Adaptive Traffic Assignment in Stochastic Dynamic Networks

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***Abstract*— This paper establishes a user-equilibrium dynamic traffic assignment (DTA) model where users make adaptive routing decisions, denoted as routing policies, in a stochastic time-dependent network. A routing policy is defined as a decision rule which specifies what node to take next out of the current node based the current time and online information, essentially a mapping from network states to decisions on next nodes. A general definition of routing policy is given to allow for a wide variety of information accessibility situations, thus excluding the usually simplified assumptions such as either no information or full information. In the proposed DTA model, a routing policy is treated as an element of a traveler's route choice set. The key advantage of this approach is that online information is embedded in a traveler's route choice alternatives and, thus, systematic methods can be designed independent of online information formats. A generalization of Wardrop's First Principle is used as the equilibrium condition: each user follows a routing policy with minimum perceived disutility at his/her departure time and no user can unilaterally change routing policies to improve his/her perceived disutility. A general framework is provided and the equilibrium model is formulated as a fixed point problem with three components: the routing policy generation module, the routing policy choice model and the policy-based dynamic network loader. An MSA (method of successive averages) heuristic is designed. Computational tests are carried out in a hypothetical network, where a random incident is the source of stochasticity. The heuristic converges satisfactorily in the test network under the proposed test settings. The adaptiveness in the routing policy based model leads to shorter expected travel times at equilibrium compared to DTA models where users make non-adaptive routing choices. As a byproduct, travel time reliability is also enhanced. The value of online information is an increasing function of the incident probability. Travel time savings are high when market penetrations are low. However, the function of travel time saving against market penetration is not monotonic. This suggests that in a traveler information system or route guidance system, the information penetration needs to be chosen carefully to maximize benefits.**

I. Introduction

Stochasticity in transportation systems is both intuitively prevalent and experimentally shown. Travelers' routing decisions in a stochastic network with online information is conceivably different from those in a deterministic network. It is generally believed that adaptive routing will save travel time and enhance travel time reliability. For example, in a network with random incidents, if one does not adapt to an incident scenario, he/she could be stuck in the incident link for a very long time. However, if adequate online information is available about the incident and the traveler adapts to it by taking an alternative route, he/she can save travel time compared to the non-adaptive case. The adaptiveness also ensures that the travel time is not prohibitively high in incident scenarios, and thus provides a more reliable travel time. The problem of optimal adaptive routing decision making for individual travelers has been studied by various researchers [1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [11] [12] [13] [14] [15] [16] [17] [18] [19], and a complete literature review can be found in Gao and Chabini [9]. A general conclusion from the above studies is that in a flow-independent stochastic time-dependent (STD) network, an individual user's expected travel time from being adaptive (in one way or another, depending on the problems studied specifically) is always no higher than that from being non-adaptive, i.e. following a simple path.

After understanding how an individual traveler makes adaptive routing decisions, another research question would be: what will be the network-level impact if many travelers make adaptive routing decisions? The interaction between supply and adaptive demand in a stochastic dynamic network needs to be captured to answer the question. This interaction in a deterministic network (with possible perception errors from the demand side) is captured by a conventional dynamic traffic assignment (DTA) model. This paper establishes a user-equilibrium traffic assignment model where users make adaptive routing decisions in a general stochastic time-dependent network with online information.

A routing policy is defined as a decision rule which specifies what node to take next out of the current node based the current time and online information, essentially a mapping from network states to decisions on next nodes. The critical difference between a routing policy and a path lies in the way adaptive behavior is modeled: a routing policy can manifest itself as various paths depending on the underlying stochastic process that drives a traffic network, while a path is fixed regardless of random disturbances to the network and available online information. The adaptive DTA model has the following distinctive features to contribute to the state of the art:

- Users' choice sets are composed of routing policies, rather than simple paths. The definition of routing policy is general and can handle a wide range of information accessibility situations, and thus avoids the usual simplified assumptions such as no information or full information.
- Link travel times are random variables with time-dependent distributions. A joint distribution of all random variables is used such that both link-wise and time-

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wise stochastic dependencies of link travel times are modeled.

- The equilibrium is in terms of distribution of link travel times, flow and other traffic quantities of interest, and generalizes the conventional equilibrium concept.

There is quite limited study of equilibrium dynamic traffic assignment models in the literature, where adaptive routing decisions are an integral part of a user's behavior model. Hamdouch et al. [20] proposed a strategic model for dynamic traffic assignment, as an extension to the static model studied by Marcotte et al. [21]. The model assumes that travel delays happen only at nodes, when the arc that a traveler wants to access has reached its rigid capacity. Randomness in travel time comes from the fact that the position of any traveler in the vertical queue at a node is random, while link costs are not random in terms of day-to-day fluctuations. Ukkusuri et al. [22] proposed an equilibrium static assignment model where link travel times are independent static random variables, and users learn the actual realizations of outgoing links when reaching a node. A sequential Logit model is employed to do the loading.

