Routing Technicians under Stochastic Service Times: A Robust Optimization Approach

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Extended Abstract

1. Introduction

In this research, the objective is to solve the problem of routing repair technicians to service equipment distributed in client locations. Provided a list of requests for service, we can assign any one of K technicians to service them. Each request for service includes a deadline to begin the service W_i, and an estimated service time s_i, decided from the diagnosis of the problem at the time of the request. This problem is inspired by a real industrial application, in which about 40 to 50 requests have to be attended by 10 technicians, every day. This problem is naturally formulated as a classic Vehicle Routing Problem with Soft Time Windows (VRPTW).

The focus of this paper is the treatment of the stochastic aspects associated to this sequencing problem. In this application, the only significant uncertainty we identify is the uncertainty due to service times. The demand and number of customers are considered as deterministic and determined by the requests for service. In this particular problem, travel times between locations are much smaller than service times. In addition, variations on travel times are negligible, thus they are also considered as deterministic. On the other hand, the estimation of service times can be significantly different from the real observed service times. Actually, in this motivating application about 30% of the requests have estimates for service time that are incorrect due to a misdiagnosis of the problem. In such cases the service times typically raise considerably (in the order of hours).

In order to deal with the problem, we propose a robust optimization approach, where by robust solution we mean a solution that is efficient for all realizations of the problem uncertainty. The robust solution is obtained by solving the robust counterpart problem, whose goal is a solution that optimizes the worst case value over all data uncertainty. Robust solutions have the potential to be viable solutions in practice, since they tend not to be far from the optimal solution and significantly outperform the optimal solution in the worst case [6].

The formulation and solution of the robust problem is based on the robust counterpart for convex optimization problems from the current literature in mathematical programming [2, 3, 1]. This approach has been recently extended to integer programming problems [5, 4]. The general approach of robust optimization is to optimize against the worst instance that might arise due to data uncertainty by using a min-max objective. The resulting solution from the robust counterpart problem is insensitive to the data uncertainty, as it is the one that minimizes the worst case, and therefore is "immunized" against this uncertainty. The robust optimization methodology assumes that the uncertain parameters belong to a bounded uncertainty set, without additional distribution assumptions. Clearly the size of the uncertainty set influences the deviation from optimality of the robust solution. For fairly general uncertainty sets, the resulting robust counterpart problem is modestly larger than the original problem, and therefore has a comparable complexity.

2. Notation and Problem Definition

In this section we present the repair scheduling problem considered. Let $I_1 = \{1, \ldots, K\}$ represent the set of locations currently being serviced by the K technicians, and a set $I_2 = \{K+1, \ldots, I\}$ of locations that need to be scheduled. We assume that all technicians are sent to a dummy depot location $I+1$ after their schedule is finished. To simplify notation, let us define the arc set $A = \{(i,j) \mid i \in I_1 \cup I_2, j \in I_2 \cup \{I+1\},\ i \neq j\}$ that represents all feasible trips among locations. Let W_i be the upper bound of the time window for service at location i, and s_i the estimated service time at that location. The travel time from location i to location j is t_{ij} and all jobs have to be scheduled before the end of the work day L. Let variable x_{ij}^k indicate whether technician k services location i and then location j, or not. In addition, we use some auxiliary variables to represent the starting time of service at location i by technician k, w_{ik}, and a penalty for having technician k violating the soft time window constraint at location i, δ_{ik}. The deterministic problem (P) can be expressed as follows:

