Investigating the reliability of the o-d matrix correction procedure using traffic counts

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Abstract

The estimation/correction of the o-d matrix from traffic counts is a classical procedure usually adopted in transport engineering by practitioners for improving the overall reliability of transport simulation models. Recently, Marzano and Papola (2006) has shown as this procedure is generally not able to effectively correct the o-d matrix through laboratory experiments. This result can be justified from a theoretical standpoint because of the lower number of (stochastic) equations (equal to the number of independent observed link flows) with respect to the number of unknowns (equal to the number of o-d pairs). Indeed, the paper confirms firstly that this circumstance represents the main reason of failure of this procedure, by showing that a very good correction is generally obtained when the number of equations is greater than the number of unknowns. Then, since this does not happen usually in practice, (being the number of o-d pairs usually much grater than the number of link counts), the paper explores alternative application fields allowing for a proper balance between unknowns and equations. This can be achieved by moving to within-day dynamic contexts, wherein a much larger number of equation is generally available (i.e. traffic counts for each time slice within the modelling period). Obviously, in order to bound the corresponding increase in the number of unknowns (i.e. o-d flows for each time slice), specific reasonable hypotheses in o-d flow variation across time slices must be introduced. In that respect, the paper analyzes the effectiveness of the o-d matrix correction procedure in the usually adopted linear hypothesis on the dynamic process evolution of the o-d flows and under the assumption of constant distribution shares.

1. Introduction

Transport systems planning is usually based on the application of systems of models, whose reliability and goodness-of-fit strongly influence the results and the quality of the planned/designed interventions. The reliability and effectiveness of these model systems should be achieved through a disaggregate estimation of each model component (i.e. supply, demand and assignment) and can be checked through an overall validation, based on comparisons between model outputs and corresponding observed measures (normally link flows). Mainly in virtue of the inherent approximation of each model component, this aggregated validation generally fails and, therefore, the observed measures are used to correct part of the model trying to improve its reliability. This correction normally involves the o-d matrix.

After this correction, the model system would need new data for a further validation, normally either part of the data used for model correction (hold-out sample) or data related to future scenarios (before and after study) wherein the prediction reliability of the model system can be directly observed (Cascetta et al. 2005). For a number of reasons - mainly the lack of data - this important validation is normally not carried out and therefore the model reliability is almost entirely addressed by means of the model correction through traffic counts. Moreover, this procedure is so widely applied and trusted in practice that researchers and practitioners often adopt sub-models already estimated in different contexts, therefore leading to a further approximation in the model system.

In spite of that, a systematic analysis of to what extent this procedure is able to provide for an effective correction of the whole model system and consistently guarantee its forecast reliability has not been carried out in the literature yet. Therefore, this paper reports the results of a research project focused on a thorough investigation of the reliability of the methods for o-d matrix correction through laboratory experiments.

Some preliminary results were reported in Papola and Marzano (2006), who analysed the static un-congested context by means of a set of laboratory experiments wherein a demand matrix, a supply and an assignment model as well as the corresponding link flows (i.e. resulting from demand assignment to the network) are all assumed to be "true", i.e. given a true o-d matrix $\mathbf{d}_{true}$, the corresponding true link flows $\mathbf{f}_{true}$ are determined through a SNL-Probit assignment. This allows carrying out a series of experiments wherein both the true o-d matrix and the whole set of (observed) unbiased link flows are available for the o-d matrix correction. In more detail, the following experiments were carried out: (a) given a specific perturbation of the true o-d demand (mimicking both generation and distribution demand biases), checking the capability of different subsets (included the whole set) of link flows to reproduce the starting true demand through the GLS estimator; (b) the same as the preceding point plus introducing a random perturbation of link flows so as to mimic assignment/sampling errors. In general, results of all experiments showed the correction not to be satisfactory, as briefly reported in the next section. On the other hand, this result can be easily justified because from the theory it is well known that in a standard application of the o-d correction procedure through traffic counts, the number of (stochastic) equations (equal to the number of independent observed link flows) is generally much lower than the number of unknowns (equal to the number of o-d pairs).

The basic idea of this paper is therefore checking whether this may be regarded as the only/main reason of the o-d correction procedure failure and, if so, providing for an investigation of possible effective applications of this procedure. For example, in a dynamic context a much larger number of equation is generally available (i.e. traffic counts for each time slice within the modelling period). Obviously, the number of unknowns theoretically increases with the same law (i.e. o-d flows for each time slice) but it is worthy exploring whether reasonable rules can be assumed in the o-d flow variation among time slices so as to bound the number of unknowns (with respect to the increase in the number of equations) and therefore obtaining an effective correction of the time slices o-d matrices.

In accordance with that, section 3 deals with the effectiveness of the GLS estimator for the static un-congested case for different values of the ratio r between number of unknowns and number of equations. Results show that a very good correction is generally obtained when $r \leq 1$ and vice versa for $r > 1$. The possibility of interesting possible applications in the within-day dynamic context, wherein r can be handled so as to be close to one as mentioned above, is investigated in section 4, where some possible o-d flow variation laws between time slices are proposed so as to generate case studies with $r \leq 1$. For these case studies, laboratory experiments similar to the static case are carried out, together with an analysis of how effective can be the o-d correction procedure on real networks (i.e. with number of o-d pairs much greater than the number of link flows) if some assumptions on the o-d flows variation between time slices were true. Section 5 summarises research outcomes and points out further research developments.

