# TIMETABLE SYNCHRONIZATION FOR MASS TRANSIT RAILWAYS 

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#### Abstract

In most urban public transport systems, passengers may need to make several interchanges between different lines to get to their destination. Designing timetables that enable smooth interchanges with minimal delay for all passengers is a service goal of the MTR Corporation Limited (MTRCL), which runs six railway lines with many cross-platform interchange stations within the Mass Transit Railway (MTR) system in Hong Kong. Designing such co-ordinated timetables is a very difficult task. This paper describes a set of decision-support tools for this timetable synchronization problem. The core of the system is a mixed-integer-programming (MIP) optimization model that minimizes the interchange waiting-times of all passengers. A novelty in our formulation is the use of binary variables which enable the correct representation of the waiting-times for transfer to the "next available" train at the interchange stations.

By adjusting the trains’ run-times and station dwell-times during their trips, and their dispatch times, turnaround times and headways at the terminals, our system can construct highquality timetables that optimize the objective of minimizing passenger transfer waiting-times. We have tested our system for rush-hour and non-rush hour periods. Preliminary numerical results indicate that our approach improves the synchronization compared to current practice significantly. We also explore the trade-offs among different operational parameters and flexibility and the impact on overall passenger waiting-times. The preliminary results are very encouraging.


## 1 Introduction

Nowadays, mass transit usually does not provide point-topoint transportation service. Passengers are often required to interchange within a multi-modal transportation network to complete a trip. During the transfer, nothing to do but to wait for the connecting vehicle is the passenger's plight. Möhring [6] found that passengers perceived their waiting-times to be almost twice of what it actually is. To be able to design
timetable with good co-ordination between vehicles so that passengers can enjoy "immediate" transfer becomes a service goal of a transportation company and also the hope of passengers. Whilst important, this problem has not received widespread research attention.
To improve customer service, Chowhury and Chien [1] studied how to dynamically dispatch the vehicles so as to minimize the transfer waiting-times of passengers. Whilst dynamic dispatching is important for real-time operations, the focus of our paper is on pre-planning, the development of a synchronized timetable off-line so that planned dispatch of trains minimize the transfer waiting-times.

Liebchen and Möhring [5] constructed a timetable with consideration of many criteria. One of them is to maximize the number of cross-wise correspondences, which are the meeting frequency of feeder vehicle and connecting vehicle. In the paper, passengers were categorized into two transferring groups, those who may need to wait or those who do not need to wait at the interchange station. The objective of the problem was to maximize the size of the passenger group who do not need to wait. However, in reality, the length of the waiting-time is important. There is a big difference to the passengers between waiting for 10 seconds and for 200 seconds. To deal with this case, we will compare the total transfer waiting-times instead of the meeting frequency in our model.

Goverde [4] introduced the concept of primary and secondary waiting-times. The waiting-times of originating and through passengers on the current transfer station, as well as the waiting-times on subsequent stations resulting from the optimization in the service networks were considered. He studied the possible delay of connecting trains at the interchange station such that the total primary and secondary waiting-times were minimized. In our model, we do not only find the possible delay of connecting trains but also the possible earlier arrival of connecting trains. In addition, we could adjust the possible delay or earlier arrival of feeder trains at any station. As our model has more flexibility, the improvement of synchronization is better.
Vansteenwegen and Van Oudheusden [9] proposed a linear programming model and showed by simulation that their new timetable is better than the current timetable in the Belgian
railway network. Considering that vehicles always suffer delays; they calculated the ideal buffer times for each connection, which were then used in a linear programming to construct a new timetable. Their new timetable is a one-hour periodic timetable with part of the network only. They predefined all the connection patterns between feeder trains and connecting trains. This is the main difference to our model.
Daduna and Voss [2] and Pedersen [7] used heuristic approaches to adjust the dispatching times of train on a route to synchronize the timetable. However, both of their timetables must have constant headway. Suhl [8] introduced an optimization dispatching-support model where, based on the existing timetable, arrival times and departure times of trains would be adjusted upwards. The objective was to minimize the prolonging trip-times and the transfer waitingtimes, but the total transfer waiting-times are only approximated in their model.