The paper is organized as follows. In Section II, a conceptual framework for the policy-based stochastic DTA model is introduced with a fixed-point formulation. Three components of the DTA model are presented in detail: users' routing choice model, policy-based dynamic network loading model and routing policy generation module. The equilibrium condition based on a generalized Wardrop's principle is proposed next and a method of successive average (MSA) solution algorithm is described. In Section III, computational tests are set up to study the behavior of the proposed model and to compare it with models that do not model adaptive routing choices. Throughout the paper, a symbol with a $\sim$ over it is a random variable, while the same symbol without the $\sim$ is one specific value of the random variable. A "support point" is defined as a distinct value that a discrete random variable can take or a distinct vector of values that a discrete random vector can take, depending on the context. Thus a probability mass function (PMF) of a random variable(vector) is a combination of support points and the associated probabilities.

II. A Framework for the Policy-Based Stochastic DTA Model

We present a framework for the policy-based stochastic dynamic traffic assignment model to give a big picture on the input, output, model components' interaction, and data flow, as shown in Figure 1. The input to the overall DTA model is the stochastic dynamic demand $\tilde{D}$ and supply $\tilde{S}$ represented by a joint discrete distribution with R support points, each of which has a probability $p_r, r = 1, ..., R$. The demand is assumed to be inelastic, i.e. the demand distribution is fixed. In a discrete time representation, any realization of random demand is given as a matrix of time-dependent numbers of O-D trips during all time intervals. $\tilde{D} = \{D^1, D^2, ..., D^R\}$, where D^r is the demand matrix for the r^{th} support point. $D^r = \{D^r_{jd,t}, t = 0, 1, 2, ..., \forall \text{OD pair}\{j, d\}\}$, where $D^r_{jd,t}$ is the number of trips between origin j and destination d for departure time t for the r^{th} support point. The random supply can be represented through the random occurrence, duration and severity of an incident or any other random supply factors: $\tilde{S} = \{S^1, S^2, ..., S^R\}$. Note that the same probability p_r is associated with the outputs computed from S^r, D^r. In the remaining of the paper, whenever a support point has a superscript r, its associated probability is p_r, otherwise indicated. The output is an equilibrium distribution of flow-dependent link travel times $\tilde{C} = \{C^r_{jk,t}, \forall\{j, k\} \in A, \forall t, r = 1, 2, ..., R\}$, where A is the set of links of the traffic network, and the corresponding routing policy splits $f = \{f^i_{jd,t}\}$, where $\{j, d\}$ is an OD pair, t is the departure time, and i is the index of policies. Note that the distributions of all relevant traffic random variables are discrete. The framework in general does not restrict the link travel time distribution to be continuous or discrete. However, conceivably it is easier to work with a discrete distribution, based on which a routing policy is defined, and also it is not clear how to do network loading with continuously distributed demand/supply. It is an interesting future research question to work with continuous distributions.

There are three major components of the stochastic DTA model: the users' routing policy choice model, denoted as U, the policy-based dynamic network loading model, denoted as L, and the optimal routing policy algorithm, denoted as O.

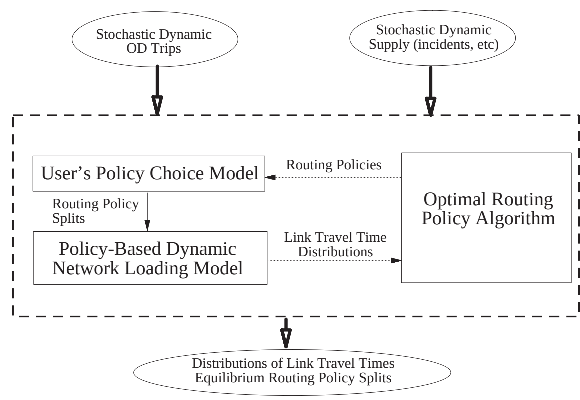

Fig. 1. A Conceptual Framework of Stochastic Dynamic Traffic Assignment Model

A. Users' Routing Policy Choice Model

The users' routing policy choice model takes as input a set of routing policies $G = \{\mu_1, \mu_2, ..., \mu_i, ...\}$ generated by the optimal routing policy algorithm, and a joint distribution of link travel times $\tilde{C} = \{C^r_{jk,t}, r = 1, ..., R\}$ generated by the policy-based dynamic network loading model. The method of generating the choice set will be discussed in Section II-D. Based on the relevant attributes of candidate routing policies, such as expected OD travel time and travel time standard deviation, a logit choice model corrected for the overlapping of different routing policies (Gao [7]) outputs policy splits f among the routing policies for each OD pair and each departure time.

$$f = U(G, \tilde{C})$$

We keep the "large sample" assumption and assume policy splits are equal to corresponding policy choice probabilities. Note that we use "splits" rather than "flows" here: policy splits are deterministic, while policy flows could be stochastic, if the demand is stochastic. Policy splits will be translated into policy flows in the network loading model. The notion of policy flow can be understood as a generalization of path flow. Since a routing policy will manifest itself as a specific path for a given realization of link travel times, a policy flow will become a path flow for each support point of link travel times. Thus a policy flow can be viewed as a set of path flows, each with some probability.