2. Literature review

The estimation/correction of the o-d matrix from traffic counts is a classical problem of transport engineering. Most of the studies proposed in the literature can be classified, according to their theoretical approach, either in the "classical" framework, i.e. the Maximum Likelihood (ML) estimator proposed by Maher (1983) and Bell (1983) and the Generalised Least Squares (GLS) estimator proposed by Cascetta (1984), or in the "Bayesian" framework proposed by Maher (1983). Following Cascetta and Nguyen (1988) and Cascetta (2001), classical estimators provide for a Maximum Likelihood estimate $\mathbf{d}_{ML}$ of the demand vector by maximizing the probability (likelihood) of observing both o-d sampling survey data and link counts (under the usually acceptable assumption that these two probabilities are independent), yielding:

$$\mathbf{d}_{ML} = \max_{\mathbf{x}\geq 0}\left[lnL(\hat{\mathbf{d}}/\mathbf{x}) + lnL(\hat{\mathbf{f}}/\mathbf{x})\right] \tag{1}$$

wherein $\mathbf{x}$ is the variable demand, $\hat{\mathbf{d}}$ is the demand by sample and $\hat{\mathbf{f}}$ the vector of link counts. Log-likelihood functions in equation (1) are specified on the basis of hypotheses on the probability distribution of demand counts $\hat{\mathbf{d}}$ and traffic counts $\hat{\mathbf{f}}$ respectively, conditional on the demand vector $\mathbf{x}$. Normally, traffic counts can be assumed as independently distributed as Poisson random variables, or following a Multivariate Normal random variable, while the statistical distribution of o-d demand counts depends on the sampling strategy. Generalized Least Squares (GLS) demand estimation $\mathbf{d}_{GLS}$ provides for an estimate of the o-d demand flow, starting from a system of linear stochastic equations, leading to the following optimization problem:

$$\mathbf{d}_{GLS} = \min_{\mathbf{x}\geq 0}\left\{\frac{1}{2}\left(\mathbf{x}-\hat{\mathbf{d}}\right)^T \mathbf{Z}^{-1}\left(\mathbf{x}-\hat{\mathbf{d}}\right) + \frac{1}{2}(\hat{\mathbf{f}}-\mathbf{M_f}\mathbf{x})^T \mathbf{W}^{-1}(\hat{\mathbf{f}}-\mathbf{M_f}\mathbf{x})\right\} \tag{2}$$

where $\mathbf{M_f}$ is the sub-matrix of the assignment matrix related to links with available traffic counts and $\mathbf{Z}$ and $\mathbf{W}$ the covariance matrices related to the sampling error underlying the demand estimation and the measurement/assignment errors respectively.

Bayesian methods estimate unknown parameters by combining experimental information (traffic counts in this case) with non-experimental information (a priori or "subjective" expectations on o-d demand, e.g. coming from an out-of-date estimation or from a model system), by maximizing the logarithm of the a posteriori probability:

$$\mathbf{d}_B = \max_{\mathbf{x}\in S}\left[ln\, g(\mathbf{x}/\mathbf{d}^*) + lnL(\hat{\mathbf{f}}/\mathbf{x})\right] \tag{3}$$

wherein $g(\mathbf{x}/\mathbf{d}^*)$ expresses the distribution of subjective probability attributed to the unknown vector given the a priori estimate $\mathbf{d}^*$ and $L(\hat{\mathbf{f}}/\mathbf{x})$ expresses the probability of observing the vector of traffic counts $\hat{\mathbf{f}}$ conditional on the unknown demand vector $\mathbf{x}$. Again, the detailed specification of a Bayesian estimator depends on the assumptions made about the probability functions $g(\mathbf{x}/\mathbf{d}^*)$ and $L(\hat{\mathbf{f}}/\mathbf{x})$. Normally, the unknown demand vector can be assumed to follow a multinomial random variable (in this case $\ln g(\mathbf{x}/\mathbf{d}^*)$ becomes the entropy function of the unknown vector $\mathbf{x}$), a Poisson random variable (in this case $\ln g(\mathbf{x}/\mathbf{d}^*)$ becomes the information function of the unknown vector $\mathbf{x}$), or a Multivariate Normal random variable.

Moreover, within this framework, a number of generalizations have been carried out. For instance, Bell (1991) explored further theoretical properties of the GLS method. Yang et al.

(1992) dealt with the hypothesis of congested network, incorporating o-d estimation and traffic assignment feedbacks in the correction procedure; this problem has been eventually studied as a bi-level optimization problem, among others, by Florian and Chen (1995), Yang (1995) and Cascetta and Postorino (2001). Lo et al. (1996) introduced an explicit representation of the stochastic nature of observed flows, eventually generalized by Vardi (1996); Lo et al. (1999) describe an optimization method for the application of this approach to large-scale networks. A further generalization is proposed in Lo and Chan (2003), who proposed a procedure for the simultaneous estimation of o-d matrix and route choice dispersion parameter for congested networks. Hazelton (2000) proposes a method which can also make use only of link counts, but it requires explicit path enumeration and is therefore practically strong time-requiring for large-size networks. Finally, as pointed out by and Hazelton (2003), a promising research development deals with considering time-series link counts (e.g. referred to several days) as a key aspect for improving the reliability of o-d matrix estimation. In more detail, he took into account the covariance matrix of link count observations taken on several days within the estimation procedure, showing its reliability in very small test networks.

Moving towards the dynamic framework, an extension of the static o-d correction procedure for obtaining time-varying o-d flows using time-varying traffic counts was provided by Cascetta et alii (1993), who proposed two different dynamic estimators, i.e. a simultaneous and a sequential estimator. The first jointly estimates all o-d matrices for all time slices using the whole set of traffic counts: this approach, requiring knowledge of the dynamic assignment matrix resulting from a dynamic traffic assignment (DTA) in order to map the relationship between o-d flows and traffic counts, represents a straightforward extension of the static case. The second is based on the estimation at each interval of the o-d demand $\mathbf{d}_h$ for that single interval *h*, expressing traffic counts of time slice *h* as a function of $\mathbf{d}_h$ and of the already estimated demands of previous intervals ($\hat{\mathbf{d}}_{h-1}, \hat{\mathbf{d}}_{h-2} \cdots$). This approach offers computational advantages, since reduces a large optimization problem into a number of smaller ones and gives the possibility of using the estimates for an interval as a priori estimates of subsequent ones. Such aspects have made this approach suitable for real-time estimation problem, while the simultaneous approach is usually used for off-line estimation.

