A MULTI-AGENT APPROACH TO DYNAMIC TRAFFIC ASSIGNMENT BASED ON ACTIVITY

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ABSTRACT

The road choice behavior of travelers is closely related to their activity planning and location choice. This approach has led to the increasing development of multi-agent and activity-oriented modeling. This work attempts to model travelers' dynamic departure time/route/destination choice behavior in a queuing network. In this respect, we propose an activity-based predictive dynamic traffic assignment model based on a multi-agent approach with two interacting levels: the travelers' adaptive reaction level and the network propagation level. For the first level, traffic conditions change according to travelers' departure time/route/destination choices, dynamic traffic information and network supply constraints. En-route dynamic traffic assignment is reflected by travelers' strategies in response to the traffic state and the activity distribution information. For the second level, traffic congestion is modeled by point queue dynamics concept on a network. As a solution method of the predictive equilibrium, we propose an ACO (ant colony optimization) algorithm based on a time-dependent multi-type pheromone scheme in order to solve the proposed dynamic traffic assignment model. This algorithm focuses on how to provide dynamic on-route information and off-route information to guide travelers to make the best travel decision in a dynamic environment. The algorithm can be adapted to communication and information exchange schemes closer to actual traveler behaviour. A numerical example is given to illustrate the performance of the proposed method.

***Keywords*:** Dynamic traffic assignment; Activity-based analysis; Departure time choice; Ant colony optimization

1. INTRODUCTION

The road choice behavior of travelers is closely related to their activity planning and location choice [1]. This approach has led to the increasing development of multi-agent and activity-oriented modeling [2], [3]. The growing interest for using intelligent agents also in applications is illustrated by [4]. This work attempts to model travelers' dynamic departure time/route/destination choice behavior in a queuing network. Previous work in the same direction includes [5] and [6]. In this respect, we propose an activity-based predictive dynamic traffic assignment model based on two interaction levels: travelers' adaptative reaction level and network propagation level. For the first level, traffic conditions change according to travelers' departure time/route/destination choice behavior, dynamic traffic information and network supply constraints. En-route dynamic traffic assignment is reflected by traveler's strategies in response to traffic states and activity distribution information. For the second level, traffic congestion is modeled by queue dynamics concept, following ideas introduced in [7]. For the solution method, we propose a multi-agent approach based on a time-dependent multi-type pheromone scheme to solve the activity-based dynamic traffic assignment problem. This algorithm focuses on how to provide dynamic on-route (local traffic condition) information and off-route information (travelers' (agents') experience) to guide agents to take the best travel decision in a dynamic environment. We develop a discrete-event traffic simulator to simulate the travelers' behavior of adaptation to network traffic characteristics while satisfying their activity requirements, and to approximate the network dynamics equilibrium.

2. MODEL FORMULATION

Notation

Network variables

$G(V, E)$	network of interest composed of a set of nodes V and a set of directed arcs E
$o(k)$	origin node of the origin-destination pair k
$d(k)$	destination node of the origin-destination pair k
$M(i)$	set of entering links of node i
$N(i)$	set of outgoing links of node i
$d_o(t)$	demand flow rate from origin o at time t
D_o	total demand from origin o
$A_{ij}(t)$	cumulative number of vehicles entering link (i, j) at time t
$D_{ij}(t)$	cumulative number of vehicles leaving link (i, j) at time t
$\sigma_{ij}(t)$	inflow capacity of link (i, j) at time t
$\delta_{ij}(t)$	outflow capacity of link (i, j) at time t
δ_{ij}^*	maximum outflow capacity of link (i ,j)
$T_{ij}(t)$	travel time over link (i, j) that results from entering link (i, j) at time t
$y_{ij}(t)$	number of vehicles on link (i, j) at time t
$k^{\max}$	maximum density per lane
L_{ij}	length of link (i, j)
n_{ij}	number of lanes at link (i, j)
t_{ij}^f	travel time with free speed on link (i, j).
$x_e(t)$	flow rate over link e at time t
$x_p(t)$	flow rate of path p at initial point of path p and at time t
p	designation of a path
P_k	set of paths between the origin-destination pair k
l	lane index
Ω_o	set of travelers (agents) departing from origin o