In this study, we propose a mixed-integer programming model for the timetable synchronization problem. Our model minimizes transfer waiting-times of all passengers in a railway system. Coordination and synchronization of the schedule is achieved by adjusting the run-times, dwell-times and dispatch-times of each train. The optimized timetables generated using this model improve the current timetables significantly.
This paper is organized as follows. Section 2 explains the model assumptions and describes the MIP formulations. Then an Optimization-based Heuristic Approach (OHM) is presented in Section 3. Section 4 discusses several case studies on the MTR system in Hong Kong. In the scenarios explored, run-times, dwell-times and dispatch-times may be changed to different degree. The impact on the reduction of transfer waiting-times will be illustrated. Finally, a summary of our findings and suggestions for further study are discussed in Section 5.

## 2 Timetable Synchronization Problem (TSP)

### 2.1 Underlying Assumptions

In actual operations, passengers move at various speeds and not all passengers catch the transfers intended. Since ours is a planning model, we make the following simplifying assumptions regarding passengers flows.

Firstly, we assume that the transfer-times is known and fixed for all transfer passengers. The transfer-times is the time for a passenger to get off the feeder train and walk across the interchange platform to get on the next appropriate connecting train. The values used in our case studies are determined by surveys conducted by the MTR Corporation.

Secondly, route choices of passengers are known and fixed. When there are alternative routes in the system, not all passengers will choose the same route. Fung [3] predicted the passenger flow for the MTR network by four criteria: invehicle time, waiting time, walking time and the number of
lines boarding. By setting different weights for those four criteria, overall passenger flows can be predicted. In our model, for simplification, we assume that passengers choose their routes by only two criteria: the number of lines boarding and the number of stops on the trip. We assume passengers prefer not to transfer so they choose a route where the number of lines boarding is as small as possible. When the numbers of transfers are the same for two alternative routes, passengers will choose the one with fewer stops. This assumption allows us to compute the number of transfer passengers at each station, based on origin-destination counts. Other more accurate methods can be used to calculate the patronage. It will not affect our mathematical model; it only changes the coefficients in our objective function.

Thirdly, we assume passenger flow is evenly distributed within a short time period. It is time-consuming to get the exact number of passengers getting on a train. In our case studies, the origin-destination matrix given by the MTR Corporation is in fifteen-minute intervals. We divide the fifteen-minute by the number of departing trains in that time interval to get the "weight" of passenger flow at the interchange station. Our model then minimizes the weighted sum of transfer waiting-times for the entire system.
Fourthly, we assume the capacity of the trains is sufficient at any time to receive all passengers who want to enter that train. Obviously, infinite capacity is unrealistic, especially for rushhour traffic in Hong Kong. However, it is a common assumption in transfer-scheduling practice. It can substantially reduce the complexity of our model.

### 2.2 Modelling Transfer Waiting-times

As noted in the previous section, not all transfer passengers get on the next possible connecting train immediately; some may prefer to skip trains and board later ones. However, the transfer waiting-time that we count is the time to the first possible train only. In other words, we assume all passengers get on the next connecting train immediately. To capture the value of this waiting time in our model, we define variables $w_{t q}^{t q}$ to represent the "waiting time" of passengers transferring from the $q^{\text {th }}$ train on Route $t$ to the $q^{\text {, th }}$ train on Route t'. Our model incorporates constraints so that the "waiting time' have the correct interpretation, as illustrated in Fig. 1.


