QUEUEING MODELS REVISTED:
ANALYSIS OF VEHICULAR DELAYS AND QUEUES USING ADAPTIVE CONTROL AT INTERSECTIONS AND RAMPS

Pitu B. Mirchandani and Ning Zou
The ATLAS Research Center
Systems and Industrial Engineering Department
The University of Arizona

ABSTRACT

Microsimulation models are normally used to evaluate traffic-adaptive signal control systems. In this paper develops an analytical approach for this evaluation based on queuing models. In particularly, the model is developed for an adaptive control strategy that was originally proposed by Newell (1998), where a signal serves two movements alternatively. In this strategy, the first movement is served until the queue dissipates, then the second movement is served until the queue dissipates, then the signal goes back to serving the first movement and this cyclic process repeats. The strategy is an approximation of the RHODES strategy (Mirchandani and Head, 2001), which essentially switches to the next phase in a given phase order after “sufficiently” serving the current phase. Newell compared such a strategy with rolling horizon using a continuous-time formulation.

There have been several continuous-time models for signal control with fixed timing plans, including Stephanopoulos and Michalopoulos (1978), Webster and Cobbe (1958) and Newell (1989). To obtain distribution of queue lengths at the end of each phase for an intersection with fixed timing, Olszewski (1990) has presented a discrete stochastic model.

In this paper we develop a new queuing model for analyzing vehicular delays and queue distributions at an undersaturated intersection operating with a two-phase signal, where the vehicle arrival process is Poisson, for both *fixed-timing signal control* and *traffic-adaptive signal control*. Numerical results for the two control strategies are provided and compared.

Briefly, the paper contains the following results:

1. *Calculation of the Busy Period Distribution at Signalized Intersection*: We model the busy period (i.e. when vehicles are passing through the intersection) distribution of the intersection given that multiple vehicles are waiting at the beginning of a green phase. Using numerical method we calculate the distributions for different *v/c* ratios.

2. *Analysis of an Intersection with Fixed-timing Signal Control*: For this case we assume that the service time per vehicle of each direction is constant and we define a cycle as Phase Duration 1(Φ_1) plus Phase Duration 2(Φ_2). Each Phase Duration is composed of a constant all-red clearance time plus a green phase serving one specific direction (see Figure 1). Signal settings for fixed-timing control system are based on the average arrival rates and service rates in the two directions. If vehicle arrival processes are deterministically uniform (i.e. at a *constant* rate), it’s quite straightforward to get the signal settings that minimize average delays. For Poisson arrivals, with the *same* average arrival rates, these fixed time settings are no longer optimal. Therefore, a multiplicative factor □ is imposed to both green-phase lengths and an optimal value of □ is found that minimizes average delay. Of course, as would be expected in this setting, the delays with given constant arrival rates are much less than when the arrivals are Poisson with same average rate. In a way, for fixed-time signals, constant arrivals is the best-case scenario and may be considered as some sort of lower bound.

When Φ_1 and Φ_2 are fixed, vehicle delays and queues in the two directions are independent to each other and the analyses for the two directions are uncoupled. Since the intersection is undersaturated the initial queue size can be set arbitrarily since it does not effect limiting distributions. Thus, with this model, we can calculate the steady-state values for several traffic characteristics, including average delays, average queue lengths, maximum queue length, their variances, and so on.

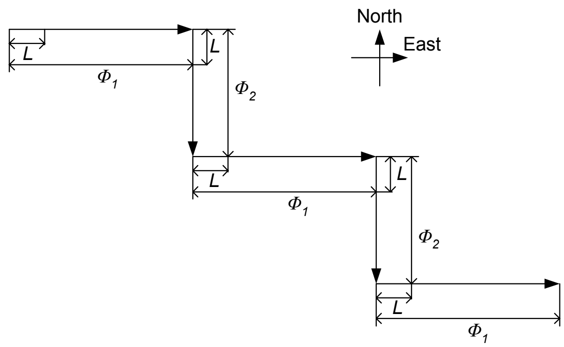
Analysis of an Intersection with Adaptive Traffic Signal: In this case the service time per vehicle and arrival process are the same, but, because of the adaptive signal strategy and the Poisson arrival processes, the cycle lengths are not fixed. Suppose the first cycle is defined as Phase Duration 1(Φ_1) plus Phase Duration 2(Φ_2), the second cycle as Phase Duration 3(Φ_3) plus Phase Duration 4(Φ_4), and so on (see Figure 2). Our adaptive signal strategy serves one direction till its queue is emptied, and then it switches to the other direction, and so on. That is, unlike the fixed-cycle case, no residual queue can exist at the end of each green phase. Thus the length of each green phase is a discrete random variable that is proportional to the total number of vehicles served.