Starting from the sequential framework, several formulations have been proposed in order to overtake the limitation inherent the dependence of o-d demand $\mathbf{d}_h$ only on traffic counts for the same interval. For example, Ashok and Ben-Akiva (1993) implemented an augmented state-space model where the state variables, relative to the deviation of vector demand from their historical estimates for a number of time interval, are estimated on the basis of traffic counts observed in different time slices. Bieralire and Crittin (2004) also proposed an efficient algorithm to deal with this problem. In many applications, the technique of interpreting demand estimates of a given interval as a priori estimates of subsequent slices has been replaced by linear combinations involving demand estimates related to more previous time slices. This leads to procedures mainly based on Kalman filters, assuming the within day evolution of demand as an autoregressive process and utilising a DTA model as measuring equation (Ashok and Ben-Akiva, 1993). Other studies dealt with the randomness of dynamic matrix assignment, e.g. Chang and Wu (1995) and Ashok an Ben-Akiva (2002), who in particular introduced a new estimator, including the dynamic assignment matrix or the variable which it depends on (travel time and path choice fraction) within the state-space formulation.

3. O-d matrix correction performances in static un-congested contexts

As stated in the introduction, in spite of the practical diffusion of the o-d model correction through traffic counts, few studies focused on a systematic analysis of its reliability and goodness-of-fit. To the authors' knowledge the most relevant was carried out by Di Gangi (1988) who considered a mesh network made up by 64 o-d pairs and 96 links. Two different sets of link counts, comprising respectively 8 and 24 links, were considered for o-d matrix correction using different estimators. A more systematic analysis was provided by Papola and Marzano (2006), whose results will be reviewed and enlarged in this section.

Following the approach shared by the aforementioned works, the performances of the GLS estimator (2) can be checked by means of laboratory experiments wherein a given origin-destination demand matrix $\mathbf{d}_{true}$ is assumed to be the "true" o-d matrix. The assignment, through a supposed "true" assignment model, of this matrix to the network determines a vector of "true" link flows $\mathbf{f}_{true}$. In a first step, link counts are assumed to be equal to the "true" link flows, i.e. without any perturbation (no measurement and/or assignment errors). From a practical standpoint, the hypothesis of unbiased link counts can be practically introduced in equation (2) either by means of a variance matrix $\mathbf{W}$ close to zero, i.e. with flow estimate variances small enough, or following the approach reported for instance by Cascetta et al. (2005), that is introducing link counts consistency as a constrain in the optimization problem:

$$\mathbf{d}_{GLS} = \min_{\substack{\mathbf{x}\geq 0 \\ \mathbf{M_f x}=\hat{\mathbf{f}}}} \frac{1}{2}\left(\mathbf{x}-\hat{\mathbf{d}}\right)^T \mathbf{Z}^{-1}\left(\mathbf{x}-\hat{\mathbf{d}}\right) \qquad (4)$$

solvable through a convex simplex algorithm (Zangwill, 1969). Notably, from a mathematical standpoint, when $\mathbf{Z}=\mathbf{I}$ (identity matrix), $\mathbf{d_{GLS}}$ is the (unique) projection (in the usual metric defined in $\mathbb{R}^{|\mathbf{d}|}$ where $|\mathbf{d}|$ is the dimension of the vector $\mathbf{d}$) of the prior demand estimate $\hat{\mathbf{d}}$ on the convex subset D_f of demand vectors compliant with constraints of problem (4). Therefore, from the properties of the projector on convex subsets in a Hilbert space, it follows that the distance of the true o-d matrix from $\mathbf{d_{GLS}}$ is always not greater than the distance from the prior matrix $\hat{\mathbf{d}}$ [1]. This property obviously does not imply that all performance indicators reported in Table 1 would indicate an "improvement" of $\mathbf{d_{GLS}}$ with respect to $\hat{\mathbf{d}}$ towards the true demand.

The performances of GLS estimator are evaluated imposing specific perturbations to the true o-d matrix, in order to mimic different modelling errors. In more detail, the following perturbations are considered:

- *amplification*: all entries of the true o-d matrix are multiplied by an amplification factor, in order to mimic errors in estimating demand generation, i.e. $d_{od}^{pert} = \alpha d_{od}^{true}\ \ \forall od$;
- *row spreading*: true demand generated by each origin o is equally split among all $n_{d/o}$ (number of destinations with nonzero flow from origin o) destinations, i.e. given $d_o^{true} = \sum_d d_{od}^{true}$ each value of the o-th row is set to $d_o^{true} / n_{d/o}$. This assumption corresponds to setting infinite variance within the distribution model, collapsing into an equiprobable model;

[1] In more detail, whatever $\mathbf{d}^* \in D_f$, a property of the projector states that $\left(\mathbf{d}^* - \mathbf{d_{GLS}}\right)\cdot\left(\hat{\mathbf{d}} - \mathbf{d_{GLS}}\right) \leq 0$ where $\cdot$ denotes the scalar product generating the projection metric. Since the true demand vector lies within D_f, from the preceding inequality it is trivial to recognize that $\mathbf{d_{GLS}}$ is always closer to the true demand vector than $\hat{\mathbf{d}}$.

- *amplification plus row spreading*: this perturbation, given by the combination of the preceding perturbations, allows simulating simultaneously errors both in demand generation and distribution;
- *random*: all entries of the true o-d matrix are independently drawn for a normal distribution (truncated to nonnegative values) with mean d_{od}^{true} and variance βd_{od}^{true}, in order to mimic randomly distributed errors in o-d flow estimation;

The distance between the true o-d matrix $\mathbf{d}_{\mathrm{true}}$ and the corrected o-d matrix $\mathbf{d}_{\mathrm{corr}}$ are measured through well-known performance indicators, reported in the following Table 1.