Figure 1: Transfer Waiting-time Declaration

Figure 1 shows the sequence of events at an interchange station. For the arrow above the timeline, arrival time of the $q^{\text {th }}$ feeder train on Route $t$ at the station is shown. For those arrows below the timeline, the departure time of the $\left(q^{\prime}-3\right)^{\text {th }}$, $\left(q^{\prime}-2\right)^{\text {th }}, \ldots\left(q^{\prime}+1\right)^{\text {th }}$ trains from interchange station on Route $t^{\prime}$ are presented. The dashed double-arrow lines represent the "possible transfer" of passengers from the $q^{\text {th }}$ feeder train on Route $t$ to Route $t^{\prime}$. Common sense dictates that passengers from the $q^{\text {th }}$ feeder train cannot get on the $\left(q^{\prime}-3\right)^{\text {th }}$ and $\left(q^{\prime}-2\right)^{\text {th }}$ train which leave before the $q^{\text {th }}$ feeder train arrives. Thus, the "waiting-times" of those connections are assigned the value zero. $e_{t^{\prime}}^{t}$ is the expected transfer-time, i.e. the expected time for passengers to walk across the interchange platform when transferring from Route $t$ to Route $t^{\prime}$. Therefore, passengers from the $q^{\text {th }}$ feeder train still cannot get on the $\left(q^{\prime}-1\right)^{\text {th }}$ train; the "waiting-time" then is also zero. Since we just consider the transfer waiting-times for passenger getting on the next possible connecting train, the waiting-times for passengers transferring from the $q^{\text {th }}$ feeder train to the $\left(q^{\prime}+1\right)^{\text {th }}$ and later trains are also assigned to zero. Thus, only the "waiting-time" for the connection between the $q^{\text {th }}$ feeder train and the $q$, th connecting train may be non-zero.

### 2.3 The Mathematical Model

## Objective

In the MIP model for train synchronization, the objective is to minimize the weighted sum of transfer waiting-time in the entire railway system:

$$
\begin{equation*}
\operatorname{Min}\left\{\sum_{\left(t, t^{\prime}\right)} \sum_{q} \sum_{q^{\prime}} c_{t^{\prime}}^{t q} w_{t^{\prime} q^{\prime}}^{t q}\right\} \tag{1}
\end{equation*}
$$

where $c_{t^{\prime}}^{\text {tq }}$ is the number of passenger transferring from the $q^{\text {th }}$ feeder train on Route $t$ to a train on Route $t^{\prime}$. ( $t, t^{\prime}$ ) is the passenger group requiring transfer from Route $t$ to Route $t^{\prime}$.

## Constraints

Constraints (2) and (3) track the arrival time $A_{j}^{t q}$ and departure time $L_{j}^{\text {tq }}$ of the $q^{\text {th }}$ train on Route $t$ at/from Station $j$ :

$$
\begin{align*}
& A_{j}^{t q}=L_{0}^{t q}+\sum_{k=1}^{j} R_{k}^{t q}+\sum_{k=1}^{j-1} D_{k}^{t q}  \tag{2}\\
& L_{j}^{t q}=A_{j}^{t q}+D_{j}^{t q} \tag{3}
\end{align*}
$$

where $R_{j}^{t q}$ is the time for the $q^{\text {th }}$ train on Route $t$ to run from Station $j$-1 to Station $j ; D_{j}^{t q}$ is the dwell time that the $q^{\text {th }}$ train on Route $t$ stays at Station $j$. Station 0 is the originating terminal of the route.

To fulfil the service requirements, there are minimum/ maximum headways at each station $\left(\underline{h}_{j}^{t} / \bar{h}_{j}^{t}\right)$, trip times ( $\underline{y}^{t} / \bar{y}^{t}$ ) and turnaround times $\left(\underline{z}^{t} / \bar{z}^{t}\right)$ for each route. These bounds are set in Constraints (4), (5) and (6)
respectively. Station $m_{t}$ is the terminal station of the trains on Route $t$.

$$
\begin{align*}
& \underline{h}_{j}^{t} \leq L_{j}^{t q}-L_{j}^{t(q-1)} \leq \bar{h}_{j}^{t}  \tag{4}\\
& \underline{y}^{t} \leq A_{m_{t}}^{t q}-L_{0}^{t q} \leq \bar{y}^{t}  \tag{5}\\
& \underline{z}^{t} \leq L_{0}^{t^{\prime} \sigma(q)}-A_{m_{t}}^{t q} \leq \bar{z}^{t} \tag{6}
\end{align*}
$$