In the analysis for this case, we need to consider both the directions simultaneously because consecutive Phase Durations are related until the both queues are emptied, then the overall process *renews* when a new "busy period" begins. For example, assuming Φ_i serves W-E direction, length of Φ_i determines the red phase length of N-S direction, thus influences the queue length distribution at the beginning of the green phase of Φ_{i+1}.

In the analysis we simply set Φ_1 equal to the length of the first Phase Duration for the fixed-timed case and then, using the resulting busy period distribution, it is straightforward to get the queue distribution in N-S direction at the beginning of the green phase in Φ_2, as well as characteristic parameters of N-S direction in the cycle Φ_1+Φ_2. From the distribution of Φ_2 we can get the queue distribution in W-E direction at the beginning of the green phase in Φ_3, which is slightly different from the first Phase Duration because Φ_2 is now a random variable. We then analyze the W-E direction for the cycle *(Φ_2+Φ_3)*. This process continues till the system converges to a steady state value.

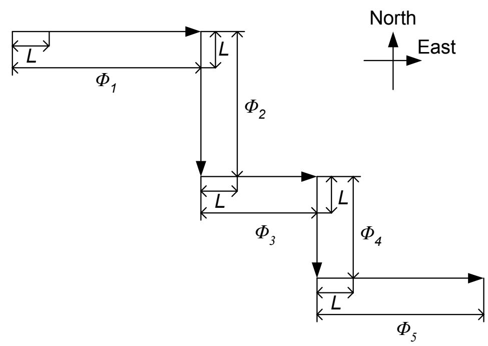
A numerical algorithm is developed to compute steady-state performance measures. Comparison of numerical results shows the benefits of adaptive signal control over optimized fixed-timing control. We show specifically that, with adaptive control, the average delays are significantly decreased over the fixed-timing case, and in fact are close to the best possible situation that may arise when vehicles arrive in constant streams. We also compare the results with simulation-based results and, indeed, the analytical models predict well the simulation results.

This type of analysis can also be used for locally adaptive ramp metering where the metering rate changes depending on arrivals and queues at an on-ramp. We develop a model to analyze a simple strategy where the metering rate increases (i.e. more cars are let through the ramp) when a spillback detector reaches a specified occupancy rate.



L : All-red clearance time,
Φ_1 : L plus green phase serving W-E direction,
Φ_2 : L plus green phase serving N-S direction

Fig. 1 Definition of Phase Durations for Fixed Timing



L : All-red clearance time,
Φ_{2i+1} : L plus green phase serving W-E direction,
Φ_{2i+2} : L plus green phase serving N-S direction,
$i = 0, 1, 2, \ldots$

Fig. 2 Definition of Phase Durations for Adaptive Timing

References

[1] P. B. Mirchandani and L. Head (2001), "A Real-Time Traffic Signal Control System: Architecture, Algorithms and Analysis". *Transportation Research Part C*, **9**.

[2] G. F. Newell (1998), "The Rolling Horizon Scheme of Traffic Signal Control", *Transportation Research Part A*, **32**.

[3] G. Stephanopoulos and P. G. Michalopoulos (1979), "Modeling and Analysis of Traffic Queue Dynamics at Signalized Intersections", *Transportation Research Part A*, **13**.

[4] F. V. Webster, and B. M. Cobbe (1958), "Traffic Signals", *Road Research Technical Paper*, **39**, Her Majesty's Stationery Office, London.

[5] G. F. Newell (1989), *Theory of Highway Traffic Signals*, UCB-ITS-CN-89-1, Institute of Transportation Studies, University of California.

[6] P. S. Olszewski (1990), "Modeling of Queue Probability Distribution at Traffic Signals", *Proceedings of the 11th International Symposium on Transportation and Traffic Theory*, Tokyo, Elsevier, New York.