[Table 1]

The same indicators have been also applied in order to measure the distance between link counts and simulated link flows, where appropriate. Moreover, in order to provide for a measurement reference, those indicators have been also computed for the initial perturbations imposed on demand and/or link counts. It is also worthy noting that the covariance matrix **Z** in equation (4) has been explicitly defined only for the random perturbation, while it has been assumed equal to the identity matrix for the other perturbations.
The first experiments are carried out under the hypothesis of using all link counts for the o-d matrix correction: this does not correspond to a real situation but can provide for a first check of the reliability of the GLS estimator. In more detail, Table 2 reports the results achieved by Papola and Marzano (2006) on an 870 o-d pairs 208 links mesh network, leading to a ratio *r* between unknown and independent equations equal to 4.94 (32 dependent equations).

[Table 2]

It is immediate recognizing that the correction performances are always very poor under all kinds of perturbations. Worse results are obviously obtained when using subset of link counts, and/or by introducing perturbations in link flows, as shown by Papola and Marzano (2006).
As mentioned in the introduction, this result can be interpreted as a consequence of the significant discrepancy between unknowns and equations, and therefore it is worthy exploring a situation wherein the ratio *r* is close to one instead. Table 3 reports the results of an experiment run on a 208 links 208 o-d pairs mesh network, so as to balance approximately the number of unknown and equations (precisely, r=1.08 due to 15 dependent equations).

[Table 3]

Thanks to the balance between unknowns and equations, results are satisfactory when starting from amplification and spreading perturbations, and correction performances tend to become poor only for randomly perturbed matrices with β>1.
This result suggests that working with *r* values close to one is a key issue for enhancing o-d matrix correction reliability. Notably, since in the static case handling much more unknowns rather than equations is a common condition, the o-d matrix correction through GLS estimator may lead to significant biases in the practice. This is obviously true even more in principle in the dynamic case, wherein the addition of a time slice in the modelling horizon provides an increase both in unknowns (o-d values for that time slice) and equations (observed flows for that time slice). As mentioned in the introduction, however, specific assumptions can be introduced in this context so as to limit the ratio *r*. This aspect will be deepened in the next

section.

4. O-d matrix correction performances in within-day dynamic contexts

As mentioned in the previous section, the difference between the number of independent equations and the number of variables is still an issue in the dynamic case, unless specific assumptions are introduced on the structure of time-varying o-d matrices so as to reduce the number of unknown variables.
Usually, these assumptions involve the dynamic evolution of o-d flows, modelling their temporal relationship or their deviations from historical estimates, by means of an autoregressive process. Alternatively, other assumptions can be introduced as well, for instance the hypothesis of constant distribution shares within the analysis horizon. Correction performances of GLS estimator will be checked in the following, under both assumptions, through laboratory experiments similar to those described in section 3.
With reference to the first assumption, true o-d profiles were generated according to the hypothesis of o-d flows following an autoregressive process of order 2, with different values of the dispersion parameters. In other terms, starting from initial o-d matrices for the first two intervals, the subsequent ones were generated from the following equations:

$$\mathbf{d_t} = A\mathbf{d_{t\text{-}1}} + B\mathbf{d_{t\text{-}2}} + \mathbf{z_t} \tag{5}$$

where $\mathbf{d_t}$ is the o-d vector at time interval t and $\mathbf{z_t}$ is the error term with $\mathbf{E}[\mathbf{z_t}] = \mathbf{0}$. In particular, matrices **A** and **B** have been assumed diagonal (r^{th} o-d flow is affected only by the two preceding r^{th} o-d flows) and the covariance matrix of $\mathbf{z_t}$ is assumed independent on time and diagonal (no correlation between error terms of different o-d cells).
In a first experiment, the covariance matrix was set equal to zero (deterministic process), then in subsequent experiments two covariance matrices were chosen and several draws of the process were generated for each case. The variances in the two covariance matrices were fixed for each o-d cell proportionally to its mean value in the first two intervals trough a dispersion parameter, chosen equal to 0.1 and 0.2 in the two experiments respectively. The true link flows are obtained from a dynamic network loading of the true o-d matrices, while perturbed o-d matrices for the first two time slices and the true evolution equation are assumed as prior information.
Notably, estimators applied in the two experiments are different. In the deterministic case, the unknowns are represented by the o-d flows belonging to the first two matrices, and the GLS estimator becomes:

$$\mathbf{d}_{GLS} = \min_{\mathbf{M}x=\mathbf{f}} \sum_{t=1}^{2} \left(\mathbf{x_t} - \mathbf{d_t}\right)\mathbf{Z_t}^{-1}\left(\mathbf{x_t} - \mathbf{d_t}\right)^T \tag{6}$$

where **M** is the overall dynamic assignment matrix and **d** and **f** the overall demand and flow vectors respectively (i.e. the vectors obtained by queuing the demand and the flow vectors related to each time slice). In the stochastic case, the unknowns are the o-d flows related to all time slices and the corresponding GLS estimator can be expressed as:

$$\mathbf{d}_{GLS} = \min_{\mathbf{M}x=\mathbf{f}} \left[\sum_{t=1}^{2} (\mathbf{x_t} - \mathbf{d_t}) \mathbf{Z_t}^{-1} (\mathbf{x_t} - \mathbf{d_t})^T + \sum_{t=3}^{n} (\mathbf{x_t} - f(\mathbf{x_{t-1}}, \mathbf{x_{t-2}})) \mathbf{Z_t}^{-1} (\mathbf{x_t} - f(\mathbf{x_{t-1}}, \mathbf{x_{t-2}}))^T \right] \quad (7)$$

where $f(.)$ is intended to be the linear stochastic equation (5). Table 4 reports the results of this experiment on a 200 o-d pairs 120 links mesh network, considering 8 time slices from the demand side, leading to 11 time slices for the supply side (i.e. further three time slices in order to carry out the network flows clearance). Consequently the number of unknowns is 400 (all the elements of the first two o-d matrices) and the number of equation is 1320 leading to r=0.30 (the actual r value is higher due to the presence of dependent equations). Notably, results are satisfactory only if the true evolution of o-d flows is close to a deterministic process.