To enable the correct representation of the waiting-times for transfer to the "next available" train at the interchange stations, we introduce binary variables $\alpha_{t^{\prime} q^{\prime}}^{\text {tq }}$ and Constraints:

$$
\begin{equation*}
F\left(\alpha_{t^{\prime} q^{\prime}}^{t q}-1\right) \leq L_{j^{\prime}}^{t^{\prime} q^{\prime}}-\left(A_{j}^{t q}+e_{t^{\prime}}^{t}\right) \leq F \alpha_{t^{\prime} q^{\prime}}^{t q} \tag{7}
\end{equation*}
$$

where $F$ is a large positive number. $\alpha_{t^{\prime} q^{\prime}}^{t q}$ is equal to one only if the $q^{\text {th }}$ train on Route $t$ arrives early enough so passengers can transfer to the $q^{\text {, th }}$ train on Route $t^{\prime}, 0$ otherwise.

With the binary variables $\alpha_{t^{\prime} q}^{t q}$, we can capture the appropriate transfer waiting-time by using Constraints (8):

$$
\begin{equation*}
L_{j^{\prime}}^{t^{\prime} q^{\prime}}-\left(A_{j}^{t q}+e_{t^{\prime}}^{t}\right)-F \alpha_{t^{\prime}\left(q^{\prime}-1\right)}^{t q} \leq w_{t^{\prime} q^{\prime}}^{t q} \tag{8}
\end{equation*}
$$

During the morning-peak hours, more trains are needed to accommodate peak traffic in the centre of town. They depart from a non-terminal station. Collision with trains running on the same track has to be avoided. This is done by using Constraints (9a) and (9b).

$$
\begin{align*}
& A_{j}^{t q}-\left(L_{j^{\prime}}^{t^{\prime} q^{\prime}}+b_{t^{\prime} j^{\prime}}^{t j}\right) \leq F \beta_{t^{\prime} q^{\prime} j^{\prime}}^{t q}  \tag{9a}\\
& A_{j^{\prime}}^{t q^{\prime}}-\left(L_{j}^{t q}+b_{t^{\prime} j^{\prime} j^{\prime}}^{t}\right) \leq F\left(1-\beta_{t^{\prime} q^{\prime} j^{\prime} j^{\prime}}^{t q j}\right) \tag{9b}
\end{align*}
$$

$\beta_{t^{\prime} q^{\prime} j^{\prime}}^{t q j}$ is a binary variable. $b_{t^{\prime} j^{\prime}}^{t j}$ is the minimum time gap between the arrival of train on Route $t$ at Station $s_{j}^{t}$ and the departure of train on Route $t^{\prime}$ at Station $s_{j^{\prime}}^{t^{\prime}}$.

## Bounds

Bounds on run-times, dwell-times, maximum transfer waiting-time and the arrival time of last train during the testing horizon are set by using Constraints (10), (11), (12) and (13) respectively.

$$
\begin{align*}
& \underline{r}_{j}^{t q} \leq R_{j}^{t q} \leq r_{j}^{t q}  \tag{10}\\
& \underline{d}_{j}^{t q} \leq D_{j}^{t q} \leq \bar{d}_{j}^{t q}  \tag{11}\\
& 0 \leq w_{t^{\prime} q^{\prime}}^{t q} \leq \bar{w}_{t^{\prime}}^{t q}  \tag{12}\\
& 0 \leq A_{m_{t}}^{t\left|N_{t}\right|} \leq P \tag{13}
\end{align*}
$$

where $\underline{r}_{j}^{t q} / \bar{r}_{j}^{t q}$ and $\underline{d}_{j}^{t q} / \bar{d}_{j}^{t q}$ are the allowable minimum / maximum run-times and dwell-times of trains on Route $t$ running to and staying at Station $s_{j}^{t}$ respectively. (12) is a soft constraint; $\bar{w}_{t^{\prime}}^{t q}$ is the allowable maximum transfer waiting-time when passengers transferring from the $q^{\text {th }}$ feeder
train on Route $t$ to Route $t^{\prime} . P$ in (13) is the length of the testing horizon. The $\left|N_{t}\right|^{\text {th }}$ train is the last adjustable train on Route $t$ during the testing horizon.

