

An Optimization Model for Empty Container Reposition under Uncertainty

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Introduction.

Container shipping companies provide scheduled maritime transportation world-wide. A significant factor for their competitiveness is the availability of empty containers in ports to meet customer orders. Due to the global trade imbalance, some ports tend to accumulate empty containers, resulting in unnecessary storage costs, while others face shortages that expose shipping companies to the risk of competitors providing containers as requested. As a consequence, shipping companies must be reactive to meet customer needs and perform the maritime repositioning of empty containers.

A major difficulty in this operation is the many sources of uncertainty regarding, e.g., the number of containers that may be requested in the future, the time when empty containers become available, and the vessel capacity for empty containers. Several deterministic models were proposed (e.g., Choong et al., 2002), but they take into account a single realization of uncertain parameters. Stochastic optimization models were presented as well (e.g., Cheung and Chen, 1998). However, they require a good knowledge of random variable distributions to avoid low quality solutions.

We present a description of a general transportation network over which empty container repositioning is performed. We then propose an optimization model to solve this issue for a heterogeneous fleet of empty containers, taking into account uncertainty through a set of representative scenarios. A weight can be assigned to each scenario to characterize its relative importance. Weights may represent probabilities of occurrence or subjective parameters assigned by managers according to the particular application. Finally, the most significant results of the study are introduced and discussed.

Problem description.

Empty containers can be available in ports due to past inventory, trucks and trains arriving from the landside, and vessels arriving from the seaside. They can be dispatched to the landside to serve exporters, loaded on vessels to reach shortage ports. Empty containers can be stored in ports once assigned to scheduled vessels or kept as not-yet-assigned inventory. In the first case, shipping companies must decide “now” to which vessels scheduled at ports in future time-periods to assign the arriving empty containers. In the second case shipping companies can store arriving empty containers and decide later.

Several container types are considered and their different sizes result in different utilizations of the available space. In order to take into account the impact of current decisions on the future state of the system, we need to explicitly consider the time perspective. Moreover, some decisions must be made when there is only a partial knowledge of some crucial parameters. For instance, vessels traveling over short maritime distances do not offer sure information about their composition. Indeed, while they are stopping in a given port, shipping companies must decide how many containers will be loaded and unloaded in the next one. Finally, another major source of uncertainty is represented by the so-called “Cut and Run” policy, that is, when berthed vessels have delays in

their schedule, sometimes terminal operations are concluded without loading empty containers. For instance, due to adverse climatic conditions, some ports interrupt their activity and, when they restart their operations, empties are “cut” to gain time.

Figure 1 shows as an example of a time-extended network made up of ports, denoted by letters A and C, where empty containers must be assigned to vessels, and a port, indicated by letter B, where they can be kept unassigned. Numbers from 1 to 4 indicate four lines operated by four different vessels. For instance, vessel 1 will arrive in period 4 at port B. This port can load on this vessel empty containers available from the third period and unload empties, which become available from the fifth period. It is worth noting that shipping companies cannot decide now vessels for empty containers available in the first period at ports A and C, because such a decision was made before their arrival. As regards port B, shipping companies must decide “now” the utilization of containers that have become available before the first period.