[Table 4]

The second assumption, that is the hypothesis of constant distribution shares within the analysis horizon (i.e. an analysis horizon wherein the dynamic evolution of the distribution shares is slower than the generation), allows also handling the ratio r between unknowns and equations. In more detail, given a time interval T wherein the distribution shares are assumed constant, the number of unknowns becomes $n_T{\cdot}n_o{+}n_{od}$ being n_T the number of time slices included in T, n_o the number of origins and n_{od} the number of o-d pairs. The corresponding GLS estimator is therefore of the type (4), wherein variables are actually demand generation for each time slice and distribution shares .
The laboratory experiment in this case is based on perturbations of the distribution shares (spreading row) and/or of the generation profiles (amplification). Table 5 reports the results obtained in the network used in the previous case using all link counts. In this case the number of unknowns is 8*33+200=464 and the number of equation is 1320 with r=0.35 (by not taking into account the number of dependent equations). Notably, correction results are also compared to those obtained by relaxing the hypothesis of constant distribution shares, i.e. using a classical simultaneous estimator wherein the number of variables is 200*8=1600:

$$\mathbf{d}_{GLS} = \min_{\mathbf{M}x=\mathbf{f}} \sum_{t=1}^{n} (\mathbf{x_t} - \mathbf{d_t}) \mathbf{Z_t}^{-1} (\mathbf{x_t} - \mathbf{d_t})^T \quad (8)$$

[Table 5]

It is worthy underlining that the proposed estimator always provides very satisfactory results while the simultaneous estimator (9) exhibits poor performances, i.e. it does not recognize the underlying constancy of distribution shares.
The GLS estimator has been also applied on subsets of link counts chosen accordingly to the maximum flow selection method (Yang et al. 1998). Results are reported in Table 6, together with the outcomes of the simultaneous estimator.

[Table 6]

Notably, results are satisfactory only for values of the ratio r lower than one and get further improved for decreasing r.
Another experiment has been carried out by assuming for each time slice a true distribution share matrix obtained through random perturbations of the distribution share matrix of the

previous experiment. This mimics a more real situation wherein distribution shares are not constant but slightly variable within the analysis horizon. Results are reported in Table 7 and once again compared with those obtained by applying the simultaneous estimator (8).

[Table 7]

Once again, assuming that analyst is not perfectly able to reproduce the law underlying o-d flows variation across time slices, results become worse but are still better with respect to those obtained with the simultaneous estimator (8) for small perturbations of the distribution shares.

5. Conclusions and research perspectives

The paper dealt with a thorough investigation of the o-d matrix correction procedure by means of real-size laboratory experiments. The paper starts from the drawbacks of the static un-congested o-d correction, whose main failure has been shown to depend strictly on the ratio *r* between unknowns and equations. Then, alternative application fields allowing for a proper balance between unknowns and equations are explored. In more detail, this is achieved by moving to within-day dynamic contexts, wherein a much larger number of equation is generally available (i.e. traffic counts for each time slice within the modelling period). Obviously, in order to bound the corresponding increase in the number of unknowns (i.e. o-d flows for each time slice), specific reasonable hypotheses in o-d flow variation across time slices must be introduced. Namely, the paper analyzed the usually adopted linear hypothesis on the dynamic process evolution of o-d flows and the assumption of constant distribution shares.

Results of the within-day laboratory experiments provide for a further evidence that the key issue for an effective o-d matrix correction is handling contexts with a *r* value close to one, wherein very good o-d estimates are always obtained whatever o-d prior estimates available. This suggests that, regardless of the specific hypotheses adopted throughout the paper (to be obviously checked in their practical validity on real data), the within-day dynamic context seems to represent the only background allowing for effective o-d correction, provided the existence of real and identifiable rules describing demand evolution.

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Euclidean distance	$\sqrt{\sum_{od}\left(d_{od}-\hat{d}_{od}\right)^2}$
MSE	$\frac{1}{n_{od}}\sum_{od}\left(d_{od}-\hat{d}_{od}\right)^2$
RMSE	$\sqrt{\frac{1}{n_{od}}\sum_{od}\left(d_{od}-\hat{d}_{od}\right)^2} = \sqrt{\text{MSE}}$
MAPD	$\frac{1}{n_{od}}\sum_{od}\frac{\left\|d_{od}-\hat{d}_{od}\right\|}{d_{od}}$
MSPE	$\frac{1}{n_{od}}\sum_{od}\left(\frac{d_{od}-\hat{d}_{od}}{d_{od}}\right)^2$
RMSPE	$\sqrt{\frac{1}{n_{od}}\sum_{od}\left(\frac{d_{od}-\hat{d}_{od}}{d_{od}}\right)^2} = \sqrt{\text{MSPE}}$