$$
\begin{aligned}
\text{(P)} \quad & \min_{x,w,\delta} \beta \sum_{k\in K}\sum_{i\in I_2} \delta_{ik} + (1-\beta)\sum_{k\in K}\sum_{(i,j)\in A} t_{ij}x_{ij}^k & & (1)\\
\text{s.t.} \quad & \sum_{k\in K}\sum_{j:(i,j)\in A} x_{ij}^k = 1 & i \in I_1 \cup I_2 \quad & (2)\\
& \sum_{j:(i,j)\in A} x_{ij}^k - \sum_{j:(j,i)\in A} x_{ji}^k = b_i^k & i \in I_1 \cup I_2 \cup \{I+1\},\ k\in K \quad & (3)\\
& x_{ij}^k \in \{0,1\} & (i,j)\in A,\ k\in K \quad & (4)\\
& w_{ik} + s_i + t_{ij} - w_{jk} \le (1-x_{ij}^k)M & (i,j)\in A,\ k\in K \quad & (5)\\
& w_{ik} \le L \sum_{j:(j,i)\in A} x_{ji}^k & i\in I_2,\ k\in K \quad & (6)\\
& w_{ik} - \delta_{ik} \le W_i & i\in I_2,\ k\in K \quad & (7)\\
& w_{ik}, \delta_{ik} \ge 0 & i\in I_2,\ k\in K\ . \quad & (8)
\end{aligned}
$$

Where $b_i^k = 1$ if $i = k$, $b_i^k = -1$ if $i = I+1$ and 0 otherwise. We also need to define $w_{ik} = 0$ for all locations $i \in I_1$ and $k \in K$ as these values appear in the constraint (5).

The robust model considers the uncertainty by a closed, convex and bounded uncertainty set. Let $s \in \mathcal{U}$, where $\mathcal{U}$ is a given uncertainty set. Then the robust model is the same as the previous one, but replacing the constraint (5) by (9):

$$w_{ik} + s_i + t_{ij} - w_{jk} \le (1-x_{ij}^k)M \quad (i,j)\in A,\ k\in K,\ s\in\mathcal{U} \qquad (9)$$

Therefore the robust version of the problem has infinitely many constraints, one per $s \in \mathcal{U}$, in addition to the difficult routing problem it entails. We then propose an uncertainty set that leads to a compact representation of these infinitely many constraints for solving the robust problem, as presented in the following section.

3. Solution of the Robust Problem

We assume a linear uncertainty in service times, i.e. each service time can vary between $\bar{s}_i \le s_i \le \bar{s}_i + \gamma_i$, However, statistics show that a realistic worst case scenario faced by technicians in a day has service times that are far smaller than the sum of the worst case service times of every client in the route. Therefore, we will define an upper bound U_k on the total deviation of service times faced by technician k.

In this setting, the uncertainty set depends on the routing solution $x_r \in \mathcal{X}$. Let $P^k(x_r)$ denote the path followed by technician k under the routing solution $x_r \in \mathcal{X}$. We define the uncertainty set $\mathcal{U}(x_r)$ by letting service times deviate in $[\bar{s}_i, \bar{s}_i + \gamma_i]$ such that every technician faces up to $U_k = \alpha \sum_{i\in P^k(x_r)} \gamma_i$, where α is a parameter that controls the level of robustness:

$$\mathcal{U}(x_r) = \left\{ s \mid \bar{s}_i \le s_i \le \bar{s}_i + z_i,\ 0 \le z_i \le \gamma_i,\ i \in I_2,\ \sum_{(q,l)\in P^k(x_r)} z_q \le U_k \right\} \tag{10}$$

Then, a worst case scenario exhibits two properties: it uses up all possible variation, which is U_k, and the delays in service occur in the first clients visited. In order to satisfy these conditions, we replace constraint (5) by (11) and add the new constraints (12)-(16) into the model (P). We then define $y_{ji} \in \{0,1\}$, which is equal to one when both, machine j satisfied $z_j = \gamma_j$ and there is a path that visits i right after j. Analytically,

$$w_{ik} + s_i + z_i + t_{ij} - w_{jk} \le (1 - x^k_{ij})M \ (i,j) \in A,\ k \in K \tag{11}$$

$$\textstyle\sum_{(q,l)\in P^k(x_r)} z_q \ge \alpha \sum_{i\in P^k} \gamma_i \tag{12}$$

$$0 \le z_i \le \gamma_i \textstyle\sum_{j:(j,i)\in A} y_{ji} \qquad i \in I_2 \tag{13}$$

$$y_{ji} \le x^k_{ji} \qquad (j,i) \in A \tag{14}$$

$$y_{ji} \le \tfrac{z_j}{\gamma_j} \qquad (j,i) \in A \tag{15}$$

$$y_{ji} \in \{0,1\} \qquad (j,i) \in A \tag{16}$$

Constraint (13) ensures that variations are possible on a client that is immediately preceded by a machine with full variation in service time and constraints (14)-(16) enforce the definition of y_{ji}.