B. Policy-Based Dynamic Network Loading Model

The demand is then loaded onto the network according the policy flow splits, by the policy-based dynamic network loading model. The stochastic demand and supply play their roles in the loading process. For each support point of the random demand and/or supply, the network loading model outputs a single realization of the link travel time distribution. The loading is deterministic, given a support point of demand and/or supply, and thus any existing network loading method can potentially be extended to carry out the policy-based loading (Gao [7]). Therefore through the loading, we obtain the PMF of link travel times from the PMF of demand/supply. Note that although the input demand/supply support points are distinct from each other, the output link travel time realizations are not necessarily distinct. This is why the word "realization" is used here, rather than support point. Nevertheless, the PMF of link travel times is still expressed through the R realizations with the corresponding probabilities.

$$\tilde{C} = L(f, \tilde{D}, \tilde{S})$$

C. Optimal Routing Policy Algorithm

The routing policy generation algorithm then takes as input the link travel time distribution and produces an optimal routing policy for each destination, which again will be used to generate the choice set for the users' policy choice model.

$$\mu_i = O(\tilde{C})$$

$$G = G \cup \mu_i$$

The two equations can be combined as

$$G = G(\tilde{C}).$$

What follows is a summary of optimal routing policy problems described in Gao and Chabini [9]. Let $G = (N, A, T, P)$ be a **stochastic time-dependent network**. N is the set of nodes and A is the set of links. The number of nodes and links are denoted respectively as $|N| = n$ and $|A| = m$. The network has a single destination node d. T is the set of time periods $\{0, 1, ..., K-1\}$. Travel time on each link (j, k) during each time period t is a random variable $\tilde{C}_{jk,t}$ with finite number of discrete, positive and integral support points. Beyond time period $K-1$, travel times are static and deterministic, i.e. the travel time of link (j, k) at any time $t \geq K-1$ is equal to $C_{jk,K-1}$.

P is the probabilistic description of link travel times. Let $P = \{v_1, v_2, ..., v_R\}$ be the set of support points of the link travel time distribution. The rth support point has a probability p_r, and $\sum_{r=1}^{R} p_r = 1$. $C^r_{jk,t}$ is the travel time on link (j, k) at time t for the rth support point.

We assume the traveler knows *a priori* the probabilistic description P of the network. The traveler can make decisions only at nodes. The decision is what node k to take next, based on the *current state* $x = \{j, t, I\}$, where j is the *current node*, t is the *current time*, and I is the *current information*. Current information I is defined as a set of available realized link travel times at the current time and current node that are useful for making inferences about future link travel times. It represents the traveler's knowledge about the network conditions. This knowledge could be dependent on time, location of the traveler, mode of transportation, etc. Current information I therefore should be regarded as $I(j, t)$, but we usually use I only since I is always associated with a state where j and t are well defined. An ideal case is when travelers have perfect online information, where all link travel time realizations up to the current time are available, but generally the information is local, e.g. one learns the travel time realization of some downstream links when he/she passes a Variable Message Sign (VMS). One can be in many different states traveling in the stochastic time-dependent network, and we have the following definition.

Definition 2.1 (Routing Policy): A routing policy $\mu(x)$ is a mapping from network states to decisions (next nodes specifically).

This definition indicates that the routing decision in a stochastic time-dependent network is far from being set *a priori*. Rather, it is closely related to the network conditions, and this notion is critical in any ATIS application. The generic optimality condition for optimal routing policy problems and an operational algorithm for the perfect online information variant can be found in Gao and Chabini [9], where an optimal routing policy is defined as a routing policy that minimizes the expected travel time from any initial state to a given destination. Other optimization criteria, such as reliability and expected schedule delay are discussed in Gao [7].

D. Policy-Based Equilibrium

The three components interact with each other, and a fixed-point formulation of the policy-based equilibrium can be derived based on the interaction.

$$\tilde{C} = L\left(U\left(G\left(\tilde{C},\right), \tilde{C}\right), \tilde{D}, \tilde{S}\right) \tag{1}$$

The equilibrium can be described by the following generalized Wadrop's First Principle: a traffic network is in policy-based stochastic dynamic equilibrium, if each user follows the routing policy with minimum perceived disutility at his/her departure time, and no user can unilaterally change routing policies to improve his/her perceived disutility.