Table 1 – Performance indicators

PERTURBATION			Performance indicator					
			Euclidean	MSE	RMSE	MAPD	MSPE	RMSPE
amplification	10%	initial	55	3,46E+00	1,9	10,00%	1,00%	10,00%
		final	25	7,11E-01	0,8	7,40%	1,91%	13,81%
	20%	initial	110	1,38E+01	3,7	20,00%	4,00%	20,00%
		final	49	2,75E+00	1,7	14,19%	5,97%	24,44%
	30%	initial	165	3,11E+01	5,6	30,00%	9,00%	30,00%
		final	71	5,86E+00	2,4	19,85%	9,89%	31,44%
	40%	initial	219	5,53E+01	7,4	40,00%	16,00%	40,00%
		final	93	9,89E+00	3,1	24,77%	13,65%	36,95%
	50%	initial	274	8,64E+01	9,3	50,00%	25,00%	50,00%
		final	113	1,47E+01	3,8	29,16%	17,33%	41,63%
spreading		initial	251	7,25E+01	8,5	86,81%	249,41%	157,93%
		final	224	5,76E+01	7,6	72,04%	175,73%	132,56%
ampSpreading	10%	initial	256	7,53E+01	8,7	96,01%	315,68%	177,67%
		final	224	5,76E+01	7,6	72,38%	176,98%	133,04%
	20%	initial	269	8,34E+01	9,1	106,46%	391,27%	197,81%
		final	224	5,78E+01	7,6	72,81%	178,74%	133,70%
	30%	initial	291	9,71E+01	9,9	117,79%	476,20%	218,22%
		final	225	5,83E+01	7,6	73,30%	181,01%	134,54%
	40%	initial	318	1,16E+02	10,8	130,05%	570,46%	238,84%
		final	227	5,91E+01	7,7	73,82%	183,77%	135,56%
	50%	initial	350	1,41E+02	11,9	143,18%	674,05%	259,62%
		final	229	6,01E+01	7,8	74,46%	187,03%	136,76%
random	0,25	initial	129	1,93E+01	4,4	18,90%	5,62%	23,71%
		final	109	1,36E+01	3,7	17,41%	4,81%	21,94%
	0,50	initial	276	8,78E+01	9,4	40,20%	24,94%	49,94%
		final	227	5,90E+01	7,7	36,60%	21,10%	45,93%
	0,75	initial	398	1,82E+02	13,5	56,37%	49,57%	70,40%
		final	324	1,21E+02	11,0	49,85%	38,10%	61,72%
	1,00	initial	455	2,38E+02	15,4	68,70%	69,75%	83,52%
		final	358	1,47E+02	12,1	57,52%	50,61%	71,14%
	1,25	initial	533	3,26E+02	18,1	82,67%	105,27%	102,60%
		final	395	1,80E+02	13,4	69,47%	75,43%	86,85%
	1,50	initial	697	5,58E+02	23,6	97,13%	148,28%	121,77%
		final	504	2,92E+02	17,1	78,71%	92,14%	95,99%
	1,75	initial	765	6,73E+02	25,9	107,47%	192,52%	138,75%
		final	534	3,28E+02	18,1	84,62%	118,14%	108,69%
	2,00	initial	839	8,10E+02	28,5	117,83%	233,69%	152,87%
		final	554	3,52E+02	18,8	88,74%	123,73%	111,23%
	2,25	initial	857	8,43E+02	29,0	118,98%	244,34%	156,31%
		final	573	3,77E+02	19,4	89,87%	128,95%	113,56%
	2,50	initial	1078	1,34E+03	36,5	143,28%	374,28%	193,46%
		final	621	4,44E+02	21,1	99,27%	164,56%	128,28%
	2,75	initial	1189	1,62E+03	40,3	144,54%	400,24%	200,06%
		final	653	4,90E+02	22,1	98,27%	157,17%	125,37%
	3,00	initial	1209	1,68E+03	41,0	156,21%	452,68%	212,76%
		final	698	5,60E+02	23,7	106,08%	185,88%	136,34%
	3,25	initial	1375	2,17E+03	46,6	181,55%	653,73%	255,68%
		final	726	6,06E+02	24,6	118,22%	264,87%	162,75%
	3,50	initial	1246	1,79E+03	42,3	170,94%	576,99%	240,21%
		final	708	5,77E+02	24,0	114,00%	239,69%	154,82%
	3,75	initial	1567	2,82E+03	53,1	194,28%	774,10%	278,23%
		final	809	7,53E+02	27,4	122,44%	274,89%	165,80%
	4,00	initial	1561	2,80E+03	52,9	202,34%	868,09%	294,63%
		final	774	6,89E+02	26,2	123,31%	290,15%	170,34%
	4,25	initial	1617	3,01E+03	54,8	215,43%	944,87%	307,39%
		final	773	6,87E+02	26,2	121,09%	260,92%	161,53%
	4,50	initial	1879	4,06E+03	63,7	228,88%	1164,10%	341,18%
		final	828	7,89E+02	28,1	129,16%	317,36%	178,14%
	4,75	initial	1995	4,57E+03	67,6	239,68%	1273,40%	356,84%
		final	804	7,44E+02	27,3	127,61%	320,60%	179,05%
	5,00	initial	1837	3,88E+03	62,3	224,16%	1105,40%	332,47%
		final	824	7,80E+02	27,9	129,12%	339,04%	184,13%
	5,25	initial	2138	5,25E+03	72,5	254,33%	1457,70%	381,80%
		final	888	9,07E+02	30,1	136,84%	376,89%	194,14%
	5,50	initial	2033	4,75E+03	68,9	262,69%	1481,60%	384,92%
		final	829	7,89E+02	28,1	130,81%	343,38%	185,30%
	5,75	initial	2202	5,57E+03	74,7	278,45%	1685,10%	410,50%
		final	897	9,25E+02	30,4	139,12%	402,96%	200,74%
	6,00	initial	2227	5,70E+03	75,5	276,01%	1743,00%	417,49%
		final	874	8,77E+02	29,6	136,48%	379,24%	194,74%

Table 2 – Correction results for the 870 links 208 o-d pairs mesh network (perturbation parameters α and β indicated beside perturbations)