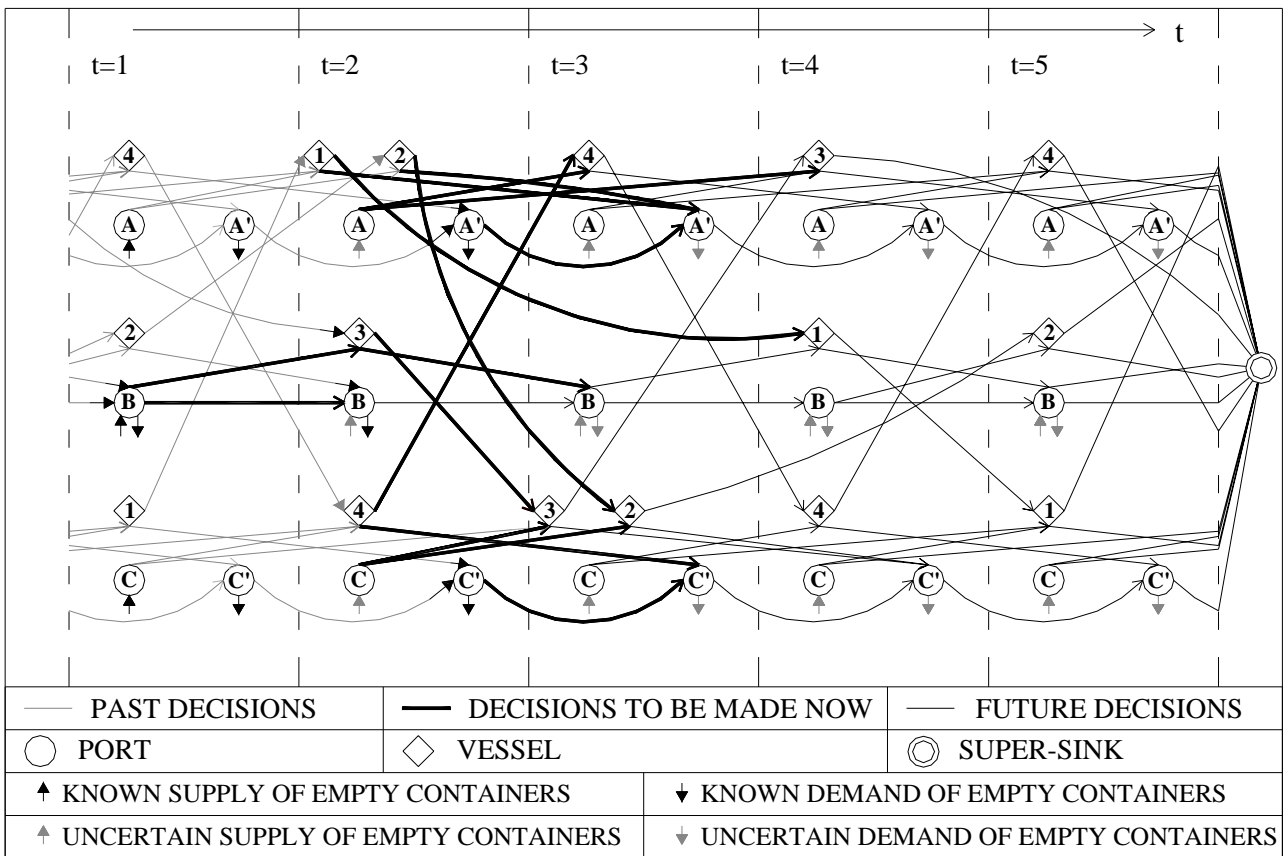


Figure 1. Sample time-extended network.

Optimization model.

Shipping companies must decide the number of empty containers repositioned, stored, loaded, unloaded, and kept on vessels. As indicated before, we take into account two categories of decisions for empty containers stored in ports, depending on the requirement to be assigned or unassigned to scheduled vessels. We consider a multi-period network and assume that decisions are implemented in a rolling horizon fashion.

Regarding notation, we consider a set P of container types, a set T of contiguous time-periods, a set V of vessels, and a set G of scenarios associated with weights $w_g, g \in G$. Let H_1 be the set of ports in which unassigned empty containers can be stored and let H_2 represent the set of ports in which this option is not allowed. Moreover we indicate by θ the time up to which decisions must be the same for every scenario. Furthermore, we denote by t' the interval of time between the arrival of empty containers at ports of set H_2 and the berthing of vessels, to which such containers have to be assigned.