4. Implementation and Solution of the Problem

The above robust counterpart is solved by using column generation, where the master problem is a set partitioning model which selects the optimal routes among a pool of routes. The subproblem generates new routes,and is formulated under a Constraint Programming framework (CP).

The CP problem is NP-hard, and for instances of large size the computation time for setting the optimal solutions could be very high. However, in this case the size of each column generation problem is in the range of three to six clients, which allows us to obtain the optimal solutions with little computational effort. The CP constraints are based on the logical relations of the problem. In particular, the time windows constraint aid significantly in reducing the number of feasible solutions. This implies that in the search tree a large percentage of nodes are eliminated.

In order to test the real quality of the above mentioned column generation approach, we improved the solution scheme by implementing a Branch and Price method, which allow us to find additional negative reduced cost routes along the Branch and Bound tree, by inspecting each node of the tree, and not only the root. Here, we implement the Branching Strategy proposed by Ryan and Foster [7] for generic set partitioning problems in order to speed up the search over the tree.

The model was coded in ILOG Concert Technology and solved using ILOG Cplex 9.0 for the master problem and ILOG Solver 6.0 for the subproblem.

5. Preliminary Results

In order to estimate the benefits of the robust approach, we compare the routes obtained with the robust model against those obtained by running the deterministic version. From these two routing solutions, we simulate 1000 scenarios, each with the same travel times and time windows but different service times following a Weibull Distribution (which nicely represents service time).

The figure 1 shows the expected value of the deterministic objective function minus the robust one, for one problem instance over 1000 simulations. We show the sensitivity for $\beta = \{0.3, 0.6, 0.9\}$, and for different values of the robust parameter $\alpha = \{0.3, 0.6, 0.9, 1.2\}$.

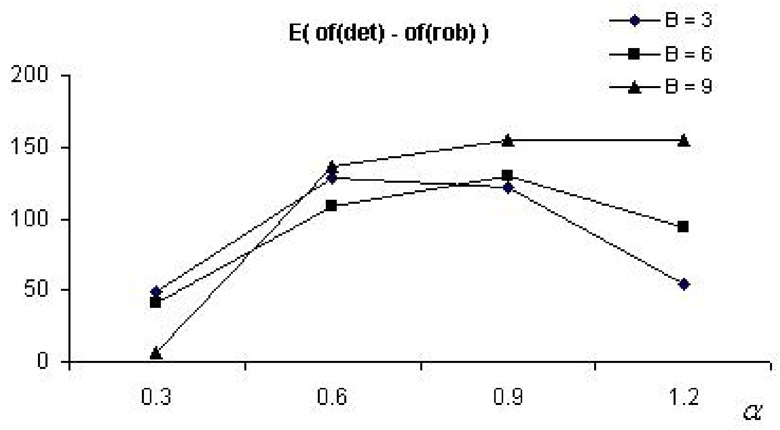

Figure 1: Expected value of the difference between the deterministic and the robust solution for 1000 simulations.

Note that when the violation of the time windows is more relevant than the travel time component in the objective function (say $\beta = 0.9$), the robust solution is more likely to be lower than the determinist one for different α values. That because $E[of(det)] - E[of(rob)]$ is always greater than 0. And for lower $\beta's$ the result is similar for all α values up to 0.9. This result shows the benefits of the robust model with respect to the deterministic routing, in cases of the same type as those studied here, where service times are highly uncertain. We observed the same behavior in most of the instances we solved.

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