The idea of the solution algorithm is to find a solution to the fixed point problem (Equation 1) by an iterative process on policy splits. At each iteration, the policy splits are updated by combining the results from the current iteration and previous iterations. Since no proof of convergence is available at this moment, the method is heuristic for the DTA problem. The algorithm is presented as follows:

Policy-Based Stochastic DTA Heuristic

Step 0 (Initialization)
0.1: N = maximal number of iterations;
0.2: MSA counter $i = 1$
0.3: $C^r_{(0)}$ = free flow link travel times, $r = 1, ..., R$
0.4: Policy choice set $G_{(0)} = \{paths\}$
0.5: Policy splits $f_{(0)} = 0$
Step 1 (Main Loop)
1.1: Generate an optimal routing policy $\mu_i = O\left(\tilde{C}_{(i-1)}\right)$
1.2: Choice set update $G_{(i)} = G_{(i-1)} \cup \{\mu_i\}$
1.3: Users' choice model $f' = U\left(G_{(i)}, \tilde{C}_{(i-1)}\right)$
1.4: MSA update $f_{(i)} = (1-\alpha) f_{(i-1)} + \alpha f'$, where $\alpha = 1/i$
1.5: Loader $\tilde{C}_{(i)} = L\left(f_{(i)}, \tilde{D}, \tilde{S}\right)$
Step 2 (Stopping Criterion)
If $i = N$, STOP
Otherwise, $i = i + 1$, and go to Step 1

A reasonable value for the maximum number of iterations will be obtained by running the heuristic for a sufficiently large number of iterations and observing the convergence property. Experimental results on this topic will be presented in the next chapter. In Step 0.4, we initialize the policy choice set to include all paths that would have been included in a choice set for a path-based DTA model. Note that for each OD pair and each departure time, there is a choice set, and the initialization is done for all choice sets. The subscripts for OD pair and departure time are omitted to avoid heavy notation. In Step 0.5, we initialize policy splits to be zeros for all OD pairs and departure times. These are infeasible policy splits, and the initialization is just for the convenience of writing a formula in Step 1.4. $f_{(0)}$ is not taken into account in the MSA update, as when $i = 1$, its coefficient is zero.

III. Computational Tests

A. Comparison of Four Models

The motivation for the policy-based DTA model is to be able to model users' adaptive choices and analyze the effects of online information in a truly stochastic network. We develop four models for comparison purposes as shown in Table I.

	Base	Path	Online Path	Policy
Distributions of demand/supply	No	Yes	Yes	Yes
Online information	No	No	Yes	Yes
Optimal online choice	No	No	No	Yes

TABLE I
FOUR EQUILIBRIUM MODELS

In each column, we have one equilibrium model: base model, path model, online path model, and policy model, respectively. We have three features listed: distributions of demand/supply, online information, and optimal online choice, which specify respectively whether distributions of random demand/supply are considered, whether online information is utilized in routing decision making, and whether online information is utilized optimally by applying the optimal routing policy algorithm developed in Gao and Chabini [9]. We elaborate on the models one by one.

The first model is the base equilibrium model. It is an assignment model in a deterministic network with deterministic demand. It ignores stochastic disturbances in supply, e.g. assumes no incidents at all in a network. On the other hand, the demand is set at its expected value, if any stochasticity in demand exists. This corresponds to the case where users have no idea about the incident at all and just follow their habitual paths in a normal network. After the equilibrium path flows are obtained, they are loaded onto the true network with stochastic demand and supply, and the resulting measures of effectiveness are calculated.

The second model is the path based model with equilibrium in distribution. In this model, the distributions of both demand and supply are known and are used in the assignment. We seek equilibrium in the distribution of link travel times. Users are assumed to take paths with minimum expected travel time. We emphasize that a path is a fixed set of concatenated links. If a user follows a path, then s/he will traverse this set of links one by one, regardless of any online information. Note that online information includes information at the origin node and it should not be restricted to information collected *en route*. Basically it is any information beyond the *a priori* knowledge about the distribution of link travel times.