### 2.4 Optional Operational Constraint

## Improve Regularity of Dwell-times

Our model allows dwell-times of each train at each station to be adjusted individually thus dwell-times at the same station may be different from train to train. To improve the regularity of dwell-times, we could replace Bound (11) by Constraints (14) and (15):

$$
\begin{align*}
& \underline{d}_{j}^{t}+\frac{d}{2} \leq \tilde{D}_{j}^{t} \leq \bar{d}_{j}^{t}-\frac{d}{2}  \tag{14}\\
& \tilde{D}_{j}^{t}-\frac{d}{2} \leq D_{j}^{t q} \leq \tilde{D}_{j}^{t}+\frac{d}{2} \tag{15}
\end{align*}
$$

where $\underline{d}_{j}^{t}$ and $\bar{d}_{j}^{t}$ are the allowable minimum and maximum dwell-times of train on Route $t$ at Station $s_{j}^{t}$ respectively. $\tilde{D}_{j}^{t}$ is a variable representing the median of dwell-times of train on Route $t$ at Station $s_{j}^{t}$. $d$ is the maximum allowable dwelltimes variance at the same station. Constraint (14) is used to fix the median of dwell-times, $\tilde{D}_{j}^{t}$, and Constraint (15) restricts the dwell-times of each train to vary from the median by no more than $d / 2$ seconds. Fig. 2 shows the relationship graphically.


Figure 2: Possible Range of Dwell-times at Each Station

## Improve Regularity of Headways

Similar to the above requirement for dwell-times, if the variation of headway is considered at each station, Constraint (4) should be replaced by Constraints (16) and (17):

$$
\begin{align*}
& \underline{h}_{j}^{t}+\frac{h}{2} \leq \tilde{H}_{j}^{t} \leq \bar{h}_{j}^{t}-\frac{h}{2}  \tag{16}\\
& \tilde{H}_{j}^{t}-\frac{h}{2} \leq L_{j}^{t q}-L_{j}^{t(q-1)} \leq \tilde{H}_{j}^{t}+\frac{h}{2} \tag{17}
\end{align*}
$$

where $\underline{h}_{j}^{t}$ and $\bar{h}_{j}^{t}$ are the allowable minimum and maximum headway of on Route $t$ at Station $s_{j}^{t}$ respectively. $\tilde{H}_{j}^{t}$ is a variant representing the median of headway of train on Route $t$ at Station $s_{j}^{t}$. $h$ is the maximum allowable headway variance. If $h$ is equal to zero, headway on that route is constant.

## 3 Solution Approach

In our MIP model, to synchronize a one hour schedule for the entire railway system (with 154 trains), there are about forty thousands constraints and about thirty thousands variables in which about ten thousands variable are binary. In our initial run using a standard Branch-and-Bound approach in CPLEX, getting a feasible solution took three days and nights! To reduce the computational time, we investigated an Optimization-based Heuristic Method (OHM). The flow chart for OHM is shown in Fig. 3.


Figure 3: Optimisation-based Heuristic Method, OHM
In our computational experience, OHM fixed about $87 \%$ of binary variables. In one instance, the solution time of getting a feasible solution by using the OHM is reduced to only thirty minutes, compared to nineteen hours using standard Branchand Bound approach. Our approach reduces the solution time by nearly $90 \%$ in most of our computational experiments.

