HYBRID PREDICTIVE CONTROL STRATEGY FOR A PUBLIC TRANSPORT SYSTEM WITH UNCERTAIN DEMAND

D. Sáez, C. E. Cortés*, E. Sáez**, M. Riquelme**, A. Núñez** and A. Tirachini***

*Civil Engineering Department, Universidad de Chile,
Blanco Encalada 2002, Santiago, Chile.
Phone: 56-2-9784380 Fax: 56-2-6894206, Email: ccortes@ing.uchile.cl
**Electrical Engineering Department , Universidad de Chile,
Av. Tupper 2007, Santiago, Chile.
Phone: 56-2-9784207, Fax: 56-2-6720162, Email: dsaez@ing.uchile.cl

EXTENDED ABSTRACT:

In most public transport systems, the movement of transit vehicles is affected by different disruptions as the day progresses, such as traffic congestion and unexpected delays, randomness in passenger demand (both spatial and temporal), irregular vehicle dispatching times, incidents and so on. These events hinder the dispatch of vehicles either following a pre-planned schedule or trying to maintain a regular headway. As an attempt to reduce the negative effects of service disturbance, researchers have devoted significant effort to develop flexible control strategies, either in real-time or off-line, depending on the specific features of the problem.

The most studied transit control strategies can be classified into three categories (Eberlein, 1995): station control, interstation control and other strategies. Station control strategies are of two types: holding and station skipping. Interstation control strategies include speed control, transit signal priority and so on. The holding strategy has been intensely studied in the literature in the last years. Among the most remarkable contributions we can mention Turnquist and Blume (1980), Eberlein (1995), Eberlein et al. (2001), Hickman (2001) and Sun and Hickman (2004). The operation of express services (expressing) as a pre-planned strategy is studied by Jordan and Turnquist (1979) and Furth (1986). Other novel research topic is the *Transit signal priority* (TSP) that studies the utilization of traffic signals to regularize bus headways (Furth and Muller, 2000; Ling and Shalaby, 2004; and Lee et al., 2005)

The real-time control strategies as found in the public transport literature have been treated mostly heuristically, lacking of a dynamic control framework. In this paper, we develop a model integrating three of the aforementioned strategies (holding, expressing and transit signal priority) to solve a real-time public transport control problem with uncertain passenger demand, relying on online information of system behaviour. The model is formulated as a predictive control problem, since the theory nicely fit the dynamic conditions of typical public transport problems. Predictive models permit to estimate the effects of the control actions on the behaviour of the transit system, and also allow the inclusion of complex system constraints. They also have the capability of optimizing the system performance in real time based on an objective function properly chosen (Hegyi, 2004).

Specifically in this research, we propose to design and evaluate a predictive control strategy of a bus system with uncertain passenger arrival at bus stops, in a real-time framework. The generic predictive control strategy to be applied is shown in Figure 1 below. For this problem, the predictive controller corresponds to the bus dispatcher, who dynamically provides the optimal control actions to the bus system in order to regularize the headway between buses according to the assumed objective function. The real-time passenger demand, which is unknown and uncertain, is modelled as a disturbance for the predictive scheme. That because, different passenger arrival patterns could significantly affect the estimated bus travel time from longer passenger transfer operations at bus stops.

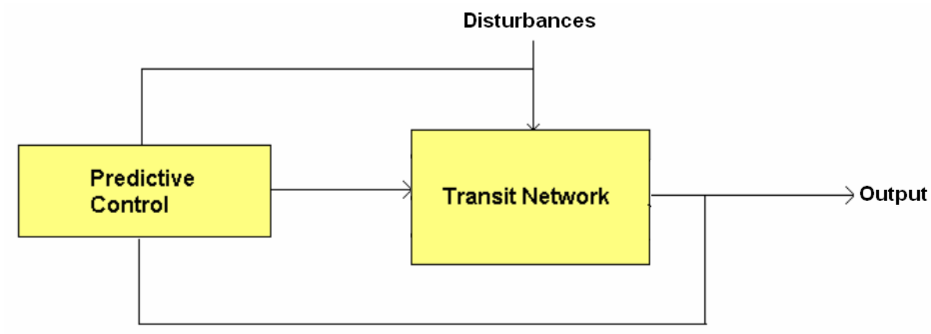

Figure 1. Predictive control scheme for a bus transport system.

