# Trip-based route choice models – A method to eliminate aggregation bias in activity-based models

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## SHORT ABSTRACT

The paper compares trip-based assignment procedures with traditional matrix-based procedures. The main benefits of trip-based procedures are 1) that the full information about trips from the demand modelling can be utilised in the assignment, and 2) that the calculation of Level of Services (LoS) are consistently feed back into the demand model. This is especially beneficial in activity-based models, as the detailed casual relationships in the demand model can then be reflected in the assignment procedure.

Traditionally, trip-based assignment models are rejected due to calculation times. In the paper it is shown, that this indeed may not be a valid argument concerning the tendency to increase the number of zones, time of day intervals and trip purposes in demand models, including especially activity-based models. It is shown that the theoretical calculation complexity of large-scale models may indeed be comparable or even smaller in trip-based assignment procedures than in traditional matrix-based. This is exemplified on the Copenhagen traffic model.

The core issue concerning calculation complexity is that the trip-based assignment depends on the number of trips and the network size. The zone-based models depend on the number of matrices, zones and the network size. This means, that the trip-based models are slower than zone-based, if the network is small or the number of zones and trip-matrices is low. In small cases calculation time is usually not an issue though. If – however – the model consists of many zones and matrices, then the trip-based assignment seams to be more efficient in terms of calculation time.

## LONG ABSTRACT

#### 1. Background and discussion

Activity-based models provide much more insight in persons' trip making than traditional trip- or tour-based models. The newest types of activity-based models simulate individuals' activity patterns over the day, i.e. the fundamental modelling unit is the individual (integer). An example is the Sacramento Model (Bowman & Bradley, 2006). This individual-based disaggregate approach is different from traditional matrix-based models (whether activity-based or not), where the fundamental unit is the cells in the matrices and the unit is a floating point. With the increasingly segmentation of models into time-of-the-day, trip purposes and number of zones, the size of this fundamental unit may be quite small – often even much lower than one on average, whilst one (integer) is the fundamental unit in disaggregate activity-based models). An example is the Copenhagen model (OTM), where there are 2.2 million car trips, distributed between 835x825 zones, 7 time-periods and 6 trip purposes (Nielsen et.al. 2006). This equals 42 trip matrices and 0.07 trips in average per cell.

The new generation of activity-based models have the person as the fundamental unit. The insight of the characteristics of each trip is therefore higher; Income, Value of Time (VoT), gender, etc., may "follow" the trip. Especially VoT is interesting, since it may depend on the person's income<sup>1</sup>, the time use of the trip<sup>2</sup>, the trip purpose<sup>3</sup>, and the time of the day<sup>4</sup>.

## 1.1 Link from demand models to assignment models

Activity-based models are often used to evaluate network effects, where the Value of Time may be a core element in the decision making. An example is congestion reducing new projects or road pricing schemes. The information on the trip characteristics in activity based models is however not used in the assignment procedures. Hence, a very often used approach is that the detailed trips from the activity-based models are aggregated into a zonal level and into trip matrices, which are then assigned by "traditional" assignment procedures. The casual relationship between for example VoT and route choice is hereby lost. Another problem is that the LoS matrices produced by the assignment procedure are also only an average over the trips.

<sup>&</sup>lt;sup>1</sup> Generally one may assume that VoT is increasing with income.

<sup>&</sup>lt;sup>2</sup> That VoT is increasing with trip length (measured in time) is e.g. shown by micro-economic theory (de Serpa, 1971, Sara-Diaz, 1997) and empirically (e.g. in Mabit & Nielsen, 2006, for Copenhagen)

<sup>&</sup>lt;sup>3</sup> The SAME person may have DIFFERENT VoT's for different purposes, e.g. having to meet at work at a fixed working hour, while having more flexibility with regards to shopping trips.

<sup>&</sup>lt;sup>4</sup> The SAME person may have DIFFERENT VoT's during the day due to variations in time-restrictions. If a mother has to collect her child from the kindergarten at a fixed hour in the afternoon, this may be a tighter time-restriction, than delivering the kid in the morning.

As an example, if the high VoT and low VoT users are aggregated into matrices before the assignment and assigned according to the average LoS, they may all choose a new toll-road. If there is high congestion on the alternative routes, there might be a deterministic User Equilibrium situation between the two routes even though the VoT's implicitly are assumed identical. If the two alternatives, given the average VoT, are fairly equal in utility, there might also be a split on the alternatives due to the error term in the model. Variation of the VoT within the assignment procedure may also result in a split between routes (Nielsen, 1996).

In the base-year scenario, the assignment may be calibrated by such approaches to provide reasonable results. It does, however, not describe the causal relationship between e.g. income and VoT, nor spatial differences of e.g. income distribution. With respect to forecasts of impacts of policy initiatives, this can turn our to be a crucial simplification of the model.