The key points of OHM are the modification of the original MIP and the "prediction" of the binary variables. In the modification, binary variables are added to the objective of the original MIP. Prediction of the binary variables is done according to rounding of their values in the LP-relaxation of the modified MIP. If we predict all the binary variables, the MIP may be infeasible. We then iteratively unfix some of the fixed binary variables until the problem becomes feasible. The binary variables are naturally sequenced in ascending order of the train sequence, and so binary variables around the "borderline" of fixed values are released at each iteration.

## 4 Case Studies

In this section, we study the quality of the synchronized timetable using several real cases in the MTR system in Hong Kong.

### 4.1 Problem Setting

The MTR Corporation runs six railway lines with thirteen interchange stations. As trains running on Tung Chung Line and Airport Express Line share part of the track, many
signalling conditions need to be considered when making adjustment on those lines. To reduce the complexity of the problem, we ignore these two lines. The problem is reduced to four lines and eight interchange stations.
The allowable adjustments based on MTR's current schedules and standards are listed in Table 1.

## Allowable Adjustments to Operational Parameters

| Operational Parameters | Allowable <br> Adjustments |
| :--- | :--- |
| Run-times, $R_{j}^{\text {tq }}$ | Same as the current <br> standard |
| Dwell-times, $D_{j}^{t q}$ | $\pm 5$ sec. to the current <br> standard |
| Headway, $L_{j}^{t q}-L_{j}^{t(q-1)}$ | $\pm 30$ sec. to the current <br> standard |
| Trip-times, $A_{m_{t}}^{t q}-L_{0}^{t q}$ | $\pm 5$ sec. to the current <br> standard |
| Turnaround Times, $L_{0}^{t^{\prime} \sigma(q)}-A_{m_{t}}^{t q}$ | $\pm 5$ sec. to the current <br> standard |
| Minimum Time Gap, $b_{t^{\prime} j^{\prime}}^{t j}$ | 60 sec. |


| Regularity |  |
| :--- | :--- |
| Regularity of dwell-times, $d$ | 6 sec. |
| Regularity of headways, $h$ | 6 sec. |

Table 1: Allowable Adjustments to Operational Parameters

### 4.2 Solution Quality during Rush Hour

The first case discussed covers the morning rush-hour period, 08:00 ~ 09:00. There are 134 number of adjustable train during this time period. The time to get an optimal solution is about ten and a half hour.

### 4.2.1 Average Transfer Waiting-times

The current timetables in use have fixed values for many operational parameters. With the added flexibility allowed in our model (as indicated in Table 1), our generated solution reduces the transfer waiting-time of passengers by $41 \%$ on average, although the average waiting times are increased for some interchange stations. Moreover, the generated schedule from our model has certain features (e.g., non-constant headways) that may impact on the operational complexity.

### 4.2.2 "Just Miss" Improvement

Passengers do not like just missing the connecting train by a few seconds, especially if they see the previous train leave. To compare the frequency of this scenario happening in our timetable and the current timetable, we introduce a rate called the "Just Miss" rate. We get the number of "Just Miss" by counting the number of occurrences when the transfer waiting-times is greater than the difference of headway of the connecting train and the "expect" cross-platform time. The relationship is shown in Fig. 4. In our case study, the
improvement of "Just Miss" is $58 \%$. It is an encouraging result.


Figure 4: Occurrence of "Just Miss"

### 4.3 Solution Quality during Non-rush Hour

Now, we take the non-rush hour, 14:00~16:00 as the testing horizon. During the testing horizon, 193 trains running on the four lines are adjusted. The improvement of average transfer waiting-times is $34 \%$ and the "Just Miss" Improvement is $100 \%$. The rate of improvement on "Just Miss" shows that there are no passengers who will miss the connecting train for a few seconds. The solution time is about four seconds only.

### 4.4 Cases Analysis

We would like to see the impact of transfer waiting-times when run-times, dwell-times or headways are adjusted to different degrees. We can then explore the trade-offs among different operational parameters and flexibility. The cases discussed below cover the morning rush-hour period, 08:00 ~ 09:00 and the afternoon non-rush hour, 14:00~16:00. The values in the brackets are the results for the non-rush hour.