For the design of the predictive controller, an objective function to be optimized is defined. In our approach, such a function includes four components as shown in expression (6) ahead. The control strategies will allow us incorporating into the design the future behaviour of the bus system, by using an on-line prediction system for the disturbances, such as number of passengers at stops. In our approach, there are discrete (traffic light states, number of passengers on the buses) as well as continuous (bus position and speed) variables. For this reason, we will use a hybrid predictive approach of the form proposed by Bemporad and Morari (1999), in which it is possible to optimize the control actions considering both kinds of variables. We propose Genetic Algorithm for solving the resulting hybrid predictive control optimization problem.

DYNAMIC MODELING FOR HYBRID PREDICTIVE CONTROL DESIGN

One major objective of this paper is to formulate the real-time transit system optimization problem under a hybrid predictive control (HPC) approach. For the sake of simplicity, the HPC framework is constructed for a single loop bus system, although is extensible to more complex systems too.

The system is represented in Figure 2a). The network is a one-way loop route, with N equidistant stations and B buses running around the loop, under the control of the dispatcher. Station 1 is the terminal of the bus route. All passengers have to get off the bus there. Passengers arrive to each station at certain rate, with destination among the stations ahead of the station the passenger is getting on. The problem is formulated as a hybrid predictive system, where events are triggered by specific actions, resulting in a variable stepsize as a proxy for expected arrival time at either bus stops or traffic lights.

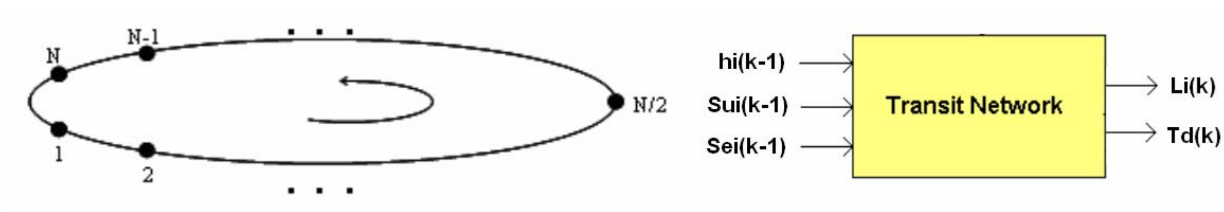

Figure 2. Transit network a) and the dynamic model of the bus system/traffic light b).

One major feature of this particular HPC approach, different from typical HPC schemes, is the double dimensionality of this specific dynamic modeling framework: spatial and temporal. The events are triggered when a bus arrives at either a bus stop or a traffic light (spatial dimension), and when that happens, the status of each bus is updated every time any other bus arrives at a specific spatial location (station or traffic light), which defines the corresponding variable time step (temporal dimension). Figure 2b) shows the main variables for the dynamic modeling of the system, which are function of a continuous and discrete time. Input variables at the instant before (k-1) generate the outputs at either continuous instant t or when a new event occurs (discrete instant k). As shown in Figure 2b), passengers boarding and alighting (demand) are modeled as a disturbance in the dynamic model scheme. Moreover, we define two state space variables in order to check the bus status and consequently trigger the events. These are the position of the bus at any instant t, $x_i(t)$, and the remaining time for the bus to reach the next stop, $T_i(t)$.