The third model is the online path based model with equilibrium in distribution. It makes use of online information as compared to the previous model. With the equilibrium link travel time distribution, a user makes routing decisions as follows. For any given state, i.e. node, time and current information, the conditional link travel time distribution is obtained and a path with minimum expected travel time is sought. The user then takes the first link of the path. When s/he arrives at the next node with an arrival time and updated online information, a new conditional distribution is obtained with a new minimum expected time path. The user continues on the first link of this new path. The above steps are repeated until the destination is reached. We remark that the outcome of the process is also a routing policy, in the sense that it is a mapping from any state to a next node. It is just that the routing policy is not generated optimally, as each

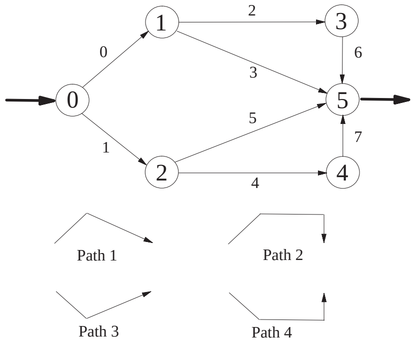

Fig. 2. Test Network

	Link 0(1)	Link 2(4)	Link 3(5)	Link 6(7)
Length (mi)	0.5357	0.7576	0.8470	0.3788
# of Lanes	2	1	1	1
Free Flow Speed (mph)	40	30	20	30
Free Flow Time (sec)	48	91	152	45
Jam Density (veh/link/meter)	0.30	0.15	0.15	0.15
Output Capacity (veh/link/sec)	1.1	0.5	0.5	0.5

TABLE II
LINK DATA OF THE TEST NETWORK

decision is made assuming that no further information will be available. We term a routing policy generated in the above stated process as “online path”. On the other hand, since the information is updated quite often (at the same pace as in an optimal routing policy algorithm), the online path could be a good approximation to an optimal routing policy.

The last model is the optimal routing policy based model with equilibrium in distribution. It is different from the online path model, in the sense that it makes optimal use of online information. We note that the routing decisions in both models are link based, in the sense that only a next link is chosen at each decision node. However, the attractiveness or utility of a link is evaluated differently in these two models. In the online path choice model, the utility of a link is based on only one path; while in the optimal routing policy model, the utility of a link is based on a set of paths that have this link in common. Intuitively the second approach should lead to better decisions.

By comparing results of the base model and the path model, we can study the value of *a priori* information on stochasticity of demand/supply, as the base model ignores the stochasticity of demand/supply while the path model makes use of the *a priori* knowledge on distributions of demand/supply. By comparing results of the path model and the last two models (online path model and policy model), we can study the value of online information. Finally, by comparing results of the online path model and policy model, we can study the value of making optimal use of online information.

B. Experimental Design

1) The Test Network: We conduct computational tests on the simple hypothetical network shown in Figure 2. The network has 6 nodes and 8 directed links. It is symmetric with respect to the horizontal line passing through nodes 0 and 5. The link data is summarized and shown under the network.

We deal with one OD pair between node 0 and node 5. We assume zero flows between any other OD pair. Four paths exist for OD pair (0,5) as shown in Figure 2, with online diversion possibilities at nodes 0, 1 and 2. The study period is from 6:30am to 8:00am. The time resolution is 1 minute for the optimal routing policy algorithm and users’ behavior model. The loader works at a finer resolution (5 sec) for the simulation, but the post-processed link (path) travel times are also by minute. Therefore we have 90 time periods in the tests.

2) Random Incidents: We have random incidents in the network. An incident is defined by the segment ID, start time, duration and capacity reduction factor. A segment is part of a link, and a link can be composed of one or multiple segments. In our network, each link is composed of only one segment. If an incident starts from 8:00am and lasts for 20 minutes with a capacity reduction factor 0.5 on link 0, then the output capacity of link 0 will be $0.5 \times 1.1 = 0.55$ veh/link/sec from 8:00am to 8:20am, and will revert to the original value 1.1 veh/link/sec from 8:20am on. As the capacity reduction is with respect to output capacity, an incident could only happen at the end of a link.

The random incident is defined as follows.

- There is at most one incident during the study period for any given day;
- The incident has a positive probability of occurring on link 0, 2, 3 and 6, but zero on links 1, 4, 5 and 7, which simulates a situation where some links are much safer than others, and thus incident probabilities on those links are negligible;
- The probability of incident occurrence on a link is proportional to the link’s length (for links 0, 2, 3 and 6);
- If an incident occurs on a link, the start time can be 6:30am, 6:40am, 6:50am, ..., 7:50am with equal probability;
- The duration of any incident is fixed at 10min, and the capacity reduction factor is fixed at 0.3;
- The probability of no incident in the network is $1 - p$.

Based on the above description, the random incident can be described by the joint distribution of link ID l and start time t_0. Denote l_0, l_2, l_3, l_6 as the length of link 0, 2, 3 and 6 respectively and $L = \sum_{i=0,2,3,6} l_i$.

$$(l, t_0) = \begin{cases} (0, \text{6:30 or 6:40 ... or 7:50}), & w.p.\ p \times l_0/L/9 \\ (2, \text{6:30 or 6:40 ... or 7:50}), & w.p.\ p \times l_2/L/9 \\ (3, \text{6:30 or 6:40 ... or 7:50}), & w.p.\ p \times l_3/L/9 \\ (6, \text{6:30 or 6:40 ... or 7:50}), & w.p.\ p \times l_6/L/9 \\ (\text{non-exist, non-exist}), & w.p.\ 1 - p \end{cases}$$

3) Demand: We assume that the demand for OD pair (0, 5) is always deterministic. The flow rate is 2880 veh/hour between 6:30am and 7:00am, and 4680 veh/hour between 7:00am and 8:00am.