The notation $b_{i,t}^{p,g} \geq 0 (\leq 0)$ represents for a port $i \in H_1$ the supply (demand) of empty containers of type $p \in P$ at time $t \in T$ in scenario $g \in G$. Each port in H_2 is represented by two nodes i and i' . The first node $i \in H_2$ is related to the supply $s_{i,t}^{p,g}$ of empty containers of type $p \in P$ available in that port at time $t \in T$ in scenario $g \in G$. The node $i' \in H_2$ is associated with the demand $d_{i',t}^{p,g}$ of empty containers of type $p \in P$ requested in that port at time $t \in T$ in scenario $g \in G$. Proper capacity constraints are proposed to avoid storing and repositioning an inadmissible number of empty containers. Moreover, since we are managing a heterogeneous fleet of containers of different sizes, we consider the largest container type \bar{p} and express capacities in terms of number of available slots able to include \bar{p} -type containers. Given the container type $q \neq \bar{p}$, the available space for q -type containers can be determined using conversions factor $a_{q\bar{p}}$, introduced by Crainic, et al. (1993). In the notation adopted hereafter, $U^h(i_t^{\bar{p}}, i_{t+1}^{\bar{p}})$ represents the storage capacity of port $i \in H_1$, $U^h(i_t^{\bar{p}}, i_{t+1}^{\bar{p}})$ the storage capacity of port $i \in H_2$, and $U_{v,g}^r(i_t^{\bar{p}}, j_{t+\tau}^{\bar{p}})$ the residual capacity for empty containers carried by vessel $k \in V$ traveling among ports $i \in H_1 \cup H_2$ and $j \in H_1 \cup H_2$ in scenario $g \in G$.

The sources of uncertainty involved in the issue are $b_{i,t}^{p,g}$, $s_{i,t}^{p,g}$, $d_{i',t}^{p,g}$, and $U_{v,g}^r(i_t^{\bar{p}}, j_{t+\tau}^{\bar{p}})$. The problem is presented as an integer programming model whose decision variables are denoted by letter x and costs by letter c , where l means “loaded”, u “unloaded”, r “repositioned”, and h “hold”.

1. Variable $x_g^l(i_t^p, v_{t+1}^p)$ indicates the number of empty containers of type $p \in P$, available in port $i \in H_1$ at time $t \in T$, to be loaded on vessel $v \in V$ arriving at time $(t+1) \in T$ in scenario $g \in G$; $c^l(i_t^p, v_{t+1}^p)$ represents the related unitary cost.
2. Variable $x_g^l(i_t^p, v_{t+t'}^p)$ indicates the number of empty containers of type $p \in P$, available in port $i \in H_2$ at time $t \in T$, to be loaded on vessel $v \in V$ arriving at time $(t+t') \in T$ in scenario $g \in G$; $c^l(i_t^p, v_{t+t'}^p)$ represents the related unitary cost.
3. Variable $x_g^u(v_t^p, i_{t+1}^p)$ indicates the number of empty containers of type $p \in P$ to be unloaded in scenario $g \in G$ from vessel $v \in V$ arriving at time $t \in T$ at port $i \in H_1$, where they become available at time $(t+1) \in T$; $c^u(v_t^p, i_{t+1}^p)$ represents the related unitary cost.
4. Variable $x_g^u(v_t^p, i'_{t+1}^p)$ indicates the number of empty containers of type $p \in P$, to be unloaded in scenario $g \in G$ from vessel $v \in V$ arriving at time $t \in T$ at port $i' \in H_2$, where they become available at time $(t+1) \in T$; $c^u(v_t^p, i'_{t+1}^p)$ represents the related unitary cost.
5. Variable $x_{v,g}^r(i_t^p, j_{t+\tau}^p)$ indicates the number of empty containers of type $p \in P$, to be repositioned in scenario $g \in G$ by vessel $v \in V$ between ports $i \in H_1 \cup H_2$ and $j \in H_1 \cup H_2$ with respective berthing time $t \in T$ and $(t+\tau) \in T$; $c_v^r(i_t^p, j_{t+\tau}^p)$ represents the related unitary cost.
6. Variable $x_g^h(i_t^p, i_{t+1}^p)$ indicates the number of not-yet-assigned empty containers of type $p \in P$ to be stored in port $i \in H_1$ between times $t \in T$ and $(t+1) \in T$ in scenario $g \in G$; $c^h(i_t^p, i_{t+1}^p)$ represents the related unitary cost.
7. Variable $x_g^h(i_t^p, i'_{t+1}^p)$ indicates the number of empty containers of type $p \in P$ to be stored in port $i \in H_2$ between times $t \in T$ and $(t+1) \in T$ in scenario $g \in G$; $c^h(i_t^p, i'_{t+1}^p)$ represents the relative cost.