Time variables

t	time index
$[0,T]$	departure time window
$[0,\bar{T}]$	time window of simulation in which the right bound is the time of last vehicle leaves the network
H	time interval index
H_T	set of time intervals for T , $H_T = \{h_0,...,h_T\}$ with fixed time slice Δ_h , where T is a multiple of Δ_h
$H_{\bar{T}}$	set of time intervals for $\bar{T}$, $H_{\bar{T}} \equiv \{h_0, h_1, \ldots, h_{\bar{T}}\}$, where $\bar{T}$ is a multiple of the same fixed time slice Δ_h
W	iteration index

The basic assumptions adopted in this model are firstly discussed as follows. (i) We consider a road network represented by a directed graph with multiple origins and destinations. The profile of time-dependent origin-destination (OD) demands is a priori unknown. (ii) The departure time choice problem is considered in a fixed study time period, which is equally divided into numerous small departure time choice intervals. Each traveler, from their chosen departure time interval, selects a departure time instant leaving their origin to engage in an activity towards a yet unknown destination within a desired arrival time interval, assumed to be the same for all travelers. (iii) As in many activity-based models, the dynamical activity/destination choice problem is formulated as a utility maximization problem. However, its assumption of perfect human cognitive capacity has been questioned [8]. Following the previous work [9], we assume that activities are distributed in some nodes of the network and that each activity possesses an economic value, determined following a probability distribution. In order to facilitate the essential idea without loss of generality, we consider only homogeneous activity types and assume its perceived economic value is the same for all travelers. However, different classes of travelers/activities may have various activity value evaluations. Travelers who move to some node to engage an activity receive its economic value. The complex activity chain problem is not considered in this work. (iv) Travelers have no perfect information about activity value/location distributions, and about traffic condition throughout the whole network. The route/destination choice behavior is link/node based according to travelers' adaptive behavior with respect to congestion and activity availability dynamics. (v) The framework of our model is that each traveler tries to maximize his revenue obtained by engaging certain activity and reducing its related general travel cost. The last one is comprised of the travel cost resulting from the travel time between the origin and the destination, plus time-related penalty cost linked to early/late arrival. We describe the model as follows: 1.dynamic traffic flow propagation model; 2. activity value distribution assumption and arrival penalty; 3. activity-based dynamic user equilibrium condition.

Dynamic traffic flow propagation

We consider that the network of interest is composed of a set of nodes and a set of directed links. At a chosen time, travelers leave their respective origin into the network of interest. The conditions for the conservation of the flow at origins is defined by:

$$D_o = \int_0^T d_o(t)dt, \quad \forall o \in O \tag{1}$$

Also, the number of vehicles absorbed at destinations is constrained by their available activity capacity, N_d :

$$\sum_{e \in M(d)} \int_0^{\bar{T}} x_e(t)dt \leq N_d \text{ , } \forall d \in D \tag{2}$$

For a link (*i, j*), flow direction being from node *i* to node *j*, link travel time is valued by the difference between the arrival and departure curves at arrival time *t*:

$$T_{ij}(t) = D_{ij}^{-1}(A_{ij}(t)) - t \tag{3}$$

We consider intersections as a transition point, which does not imply extra passing time and stock space. However, the intersections can be modeled more realistically by endowing their physical characteristics. Now let us consider the flow propagation at intersections comprised of various entering (upstream) links and various outgoing (downstream) links. The flow splitting from upstream links to downstream links is limited by some dynamic inflow and outflow capacity constraints. Before addressing this issue, let us discuss the formation and dissipation of traffic queues in general intersections. Contrarily to the common point queue model, we separate different queue generations and dissipations in two or more groups of lanes within a link. First-In-First-Out (FIFO) discipline is respected within each group of lanes. Travelers' route choice is link-based, following a stochastic choice model as described in the next section. As an outgoing link choice is made, agents are added in the corresponding group of lanes. As far as links are concerned, when time-dependent traffic demand exceeds its time-dependent capacity, a traffic queue is generated. In this model, the traffic queue is separated within two or more groups of lanes if the number of lanes and the number of outgoing links exceed 1. Traffic queue in one group of lanes is assumed not to influence the fluidity of another group of lanes within the same link. For links with only one lane or one outgoing link, the common point queue model is applied. We assume that traffic propagation behaviors are different according the traffic conditions (free flow/congestion). This very basic model can easily and at little cost be improved, by implementing particle discretization of macroscopic traffic flow models, such as first order models [11] or second order models [12]. To model the traffic propagation at intersection nodes, for free flow case, the FIFO discipline is imposed to entering vehicles. By contrast, in the congestion case, when more than one entering link are congested and agents aim at the same outgoing link, then the entering order to the chosen outgoing link is given with a probability of choice expressing how conflicts between agents are resolved in the node. Different rules, based on [13], could be used: node optimization models, supply split equilibrium models etc. Vehicles' link delay depends on the number of vehicles in the front of the agent in the same queue. Below, we discuss the traffic flow constraints for diverge and merge case in intersections with multiple upstream links and multiple downstream links.