PERTURBATION			Performance indicator					
			Euclidean	MSE	RMSE	MAPD	MSPE	RMSPE
amplification	10%	initial	1131	6,15E+03	78,5	10,00%	1,00%	10,00%
		final	90	3,87E+01	6,2	0,54%	0,01%	1,17%
	20%	initial	2263	2,46E+04	156,9	20,00%	4,00%	20,00%
		final	180	1,55E+02	12,4	1,08%	0,05%	2,33%
	30%	initial	3394	5,54E+04	235,4	30,00%	9,00%	30,00%
		final	269	3,49E+02	18,7	1,62%	0,12%	3,50%
	40%	initial	4526	9,85E+04	313,8	40,00%	16,00%	40,00%
		final	359	6,20E+02	24,9	2,15%	0,22%	4,67%
	50%	initial	5657	1,54E+05	392,3	50,00%	25,00%	50,00%
		final	449	9,68E+02	31,1	2,69%	0,34%	5,83%
spreading		initial	3112	4,66E+04	215,8	30,23%	20,90%	45,72%
		final	878	3,71E+03	60,9	5,18%	1,31%	11,45%
ampSpreading	10%	initial	3297	5,23E+04	228,6	34,70%	29,01%	53,86%
		final	877	3,70E+03	60,8	5,16%	1,31%	11,44%
	20%	initial	3797	6,93E+04	263,3	41,29%	40,03%	63,27%
		final	876	3,69E+03	60,7	5,15%	1,31%	11,43%
	30%	initial	4510	9,78E+04	312,7	49,60%	53,96%	73,46%
		final	875	3,68E+03	60,7	5,14%	1,30%	11,42%
	40%	initial	5350	1,38E+05	370,9	59,36%	70,80%	84,14%
		final	874	3,67E+03	60,6	5,13%	1,30%	11,41%
	50%	initial	6266	1,89E+05	434,5	69,69%	90,55%	95,16%
		final	873	3,67E+03	60,6	5,12%	1,30%	11,41%
random	0,25	initial	2600	3,25E+04	180,3	18,53%	5,16%	22,71%
		final	662	2,11E+03	45,9	3,48%	0,50%	7,09%
	0,50	initial	5455	1,43E+05	378,2	38,05%	22,68%	47,63%
		final	924	4,10E+03	64,0	5,44%	1,39%	11,79%
	0,75	initial	7858	2,97E+05	544,8	54,63%	44,44%	66,66%
		final	1361	8,90E+03	94,4	8,90%	3,82%	19,54%
	1,00	initial	8985	3,88E+05	623,0	65,18%	63,46%	79,66%
		final	2508	3,02E+04	173,9	13,58%	7,71%	27,77%
	1,25	initial	10786	5,59E+05	747,9	80,82%	90,27%	95,01%
		final	2469	2,93E+04	171,2	12,51%	7,21%	26,85%
	1,50	initial	13723	9,05E+05	951,5	101,02%	160,11%	126,53%
		final	2855	3,92E+04	198,0	17,29%	11,20%	33,47%
	1,75	initial	18016	1,56E+06	1249,2	117,53%	234,63%	153,18%
		final	3897	7,30E+04	270,2	18,60%	13,40%	36,60%
	2,00	initial	19362	1,80E+06	1342,5	135,58%	303,24%	174,14%
		final	2863	3,94E+04	198,5	16,72%	11,57%	34,01%
	2,25	initial	20146	1,95E+06	1396,9	122,39%	263,82%	162,43%
		final	2931	4,13E+04	203,2	15,13%	9,23%	30,37%
	2,50	initial	25341	3,09E+06	1757,1	156,98%	445,54%	211,08%
		final	2910	4,07E+04	201,8	16,59%	11,25%	33,54%
	2,75	initial	22824	2,50E+06	1582,5	143,27%	390,02%	197,49%
		final	3082	4,57E+04	213,7	17,18%	12,30%	35,08%
	3,00	initial	28828	4,00E+06	1998,9	182,61%	668,77%	258,61%
		final	3653	6,42E+04	253,3	21,13%	14,98%	38,70%
	3,25	initial	25280	3,07E+06	1752,9	178,20%	576,20%	240,04%
		final	3182	4,87E+04	220,7	17,73%	11,65%	34,14%
	3,50	initial	25207	3,05E+06	1747,8	167,61%	514,76%	226,88%
		final	3039	4,44E+04	210,7	17,42%	12,51%	35,37%
	3,75	initial	32212	4,99E+06	2233,5	182,93%	653,75%	255,69%
		final	3840	7,09E+04	266,3	21,21%	17,93%	42,35%
	4,00	initial	31369	4,73E+06	2175,1	194,59%	735,63%	271,22%
		final	3799	6,94E+04	263,4	20,53%	15,56%	39,45%
	4,25	initial	33960	5,54E+06	2354,7	230,80%	1135,40%	336,96%
		final	4043	7,86E+04	280,3	24,69%	24,21%	49,20%
	4,50	initial	39181	7,38E+06	2716,7	218,10%	997,88%	315,89%
		final	3163	4,81E+04	219,3	16,79%	12,59%	35,49%
	4,75	initial	39272	7,42E+06	2723,1	234,80%	1182,50%	343,88%
		final	3830	7,05E+04	265,6	18,36%	14,50%	38,08%
	5,00	initial	43808	9,23E+06	3037,5	251,62%	1364,20%	369,35%
		final	3405	5,57E+04	236,1	17,58%	12,61%	35,51%
	5,25	initial	41007	8,08E+06	2843,3	269,70%	1594,60%	399,32%
		final	4672	1,05E+05	323,9	25,25%	22,06%	46,97%
	5,50	initial	44326	9,45E+06	3073,4	290,68%	1882,90%	433,93%
		final	4664	1,05E+05	323,4	27,29%	26,38%	51,36%
	5,75	initial	44867	9,68E+06	3110,9	261,96%	1554,60%	394,29%
		final	3841	7,09E+04	266,4	21,99%	17,72%	42,10%
	6,00	initial	47340	1,08E+07	3282,4	300,07%	1902,80%	436,21%
		final	4142	8,25E+04	287,2	24,15%	21,11%	45,94%

Table 3 – Correction results for the 208 links 208 o-d pairs mesh network (perturbation parameters α and β indicated beside perturbations)