The resulting mathematical model can be expressed as follows:

$$\begin{aligned}
& \min \sum_{g \in G} w_g \left\{ \sum_{t \in T} \sum_{p \in P} \left[\sum_{i \in H_1} \left(c^h(i_t^p, i_{t+1}^p) x_g^h(i_t^p, i_{t+1}^p) + \sum_{v \in V} c^l(i_t^p, v_{t+1}^p) x_g^l(i_t^p, v_{t+1}^p) \right) + \right. \right. \\
& + \sum_{i' \in H_2} c^h(i_t^{p'}, i_{t+1}^{p'}) x_g^h(i_t^{p'}, i_{t+1}^{p'}) + \sum_{i \in H_2} \sum_{v \in V} c^l(i_t^p, v_{t+1}^{p'}) x_g^l(i_t^p, v_{t+1}^{p'}) + \\
& \left. \left. + \sum_{v \in V} \left(c^r(i_t^p, j_{t+\tau}^p) x_{v,g}^r(i_t^p, j_{t+\tau}^p) + \sum_{i \in H_1} c^u(v_t^p, i_{t+1}^p) x_g^u(v_t^p, i_{t+1}^p) + \sum_{i' \in H_2} c^u(v_t^p, i_t^{p'}) x_g^u(v_t^p, i_t^{p'}) \right) \right] \right\} \quad (1)
\end{aligned}$$

subject to

$$\sum_{v \in V} x_g^l(i_t^p, v_{t+1}^p) + x_g^h(i_t^p, i_{t+1}^p) - \sum_{v \in V} x_g^u(v_{t-1}^p, i_t^p) - x_g^h(i_{t-1}^p, i_t^p) = b_{i,t}^{p,g} \quad \forall i \in H_1, \forall t \in T, \forall p \in P, \forall g \in G \quad (2)$$

$$\sum_{v \in V} x_g^l(i_t^p, v_{t+1}^{p'}) = s_{i,t}^{p,g} \quad \forall i \in H_2, \forall t \in T, \forall p \in P, \forall g \in G \quad (3)$$

$$x_g^h(i_t^{p'}, i_{t+1}^{p'}) - \sum_{v \in V} x_g^u(v_{t-1}^p, i_t^{p'}) - x_g^h(i_{t-1}^{p'}, i_t^{p'}) = d_{i,t}^{p,g} \quad \forall i' \in H_2, \forall t \in T, \forall p \in P, \forall g \in G \quad (4)$$

$$x_{v,g}^r(i_t^p, j_{t+\tau}^p) + x_g^u(v_t^p, i_{t+1}^p) + x_g^u(v_t^p, i_t^{p'}) - x_{v,g}^r(j_{t-\tau}^p, i_t^p) - x_g^l(i_{t-1}^p, v_t^p) - x_g^l(i_{t-1}^{p'}, v_t^{p'}) = 0 \quad \forall v \in V, \forall t \in T, \forall p \in P, \forall g \in G \quad (5)$$

$$x_g^h(i_t^{\bar{p}}, i_{t+1}^{\bar{p}}) + \sum_{q \in P | q \neq \bar{p}} a_{q\bar{p}} \cdot x_g^h(i_t^q, i_{t+1}^q) + \sum_{v \in V} x_g^l(i_t^{\bar{p}}, v_{t+1}^{\bar{p}}) + \sum_{q \in P | q \neq \bar{p}} \sum_{v \in V} x_g^l(i_t^q, v_{t+1}^q) \leq U^h(i_t^{\bar{p}}, i_{t+1}^{\bar{p}}) \quad \forall i \in H_1, \forall t \in T, \forall g \in G \quad (6)$$

$$x_g^h(i_t^{\bar{p}'}, i_{t+1}^{\bar{p}'}) + \sum_{q \in P | q \neq \bar{p}'} a_{q\bar{p}'} \cdot x_g^h(i_t^q, i_{t+1}^q) + \sum_{v \in V} x_g^l(i_t^{\bar{p}'}, v_{t+1}^{\bar{p}'}) + \sum_{q \in P | q \neq \bar{p}'} \sum_{v \in V} x_g^l(i_t^q, v_{t+1}^q) \leq U^h(i_t^{\bar{p}'}, i_{t+1}^{\bar{p}'}) \quad \forall i' \in H_2, \forall t \in T, \forall g \in G \quad (7)$$