Consequently with the three strategies integrated in the scheme, the inputs of the dynamic model correspond to the control action variables, and are defined as follows:

- $Su_i(k-1)$: Passenger boarding action of bus i, at instant k-1.

$$Su_i(k-1) = \begin{cases} 1 & \text{if passengers are allowed to board bus } i \text{ at instant k-1} \\ 0 & \text{otherwise} \end{cases}$$

- $h_i(k-1)$: Holding action of bus i at instant k-1

where $h_i(k-1) = n_i \tau$ holding action was executed at k-$1, n_i \in \mathbb{Z}^+, \tau > 0$ known

- $Se_i(k-1)$: Traffic light action when the bus i reaches a traffic light at instant k-1.

where $Se_i(k-1) = s_i \kappa(k-1)$ traffic light action was executed at k-$1, s_i \in \mathbb{Z}^+$

Note that $\kappa(k-1) > 0$ depends on the instant at which the bus reaches the traffic light.

In the scheme in Figure 2b, the required output variables are the bus i load at discrete instant k, $L_i(k)$, and the departure time of bus i when the new event occurs at time k, $Td_i(k)$.

Let us start computing he position of the bus i at continuous time t as follows

$$x_i(t) = x_i(t_k) + \int_{t_k}^{t} v_i(\vartheta)d\vartheta \tag{1}$$

where t_k is the continuous instant at which the event k is triggered. The instantaneous speed $v_i(t) = \dot{x}_i(t)$ is simplified by assuming a constant speed whenever the vehicle is moving, and speed equals to zero otherwise. For example, in Figure 3 we show the speed representation of the trajectory of bus i while it is traveling from the station it reaches at instant k until the bus arrives to the next stop in its route (which is associated with future instant $k+d$).

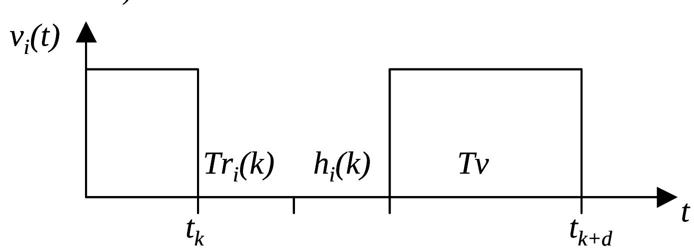

Figure 3. Example of bus trajectory between stops

In Figure 3, $Tr_i(k)$ is the time associated with passenger transference $R_i(k)$ and Tv is the travel time between two consecutive stations. As defined above, $h_i(k)$ is the holding time at the station to be decided by the dispatcher. In this context, analytically the speed can be computed as

$$v_i(t) = \begin{cases} 0 & t_k \le t \le t_k + h_i(k) + Tr_i(k) \\ v_0 & t_k + h_i(k) + Tr_i(k) \le t \le t_{k+d} \end{cases} \tag{2}$$

In order to trigger the next event of the dynamic model, the remaining time (measured from time *t*) for the bus *i* to reach the next stop is required, and can be computed as follows:

$$T_i(t) = t_k + h_i(k) + Tr_i(k) + Tv - t \tag{3}$$

With regard to the outputs of the dynamic model, first, let us define the estimated passenger load of bus *i* at instant *k+1* as follows:

$$\begin{aligned} \hat{L}_i(k+1) &= L_i(k) + R_i(k) \\ R_i(k) &= B_i(k)Su_i(k) - A_i(k) \end{aligned} \tag{4}$$

where $L_i(k)$ is the load of bus *i* at the previous instant k, and $R_i(k)$ is the net transference of passenger between instants *k* and *k+1*, that is, the number of passenger boarding bus *i* minus the number of passenger alighting from bus *i* during this interval of time, which corresponds to the disturbance of the dynamic model. $A_i(k)$ is the number of passenger alighting from bus *i* and $B_i(k)$ is the number of passenger boarding bus *i*, both defined between instants *k* and *k+1*. In addition, $L_i(k+1) \le L_{\max}\ i = 1 \ldots n$ is added as a boundary condition of the problem to check the physical capacity constraint.