Users are assumed to minimize expected travel time with perception errors. The coefficient of expected travel time is negative with a large enough absolute value (-6.0) to approximate a fastest policy (path) choice situation. All users have perfect online information in the online path model and policy model, i.e. knowledge of travel time realizations on all links up to the current time. Obviously, users have no online information in the base model and the path model.

C. Results

We discuss the solutions of the four models and compare them when appropriate. For the sake of brevity, not all results are presented. Note that we focus on the statistics collection period 7:00am through 7:30am, although statistics for all time intervals are presented. Special caution should be taken when reading statistics close to 8:00am, as there are unfinished trips during that period and the calculation of travel times could be mistaken.

We present the equilibrium OD travel time distribution of the path model (dashed lines) and the online path model (solid lines) as a function of departure time for all 37 support points in Figure 3. Each plot in the figure is of the 37 discrete supporting points for the distribution of OD travel times. The x-axis represents departure time, while the y-axis represents OD travel time. Recall that in support points 1 through 9, the incident is on link 0 and with 9 different start times from 6:30am to 7:50am. Then in support points 10 through 18, the incident is on link 2; in support points 18 through 27, on link 3; and in support points 28 through 36, on link 6; and finally in support point 37, there is no incident in the network. The incident link ID and incident start time are listed on the top of each graph in the figure. Generally, the online path model gives lower OD travel time, and the savings are quite outstanding in some cases (e.g. when incidents are on link 4 and start from 7:00 and 7:10). This is largely due to the flexibility gained through adaptive routing. Figure 4 gives time-dependent path flow distributions for path 2, and we can see flows on path 2 change from different support points, while in a path model, path flows are fixed across support points. We omit the presentation of other path flow distributions for the sake of brievity. When an incident happens, affected links will have longer travel times. Furthermore, at different stages of an incident, the realized link travel times so far are different. For example, if we are at a point when an incident just begins, then link travel times along the time axis would be flat at normal values and then jump to higher values. If we are at a point when an incident just ends, then we would see a longer period during which link travel times are at high values. If we are at a point when an incident has ended for a while, then we would be able to see link travel times first increasing and then decreasing. To sum up, the message contained in the current realized link travel times makes us adaptive to incidents. A very distinctive feature of the flows in Figure 4 is the decrease around incident across all support points. As we arrange the graphs by the start time of incident, we can see a moving "pit" in path flow. This is more intelligent than deterministic path flows as in path model. Note that policy flows are deterministic. As a policy will manifest itself as different paths in different incident support points, path flows are random and we can talk about their distributions.

The OD travel time distribution and path 2 flow distribution of the policy model are very simliar to those of the online path model, respectively, and thus are not presented here. In fact, these two models are both based on routing policies, and it is just that the methods of generating optimal routing policies are different. We expect that results from the two models are not significantly different in our simple test network, due to the limited diversion nodes. Further computational tests on larger networks are desirable to study the differences between these two models.

Next we compare expected OD travel times from all the four models in Figure 5. Expected OD travel time is the major measure of effectiveness in our tests. We observe that the path model gives lower expected OD travel times than the base model, and the two adaptive models (online path model and policy model) provide further travel time savings. Figure 6 gives the time-dependent OD time standard deviations. Although travelers are minimizing expected travel time only, their travel time variances are also reduced by taking adaptive routing choices. This is due to the fact that their travel times are reduced in incident scenarios, and thus more smooth across support points.

We just discussed in detail the results for a specific test setting (incident probability $p = 0.9$). We are also interested in learning the behavior of the models when we vary the incident probability. On the other hand, in reality online information could be provided only to part of the travelers, thus it is desirable to study how the traffic conditions change as a function of market penetration of online information. We define a single measure of effectiveness (MOE) to be compared in the sensitivity analysis, which is the expected OD travel time averaged over the statistics collection period: 7:00am through 7:29am.

First we carry out the sensitivity analysis with respect to incident probability p. We vary p from 0 to 1.0 by a step size of 0.1. The result is plotted in Figure 7. For each of the models, the average expected OD travel time increases as incident probability increases, but different model has different increasing rate. This increasing function seems intuitively correct, as a more likely incident increases the probability that a network is congested, and thus a higher expected travel time. We note that the policy model gives a higher value for $p = 0.9$ (216.06) than for $p = 1.0$ (216.00). We believe that this difference is too small to be significant, and are inclined to believe that they are the same.