## 1.2 Link from assignment models to the demand modelling

In the feedback loop from assignment to demand, the Level of Service (LoS) data – e.g. travel time – is usually averaged for all groups. If for example 30% use a toll road and 70% uses on the other road, then the average toll LoS is 30% of the toll, and the average time use is weighted as 30% toll road time (fast) and 70% non-toll road time (slow). This introduce a severe aggregation bias, since the users with high VoT should be assumed to use less time and pay more toll, than the low VoT users.

The LoS that is fed back into the activity-based model – or any other demand model for that sake - is therefore inconsistent with the insight produced by the demand models concerning the individuals and the trips.

The problem above may be solved partly by segmenting the demand into more trip purposes and by carrying out a multi-purpose assignment (such as in Nielsen et.al. 2002). A further segmentation can be to split the demand matrices according to VoT intervals. This increase – however – the number of matrices and reduce the cell-sizes further. In the case of the Copenhagen model, 5 VoT intervals would increase the number of car matrices from 42 to 210, and the average cell-size would be reduced from 0.07 to 0,015. If the trips are also to be split into trip-length segments, this will complicate this even further, and also introduce a need of sorting the matrices at cell-level before the splitting.

#### 1.3 The concept of trip-based assignment

Therefore, an intuitive improvement of assignment procedures in activity-based modelling is to assign the trips directly onto the network. The obvious benefits have been described above;

- Direct use of trip attributes in the assignment, i.e. VoT depending on the individual's income, trip length, trip purpose and time-of-the-day.
- Correct LoS calculations for feedback from assignment to demand.

Another benefit is that there is no need for a zonal level of aggregation, and the trips can therefore be assigned directly from node to node rather than from zone to zone. The need for zones and zonal connectors are therefore eliminated. And the usual problems, with too much traffic at the roads near the end of the connectors are eliminated, as well as problems to get the right distribution of traffic onto the different connectors from/to a zone.

The dis-benefit is, that it is often claimed to increase calculation time to use trip-based assignment, and that no standard software exist for this. Another complication is that the sampling of alternatives in the activity-based models would become lightly trickier at the trip-level, since this cannot be drawn directly from LoS matrices. The LoS for alternatives can however be obtained by aggregating the trip LoS into matrices - a task that is equal to the usual aggregation in activity based models - or by using a sampling procedure.

In the following it is indicated, that the calculation time of a trip-based assignment may not bee too much of a problem considering the increasing segmentation of the traditional zone-based models in practice.

## 2. Calculation complexity estimates

One of the most efficient methods to solve Stochastic User Equilibrium (SUE) assignment models is to carry out an origin-based assignment. The technique was suggested by Burrell (1967), and is e.g. used in Nielsen (1996). A later development and improvement was done by Dial (2006), which is compared with matrix based approaches in Slavin (2006). The idea of assigning rows of the matrix separately is that the tree-building process can be carried out once for the entire row, which saves as many Dijkstra path-searches as number of zones in the matrix. And the idea of not assigning the entire matrix before re-calculating the utility functions is that stochastic simulations are carried out for each row and equilibrium procedures updated accordingly, whereby the number of iterations in the outer loop (e.g. in a Method of Succesive Average procedure, Sheffi 1985) can be reduced significantly in the case of SUE (faster convergence). Such a procedure with respect of Stochastic User Equilibrium for car assignment was applied in Nielsen (1994).

The calculation complexity in this procedure is then dependent of the product of the following (with the numbers from the Copenhagen model in brackets);

- Number of trip matrices, i.e. time periods x trip purposes  $[7 \times 6 = 42]$
- Number of zones, i.e. rows [835]
- Dijkstra search + network loading per row
  - Dijkstra search; [simulation of random coefficients = 5] + path search [links (1+simulation of error term=1.1) • nodes • log(nodes);  $[5 + (30,000 + (1+1.1) + 15,000 + \log(15,000)]^5$
  - Network loading; number of zones, i.e. destinations x E(number of links per path); [835 · (approx 100 links per path) ]
- #iterations  $[200]^6$

The resulting complexity is here O[6.5xE16]

In a trip-based model, the similar numbers are;

- Number of trips [2.2 million]
- Dijkstra search + network loading per trip
  - Dijkstra search; links  $(1+\text{simulation of error term}=1.1)^7 \cdot \text{nodes} \cdot \log(\text{nodes}) [500 \cdot (1+1.1) \cdot 200 \cdot \log(200)]^8$
  - Network loading; E(number of links per path); (approx 100 links per path)
- #iterations [10]<sup>9</sup>

The resulting complexity is here O[2.4xE13]. Even in the worst-case calculation complexity of Dijkstra (full skim of the entire network for each trip assignment), the complexity is [2.0xE17], i.e. only 3 times larger than the zone-based SUE.