### 4.4.1 Varying Dwell-times and Headways

In this part, dwell-times and headways are adjusted to different degrees. They may be unadjusted or constant in some cases, i.e. both $d$ and $h$ are equal to zero. Results are shown in Table 2.

| Problem Setting | Solution <br> Time in sec | Average Transfer <br> Waiting-times <br> Improvement | "Just Miss" <br> Improvement |
| :--- | :---: | :---: | :---: |
| Constant <br> Headway | 1021 (15) | $38 \%$ (43\%) | $56 \%$ (100\%) |
| Constant <br> Dwell-times | 16879 (15) | $24 \%(43 \%)$ | $47 \%$ (100\%) |
| Constant <br> Headway <br> Regularity of <br> dwell-times, $d=2$ | 9389 (16) | $30 \%(43 \%)$ | $40 \%$ (100\%) |

Table 2: Optimization Results
In Table 2, during the rush-hour period, we find that average improvement of transfer waiting-times is highly depending on the flexibility of dwell-times. If dwell-times are set to be constant, the average improvement of waiting time sharply decreases from $38 \%$ to $24 \%$. If dwell-times cannot be
adjusted from the current practice, then there is improvement on neither average transfer waiting-times nor "Just Miss" rate.

During non-rush hour, in all cases, the average improvement of transfer waiting-times is $43 \%$. It may be due to the fact that the current schedule already has constant headway and constant dwell-times. The setting of "Constant Headway" and "Constant Dwell-times" do not influence the solution quality.

### 4.4.2 Varying Run-times

Vehicles running faster exhaust more energy. It is reasonable to recommend that run-times should not be decreased. Then, how about increasing it? Now, we allow the run-times to each station can be prolonged for one second. Results are shown in Table 3.

| Problem Setting | Solution <br> Time in <br> sec | Average Transfer <br> Waiting-time <br> Improvement | "Just Miss" <br> Improvement |
| :--- | :--- | :--- | :--- |
| Constant Headway <br> Run-times to each <br> station could be <br> prolonged by 1 sec. | 2783 (14) | $42 \%$ (44\%) | $54 \%$ (100\%) |
| Constant Headway <br> Run-times to each <br> station could be <br> prolonged by 1 sec. | $23335(7)$ | $35 \%$ (43\%) | $44 \%$ (100\%) |
| Regularity of <br> dwell-times, $d=2$ |  |  |  |

Table 3: Optimization Results (Run-times can be prolonged)
From the table, we see that if run-times can be prolonged for even just one second, the average improvement of waitingtimes sharply increases, especially for the case with constant dwell-times and headways. It should be mentioned that additional resources and/or trains may be needed to achieve the schedule with prolonged run-times.

## 5 Conclusions

The models and methods developed for train synchronization in this paper are efficient and easy to implement. As we could synchronize the existing timetable in a piece-wise fashion, we can get the suggested timetable for the entire day in a reasonable time. Our case studies show that our synchronization timetable reduces the passengers' waitingtimes sharply when comparing with the transfer waiting-times according to the current timetable in use.
Several improvements could warrant further research. Firstly, the variation of waiting-times should also be considered. Time for passengers looking for the connecting train at the same interchange station should not have a big difference. To deal with this, variance of transfer waiting-times at each platform could be added to the objective in the MIP. In this sense, the problem becomes more complicated as it becomes a non-linear programming model.

In reality, trains may miss their schedule. It may be due to two reasons. One is the late arrival of the train at its terminal; a train may not be dispatched from its origin on time. The second reason is passengers' behaviour causing trains to depart from the stations behind schedule. To consider these phenomena in our model, we could add some random variables (which could be exponentially distributed) to the dispatch-times of train; that is, modify (3). Adding the random delay scenarios makes the model to be more realistic, but introduces stochasticity and added complexity into the model.

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