The relationship for the base model is linear. The explanation is as follows. First, the path flows are the same for various incident probabilities, since the base model does not consider incidents at all. Then the OD travel time for each incident support point is calculated, and a weighted average is taken to obtain the expected OD travel time, where the weight is the probability of an incident support point. As we can see from the design of incident distribution, incident probabilities are linear functions of p. Therefore the expected OD travel time is also linear function of p. While in other three models, random incidents are considered in the equilibrium process and equilibrium path (policy) flows differ when p differs. Therefore the relationship is in general nonlinear.

In general, the path model gives less expected travel time than base model, and the two adaptive models (online path model and policy model) give less expected travel time than the path model. The savings (path over base, and adaptive over path) increase as incident probability increases, both in absolute values and in relative percentage savings. The relative saving of the path model over the base model is in the range of $0 \sim 2.9\%$, and the relative saving of adaptive models over the path model is in the range of $0 \sim 4.4\%$. This increasing function suggests that values of both *a priori* and online information are more evident when traffic conditions are worse. This could be reasonable in reality when traffic conditions without incident are not too congested, as then there is enough room for diversion. This is actually the setting of our tests, as traffic is almost in free flow state with no incident. We expect that when a network is already quite congested without incident, this function might become flat after some point.

Next we carry out sensitivity analysis with respect to market penetration of online information. For a given penetration k which is a value between 0 and 100%, we assign k of the demand to take minimum expected travel time routing policies, while the remaining $1 - k$ of the demand to take minimum expected travel time paths. Equilibrium is sought by an MSA heuristic that updates the path splits and policy splits simultaneously. We have the result for $p = 0.1$ in Figure 8. The average expected OD travel time is at its largest value when market penetration of online information is zero. At that time, if one traveler is intelligent enough and take a routing policy rather than a path, he/she can save travel time. More and more of them find the benefits of online information, and they gain travel time savings and thus bring down the average expected travel time. However, in a congested traffic network, the changing of users' behavior changes the network-wide traffic conditions through interaction between supply and demand. As seen from the figure, the saving in travel time becomes less evident when penetration goes from 20% to 40% and from 40% to 60%. Later on, higher penetration actually does not bring any more savings. We see an increase in travel time from 60% to 100%. We then conclude that the savings gained from online information is larger when market penetration is lower. After some point, more online information could actually make things worse. Therefore the function of travel time saving against market penetration is not monotonic. Despite the varying effect of online information, travel time savings are always positive with online information, compared to no-online-information case. This analysis might only be valid for the test setting, and caution should be taken if one intends to generalize the result. Future research will include market penetration tests with different incident probablities.

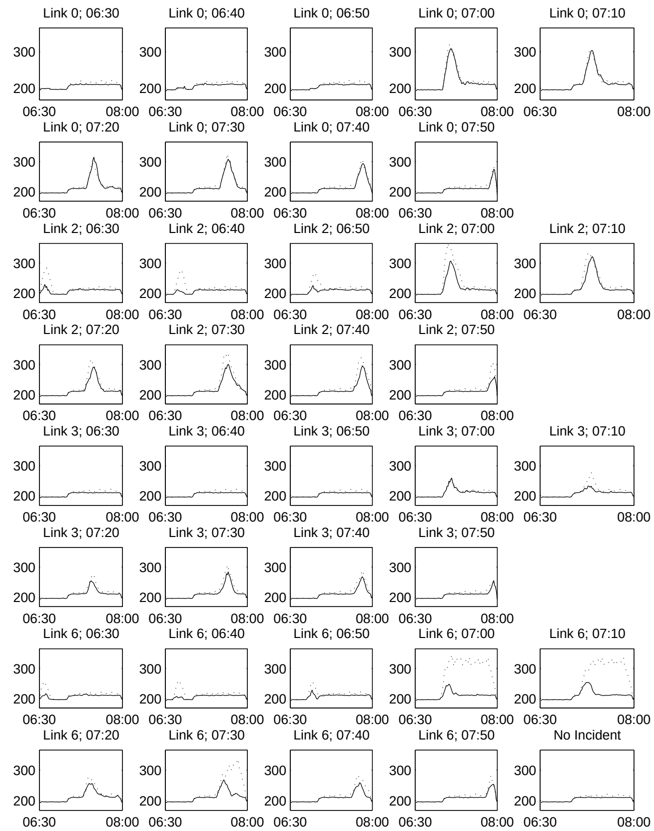

Fig. 3. OD Travel Time Distribution of Online Path Model (X-Axis: Departure Time; Y-Axis: OD Travel Time (sec); $p = 0.9$)

IV. CONCLUSIONS

This paper establishes a policy-based dynamic traffic assignment model for the analysis of effects of online information in stochastic dynamic traffic networks. The distinctive feature of the proposed model is the ability to model travelers' adaptive routing choices based on online information. Computational tests are carried out in a hypothetical network, where random incidents are the source of stochasticity. System costs derived from four models with different information accessbility situations are compared. The adaptiveness to online information leads to less expected travel time and variance at equilibrium. The value of online information is an increasing function of the incident probability. Travel time savings are high when market penetrations are low. However, the function of travel time saving against market penetration is not monotonic.