Moreover, the estimated departure time of bus *i* at future discrete instant $k+\ell$ can be computed as

$$\hat{Td}_i(k+\ell) = t_k + h_i(k) + Tr_i(k) \tag{5}$$

Finally, the system based on the dynamic model must satisfy some physical operational constraints, such as capacity load (already stated above), precedence, demand consistency, etc.

The next step is to properly define a predictive objective function in order to take the re al time decisions and optimize the dynamic system. In this case, the expression as in (6) comprises four components. The first term quantifies the total passenger waiting time (capturing the regularization of bus headways), the second component measures the delay associated with passengers on-board a vehicle when held at a control station. The third component is the extra waiting time of passengers whose station is skipped by an expressed vehicle. Finally, the fourth term shows the extra delay due to holding at traffic lights.

$$\begin{aligned} J = \sum_{\ell=1}^{Np}\sum_{i=1}^{p}\Bigg[& \frac{\lambda_i(k+\ell)}{2}\hat{H}_i(k+\ell)^2 + \theta_1 \cdot \hat{L}_i(k+\ell)h_i(k+\ell-1) + \\ & + \theta_2 \cdot \lambda_i(k+\ell)\hat{H}_i(k+\ell)\hat{H}_{i+1}(k+\ell+z)\big(1 - Su_i(k+\ell-1)\big) + \theta_3 \cdot \hat{L}_i(k+\ell) \cdot Se_i(k+\ell-1)\Bigg] \end{aligned} \tag{6}$$

θ_1, θ_2 and θ_3 are weighting parameters. *Np* is the prediction horizon, *p* is the number of the buses and *N* is the number of the stops. Besides, $\lambda_i(k+\ell)$ is the passenger arrival rate at the stop where bus *i* is located at $k+\ell$. Finally, $\hat{H}_i(k+\ell)$ represents the predicted headway of bus *i* at the station reached when event $k+\ell$ happens, and can be computed as follows:

$$\hat{H}_i(k+\ell) = \hat{Td}_i(k+\ell) - \hat{Td}_{i-1}(k+\ell-z) \tag{7}$$

where $\hat{Td}_i(k+\ell)$ (defined in (5)) is the estimated departure time of bus *i* at future instant $k+\ell$ and $\hat{Td}_{i-1}(k+\ell-z)$ is the estimated departure time of precedent bus *i-1* at future instant $k+\ell-z$. The variable *z* represents the instant at which the previous bus *i-1* reached the same stop. Note that the definition of *z* is not the same as that of *d* provided before.

PROPOSED SOLUTION METHODS: ONGOING RESEARCH

The minimization of the objective function defined in (6) corresponds to a mixed integer non linear optimization problem, which is NP-Hard. In order to solve the bus system optimization problem, we define the sequences of future control actions associated with bus *i* at stops $\left(u_i(k),u_*(k+1),\cdots,u_*(k+Np\text{-}1)\right)$ by considering the passenger boarding and holding actions. Note that the control action ahead $u_*(k+\ell\text{-}1)=\left[Su_*\left(k+\ell\text{-}1\right) \quad h_*\left(k+\ell\text{-}1\right)\right]^{\mathrm{T}}$ is associated with an unknown bus that will arrive to an unknown stop that depends on the previous control action. The potential solutions belonging to the search space set are limited for both $Su_*\left(k+\ell\text{-}1\right)$ and $h_*\left(k+\ell\text{-}1\right)$. Thus, the optimization problem of HPC strategies for the bus system using the formulation defined here could be solved by using Explicit enumeration (EE) and Branch and Bound (BB). Both allow to solve mixed integer optimization problems (MIOP) (Floudas, 1995), but the elevated computational effort, especially in the case of EE, results in inefficient solutions for real-time problems. In that sense, Genetic Algorithms (GA) has proved to be an efficient tool to solve MIOP (Man et al., 1998). So, we propose the GA as an efficient tool for optimization of the bus system.

