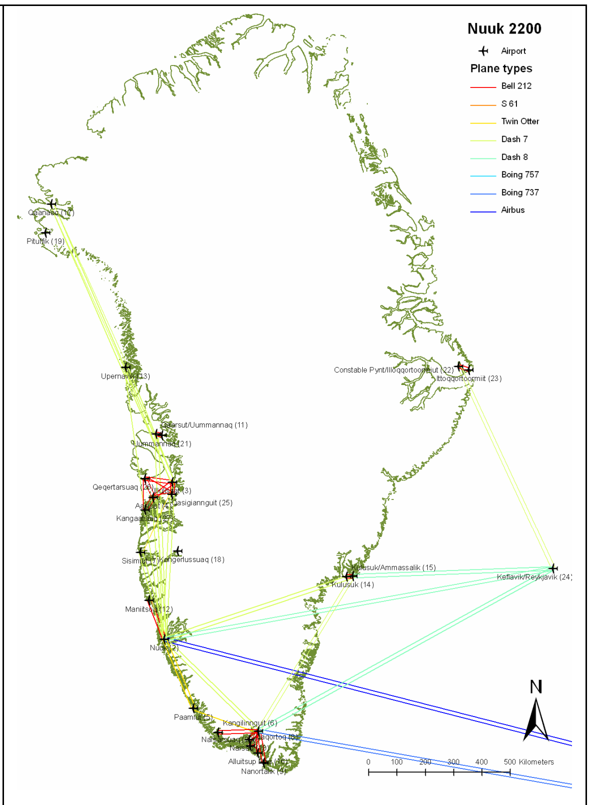

***Figure 3:*** Benefit and cost components for the three Nuuk alternatives.

The model more than indicate that significant socio-economic benefits can be expected, when the main airport are moved from Sdr.Strømfjord to Nuuk. However, when judging between a "large" (Nuuk 2200 or Nuuk 3000) and a "medium" (Nuuk 1799) size solution in Nuuk, the medium size solution turns out to be the one with the highest internal rent, due to lower costs. Clearly, the result is based on the demand estimated for 2012. If a significant increase in the number of passengers beyond the 2012 level is expected, the capacity limits may be met, and the 2200 may be preferable. However, these numbers are very difficult to forecast, which is why the Greenland home-rule has decided to forcast only to 2012.

The application shows that it has been possible to implement a model system. All though, the models is based on numerous assumptions, it provide an important decision tool and it gives a clear indication that a switch of location for the Atlantic airport is beneficial.




The use of different plane types in reference year




The use of different plane types in scenario Nuuk 2200

***Figure 4:*** Illustration of how the type of planes changes as a result of a new Atlantic airport in Nuuk.

The practical implementation and the calibration of the model turned out to be very complex. However, in the final version, the model was able to replicate the network in the reference year sufficiently well. The outer loop between the different model components converged, and the final solution was not only able to be similarly as good as the existing system, but also suggest improvements.

## 6. Conclusion

In the paper, a combined transport and optimisation model for air transport in Greenland has been presented. The structure of the model is composed as a bi-level optimisation problem. At the lower level optimisation, demand and route choice for passengers and freight is addressed, whereas at the upper level optimisation, the service network, the configuration of plane types and the flight scheduling is dealt with.

The centre of the upper level optimisation is an objective function, consisting of passenger utility, operation costs, and revenues from sale of tickets. The function may be formulated as a function that represent a socio-economic optimum or an operator-optimum (air companies), which will represent partly conflicting purposes. The present version of the model only optimises the objectives of the operator, which reflects the present decision context.

The model is solved conditional on a number of constraints, e.g. capacity constraints for plane types (weight and seat constraints), constraints for various legs (minimum service level requirements), mileage constraints for plane types, constraints concerning minimum runway lengths for certain airplane types, and finally certain “airports” (landing grounds) that can only be served with helicopters.

The model solve a complicated problem, with 28 airports, up to 1000 legs in the final flight schedules and allocation of many types of airplanes to the network. The testing shows that the model indeed is able to solve the problem and that the solution is better than the present system. More over the model is able to suggest service networks in future scenarios where the constraints are changed. Typically this is changes of air port locations and lengths of the run way. The main scenario – to build an Atlantic Airport in the Capital Nuuk – turned out to have very positive socio-economic characteristics.