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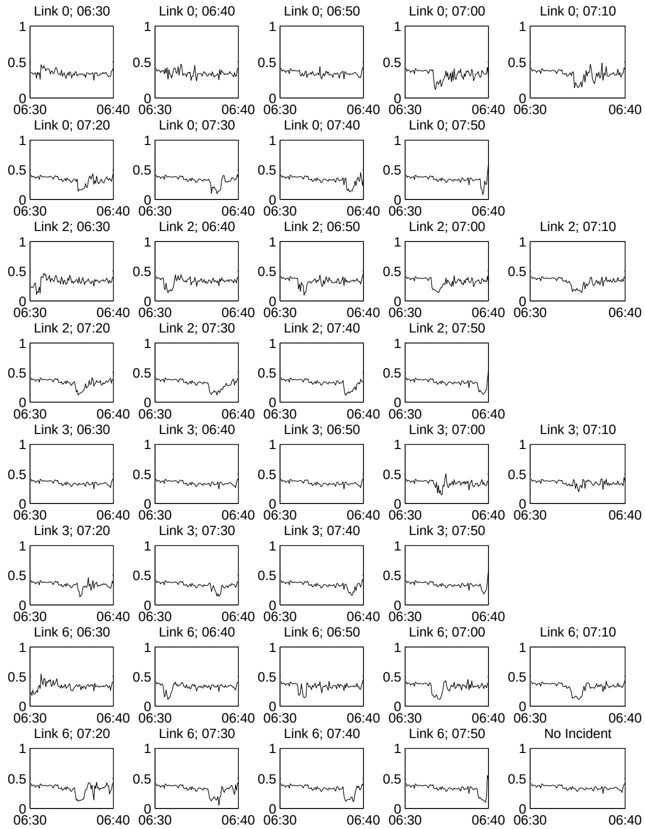

Fig. 4. Path 2 Flow Distribution of Online Path Model(X-Axis: Departure Time; Y-Axis: Path Share; $p = 0.9$)

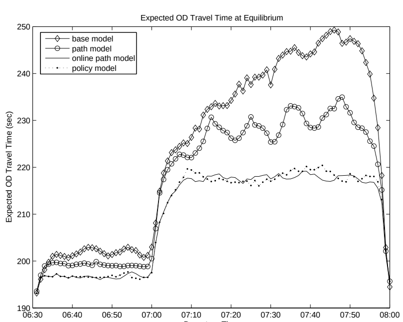

Fig. 5. Expected OD Travel Time at Equilibrium of All Four Models ($p = 0.9$)

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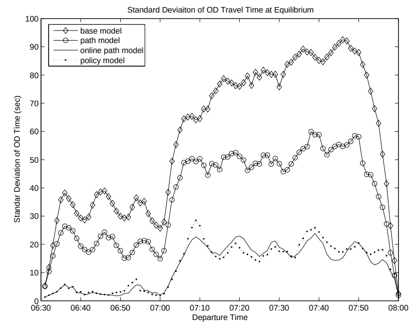

Fig. 6. Standard Deviation of OD Travel Time at Equilibrium of All Four Models ($p = 0.9$)

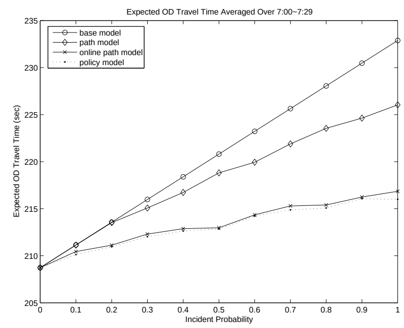

Fig. 7. Average Expected OD Travel Times as Functions of Incident Probability

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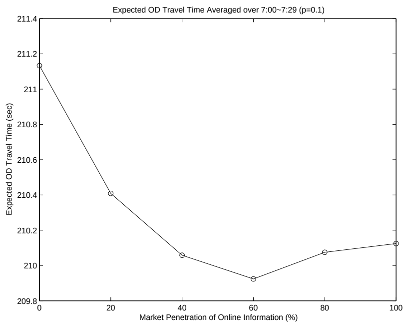

Fig. 8. Average Expected OD Travel Times as Functions of Market Penetration ($p = 0.1$)

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