SOME PRELIMINARY TESTS AND FINAL COMMENTS

This is an ongoing research, and currently we have coded the proposed HPC strategy in a discrete event simulation environment. The proposed strategy is applied to a hypothetical bus corridor of 10 [km] comprising 20 stations evenly distributed over the bus route. By using proper average boarding, alighting and load profiles, in Figures 4a) and 4b) we show the movement (time-space graph) of a small fleet of 5 buses moving around the circuit. The simulation assumes uncertain demand dynamically arriving at stations by following a Poisson process. Figure 4a) shows the movement of buses without applying any control, while in Figure 4b), a holding strategy is applied, allowing buses to stay at stations for a fixed period of 20 seconds ($\tau_{ij} = 20$) whenever needed according to the HPC formulation (note that in the general model, the holding time τ_{ij} is also an optimization variable). As a result, we can see that the strategy allows the manager to maintain much more regular headways (b compared with a), which is the result of optimizing the cost function as in (9). In Figure 4c) we specifically allow holding only in one specific station (station 6), showing how the system can dynamically react in order to keep regular headways as well.

In this paper we have shown a hybrid predictive model to optimize in real time the performance of a public transport system along a linear corridor with uncertain demand at bus stops. The optimization is conducted by applying holding, expressing and transit signal priority. Currently, we are developing GA tools to efficiently take optimal real time decisions based on the proposed framework. As further research, we plan to work over more complex systems configurations, such as trunk schemes combined with feeder transit lines connected with transfer points. We also will test our schemes under a microscopic simulation environment in order to properly capture the dynamic effects of such a transit system.

Acknowledgments
This research was partially financed by the ACT-32 Project “Real Time Intelligent Control for Integrated Transit Systems”, Fondecyt Chile grants 1061156 and 1061261, and the Millennium Institute "Complex Engineering Systems”.

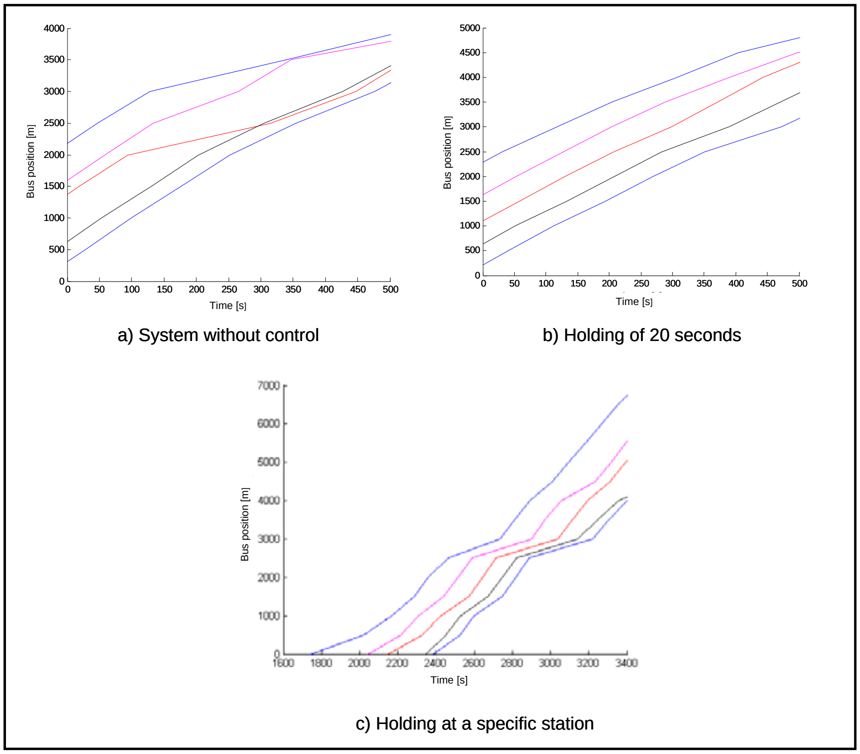

Figure 4. Illustrative HPC transit test results.

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