<sup>&</sup>lt;sup>5</sup> Empirically tests by the Copenhagen model have revealed that adding random error terms approximately double calculation time. This knowledge has been used in the estimation of calculation complexity = 1.1 of this operation compared to the link finding in Dijkstra (e.g. by heap operation).

<sup>&</sup>lt;sup>6</sup> The number of iterations is high, since UE, VoT's and error terms are simulated (Nielsen & Knudsen, 2005).

<sup>&</sup>lt;sup>7</sup> No random coefficients are simulated, since these are inherited from the activity-based model.

<sup>&</sup>lt;sup>8</sup> The core issue here is that the Dijkstra search does not need to investigate the entire network in a tripbased assignment. In the simplest case, the search is stopped when reaching the destination. Since many trips are very short (e.g. shopping, collecting kids,...) the average network skim-time is much lower than the worst-case. In addition, a number of heuristics exists, which can speed up path search even further. A conservative estimate is that 500 links on average need to be examined per path search.

<sup>&</sup>lt;sup>9</sup> Since the VoT is known per trip, and the trips are much more distributed in the network, less iteration are expected.

Theoretical calculation complexities are often indeed theoretical only, as also mentioned and shown by Slavin (2006) and in Nielsen & Frederiksen (2006). Memory needs, complexity of numerical calculations of drawing random numbers, taking the logarithm, calculating exponential functions etc are often changing the overall calculation time considerably. Furthermore, one has to guess the number of needed iterations for convergence in the estimation of calculation complexity, the average number of links per path, and needed skim-size in average for the Dijkstra trip-based path search. Finally, the number of simulated distributions and the complexity of this calculation is highly influence the overall calculation time. However, the above estimates indicates, that there is a potential for trip-based procedures in activity based models.

The core issue that the calculation complexity of the traditional zone-based SUE increase with the number of zones and trip-matrices, as well as with the network size, while the trip-based assignment "only" depends on the number of trips and the network skim per path. This is indicated in the table below, where even the worst case Dijkstra trip-based assignment seams more efficient than SUE for the new Copenhagen model size. The table shows extracts of a spread-sheet calculating the theoretical calculation complexities for 3 different models that the author uses as "test-laboratory". As indicated, trip based procedures may indeed be efficient for large models with many zones and segmentation of demand into many trip purposes and time of the day intervals.

			Model		
			Næstved	Old OTM	New OTM
Segmentation	Trip periods		3	5	7
	Trip				
	purposes		3	4	6
Data	Zones		100	618	835
	Links		2000	30000	30000
	Nodes		1000	15000	15000
Demand	Trips		200000	2200000	2200000
Elements of calculation complexity	Links per paths		50	100	100
	Links,				
	Dijkstra		100	500	500
	Nodes, Dijkstra		50	200	200
	Simulation random coefficients		5	5	5
	Simulations error terms		1,1	1,1	1,1
	Iterations, Zone based		100	200	200
	Iterations, Link-based		10	10	10
Estimates of calculation complexity	Zone-based SUE		2,6E+12	2,2E+16	6,4E+16
	Trip based SUE	Worst case Dijkstra	5,8E+13	2,0E+17	2,0E+17
		Average case			
		Dijkstra	8,2E+10	2,4E+13	2,4E+13

**Table 1.** Comparison of calculation complexity estimates for the Næstved model and the old and new OTM-model, for zone-based Stochastic User Equilibrium (SUE) and for the trip-based model.

IF the average Dijkstra is efficient which require quite some programming though – then there is a huge potential for speed up of calculation time. In the large OTM-network, i.e. a factor 1000 algorithmically, whilst this reduces to a factor 50 if the same number of iterations are needed for the convergence – i.e. still a huge gain.

## 3. Discussion

It seams like trip-based assignment procedures are feasible in a model like the Copenhagen OTM model. The core issue concerning calculation complexity is that the trip-based assignment depends on the number of trips and the network size. The zone-based SUE depends on the number of matrices, zones and the network size. This means, that the trip-based SUE is slower than a zone-based SUE, if the network is small or the number of zones and trip-matrices is low. In small cases calculation time is usually not an issue though. If – however – the model consists of many zones and matrices, then the trip-based assignment seams to be more efficient in terms of calculation time.

Due to the many potential benefits of using a trip-based assignment model, in terms of more refined casual relationships and consistency between the activities based model and the assignment, trip-based assignment seems to be a promising way forward.

The paper will discuss the methods further than in this extended abstract. In addition tests of the trip-based approaches will be carried out on the 3 cases mentioned above and compared with the existing zone-based SUE. This will include tests of convergence in order to be able to compare the two methods with respect to calculation time at an equal level of convergence. To illustrate the differences in modelling results, the zone-based and trips based models will also be run on road pricing and toll scenarios that have already been coded in the Copenhagen modelling network in a prior project.

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