INITIAL PERTURBATION	dipersion parameter		Performance indicator			
			Euclidean	MSE	RMSE	MAPD
Spreading	0.0	initial	1368.00	0.86	0.92	70.00%
		final	48.63	0.03	0.17	7.20%
	0.1	initial	2891.60	1.81	1.34	143.91%
		final	511.49	0.32	0.56	37.80%
	0.2	initial	2145.00	1.84	1.35	112.86%
		final	618.10	0.39	0.62	47.63%

Table 4 –Results of a simulation run on a 120 links 200 o-d pairs mesh network, considering 8 time slices. True o-d profiles generated according to the assumption of o-d flows following an autoregressive process

PERTURBATION			Performance indicator					
			Euclidean	MSE	RMSE	MAPD	MSPE	RMSPE
amplification	20%	initial	466.49	0.29	0.54	20.00%	4.07%	20.00%
		final	0.04	0.00	0.00	0.70%	0.05%	2.42%
		simultaneous	19.82	0.01	0.11	10.20%	7.27%	26.96%
	30%	initial	1045.70	0.65	0.80	30.00%	9.08%	30.00%
		final	0.04	0.00	0.00	0.72%	0.05%	2.42%
		simultaneous	6.10	0.00	0.06	5.66%	1.91%	13.84%
	40%	initial	1857.00	1.16	1.07	40.00%	16.09%	40.00%
		final	0.04	0.00	0.00	0.72%	0.05%	2.42%
		simultaneous	6.70	0.00	0.06	6.04%	3.10%	17.63%
	50%	initial	2896.70	1.81	1.34	50.00%	25.10%	50.00%
		final	0.04	0.00	0.00	0.72%	0.05%	2.42%
		simultaneous	9.59	0.01	0.11	9.59%	5.79%	24.07%
spreading		initial	2404.00	1.50	1.22	93.38%	305.33%	174.74%
		final	0.04	0.00	0.00	0.68%	0.04%	2.09%
		simultaneous	660.24	0.41	0.64	55.39%	166.91%	163.37%
ampSpreading		initial	4685.80	2.93	1.71	153.81%	809.70%	284.56%
		final	0.04	0.00	0.00	0.72%	0.05%	2.42%
		simultaneous	622.15	0.39	0.62	52.18%	218.60%	147.74%

Table 5 – Correction results for the 120 links 200 o-d pairs 8 time slices mesh network - the hypothesis of constant distribution shares holds exactly

PERTURBATION	Number of sensors	Number of equations	variables /equations		Performance indicator					
					Euclidean	MSE	RMSE	MAPD	MSPE	RMSPE
ampSpreading	**all link s120**	1320		initial	4685.80	2.93	1.71	153.81%	809.70%	284.56%
			0.4	final	0.04	0.00	0.00	0.72%	0.05%	2.42%
			1.21	simultaneous	622.15	0.39	0.62	52.18%	218.60%	147.74%
	100	1100		initial	4685.80	2.93	1.71	153.81%	809.70%	284.56%
			0.4	final	0.05	0.00	0.00	0.72%	0.05%	2.29%
			1.45	simultaneous	913.04	0.57	0.75	60.47%	204.47%	142.99%
	90	990		initial	4685.80	2.93	1.71	153.81%	809.70%	284.56%
			0.5	final	71.29	0,04	0.21	4.01%	6.12%	24.75%
			1.62	simultaneous	1048.10	0.65	0.81	64.05%	234.20%	153.04%
	80	880		initial	4685.80	2.93	1.71	153.81%	809.70%	284.56%
			0.5	final	71.31	0.04	0.21	4.10%	6.14%	24.80%
			1.82	simultaneous	1145.60	0.72	0.85	66.13%	218.13%	147.90%
	70	770		initial	4685.80	2.93	1.71	153.81%	809.70%	284.56%
			0.6	final	72.57	0.05	0.21	4.72%	6.49%	25.47%
			2.08	simultaneous	1314.30	0.82	0.91	70.64%	196.74%	140.27%
	60	660		initial	4685.80	2.93	1.71	153.81%	809.70%	284.56%
			0.7	final	89.68	0.06	0.24	5.59%	7.00%	26.46%
			2.42	simultaneous						
	50	550		initial	4685.80	2.93	1.71	153.81%	809.70%	284.56%
			0.8	final	263.70	0.16	0.41	14.31%	23.64%	48.63%
			2.91	simultaneous						
	40	440		initial	4685.80	2.93	1.71	153.81%	809.70%	284.56%
			1.1	final	473.46	0.29	0.54	31.10%	37.29%	61.06%
			3.64	simultaneous						

Table 6– Correction results for the 120 links 200 o-d pairs 8 time slices mesh network for different number of sensors;, the hypothesis of constant distribution shares holds exactly

PERTURBATION	dispersion parameter		Performance indicator					
			Euclidean	MSE	RMSE	MAPD	MSPE	RMSPE
Amplificationspreading	0.1	initial	4804.10	3.00	1.73	153.87%	808.39%	284.32%
		final	389.31	0.24	0.49	22.68%	11.91%	34.51%
		simultaneous	680.63	0.43	0.65	54.24%	233.89%	152.94%
	0.2	initial	4981.80	3.19	1.79	169.26%	1020..1%	319.39%
		final	650.78	0.41	0.64	29.11%	21.01%	45.84%
		simultaneous	1101.70	0.69	0.83	64.38%	388.64%	197.14%
	0.3	initial	5344.60	3.34	1.82	210.62%	8936.70%	945.34%
		final	2225.00	1.39	1.17	67.85%	198.46%	140.87%
		simultaneous	739.25	0.46	0.68	62.19%	343.57%	185.36%

Table 7– Correction results for the 120 links 200 o-d pairs 8 time slices mesh network for different dispersion parameter in drawing the true o-d matrices from the initial matrices with the constant distribution shares