$$x_{v,g}^r(i_t^{\bar{p}}, j_{t+\tau}^{\bar{p}}) + \sum_{q \in P | q \neq \bar{p}} a_{q\bar{p}} \cdot x_{v,g}^r(i_t^q, j_{t+\tau}^q) \leq U_{v,g}^r(i_t^{\bar{p}}, j_{t+\tau}^{\bar{p}}) \quad \forall v \in V, \forall t \in T, \forall g \in G \quad (8)$$

$$x_g^l(i_t^p, v_{t+1}^p) = x_f^l(i_t^p, v_{t+1}^p) \quad \forall i \in H_1, t = \{1, \dots, \theta\}, \forall p \in P, \forall g, f \in G \quad (9)$$

$$x_g^l(i_t^p, v_{t+1}^{p'}) = x_f^l(i_t^p, v_{t+1}^{p'}) \quad \forall i \in H_2, t = \{1, \dots, \theta\}, \forall p \in P, \forall g, f \in G \quad (10)$$

$$x_g^u(v_t^p, j_{t+1}^p) = x_f^u(v_t^p, j_{t+1}^p) \quad \forall i \in H_1, t = \{1, \dots, \theta\}, \forall p \in P, \forall g, f \in G \quad (11)$$

$$x_g^u(v_t^p, j_{t+1}^{p'}) = x_f^u(v_t^p, j_{t+1}^{p'}) \quad \forall i \in H_2, t = \{1, \dots, \theta\}, \forall p \in P, \forall g, f \in G \quad (12)$$

$$x_{v,g}^r(i_t^p, j_{t+\tau}^p) = x_{v,f}^r(i_t^p, j_{t+\tau}^p) \quad \forall v \in V, t = \{1, \dots, \theta\}, \forall g, f \in G \quad (13)$$

$$x_g^h(i_t^p, i_{t+1}^p) = x_f^h(i_t^p, i_{t+1}^p) \quad \forall i \in H_1, t = \{1, \dots, \theta\}, \forall p \in P, \forall g, f \in G \quad (14)$$

$$x_g^h(i_t^{p'}, i_{t+1}^{p'}) = x_f^h(i_t^{p'}, i_{t+1}^{p'}) \quad \forall i \in H_2, t = \{1, \dots, \theta\}, \forall p \in P, \forall g, f \in G \quad (15)$$

where $t \pm \tau$, $t \pm t'$ and $t \pm l$ must belong to T . All decision variables take only non-negative integer values.

The objective function (1) minimizes the cost of loading, unloading, repositioning and storing empty containers over a maritime network. Using network notation, constraint set (2) represents

flow conservation of empty containers of every type $p \in P$ in each node $i \in H_1$ at every time $t \in T$ over each scenario $g \in G$. Constraint set (3) requires to assign to vessels empty containers available in each node $i \in H_2$ at every time $t \in T$ over each scenario $g \in G$. Constraint set (4) imposes to satisfy the demand of empty containers associated with each node $i' \in H_2$ at every time $t \in T$ over each scenario $g \in G$. Constraint set (5) represents flow conservation of p -type containers for the for each vessel $k \in V$, berthing at time $t \in T$ in port $i \in H_1 \cup H_2$. Constraint sets (6) and (7) ensure that inventory level of empty containers stored does not exceed a value expressed in number of containers of a given type $\bar{p} \in P$. Constraint set (8) guarantees that containers repositioned between ports does not exceed the space available for empties on vessels. Constraint sets from (9) to (15) represent the non-anticipativity conditions.

Main results and conclusions.

We consider several reposition problems having up to 15 container types and 1500 scenarios. To solve numerical instances, the well-known solver Cplex (1995) is used. In our computational tests, problems are solved in less than 150 seconds, which is a time suitable for the operating needs of the shipping industry. The resulting reposition plan will be dispatched to ports to plan in time their internal activity (for instance they must organize the so-called *housekeeping*).

A major research perspective in this issue consists of estimating how many scenarios should be taken into account. Moreover the model exhibits strong algebraic structures (networks, commodities and scenarios) that can be exploited to develop specialized resolution techniques.

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