Diverge constraints

In the diverge case, the time-dependent departure flow capacity of lane group l_s within upstream link e_r for the next chosen downstream link e_s is constrained by (supply/demand approach [10]):

$$\delta_{e_r}^{l_s}(t) = \min(\alpha_{rs}^{l_s}\delta_{e_r}^{*}, \sigma_{e_s}(t)) \tag{4}$$

where $\alpha_{rs}^{l_s}$ is a split coefficient corresponding partial outflow capacity from upstream link e_r to downstream link e_s by lane group l_s.
Such constraints can be managed by random entrance or pointwise node models, as mentioned above.

Merge constraints
For intersection *i*, the total flow entering the downstream link e_s cannot exceed its total time-dependent inflow capacity:

$$\sum_{e_r} \delta_{e_r}^{l_s}(t) \le \sigma_{e_s}(t), e_r \in M(i) \tag{5}$$

where l_s is the lane group within upstream link e_r for the next chosen downstream link e_s.

Gross activity value distribution and arrival time penalty

For the probability distribution of gross activity value *v*, in each destination *d*, we assume in the numerical simulation that the gross activity value follows an exponential distribution function, $P_d(v) = \lambda_d \exp(-\lambda_d(v - m_d))$ where $v \ge m_d$ and $\lambda_d \ge 0$. This probability function can be replaced by any other more realistic probability function if needed. Let $\pi_{p_k}(t)$ denote travel time on route p_k for origin-destination pair *k* with departure time *t*. For an agent traveling route p_k with departure time *t*, the net activity value obtained is defined by

$$\tilde{v}_{p_k}(t) = v_{d(k)}(t + \pi_{p_k}(t)) - C_{p_k}(t) \tag{6}$$

where $v_{d(k)}(t + \pi_{p_k}(t))$ is the maximum available gross activity value when arriving at destination *d* with $t + \pi_{p_k}(t)$ being the arrival time at destination *d* and $C_{p_k}(t)$ is the general path cost.

The general path cost is composed of two parts. The first part is associated with traveling time from origin to destination. The path travel time is the sum of all links' travel time in the path. We suppose that the unit value of time is the same for all travellers. The second part is associated with the arrival time at destination. We use a piecewise early/late arrival penalty function to reflect the arrival cost associated to the ideal arrival time [14], [15] (bounded rationality). The general cost function is defined by:

$$\pi_{p_k}(t) \times \mu + \mu_\alpha \max(0, t^* - \Delta - t^{\text{arr}}) + \mu_\beta \max(0, t^{\text{arr}} - t^* - \Delta) \tag{7}$$

where μ is the unit cost of travel time, μ_α is the unit penalty linked to an early arrival, and μ_β is the unit penalty of a late arrival. Also, t^{arr} is the actual arrival time with t^* being the desired arrival time for all travellers, and Δ is the half of the tolerance interval in which penalty cost is 0. According to the experimental result [16], the condition $0 < \mu_\alpha < \mu < \mu_\beta$ holds.

The equilibrium condition

We consider a *predictive dynamic assignment* problem, i.e. a long term equilibrium with demand varying within the day. The dynamic traffic assignment based on the activity distribution considering for departure time choice can be described as (Wardrop's principle):

In dynamic user equilibrium (DUE), for travellers departing from the same origin, the net activity obtained relating to departure time, destination and route choice is equal and no less than that of any unused departure time/destinations/routes.

Let $\underline{v}_o^{\max}$ denote the maximum net activity value departing from origin o within the period of interest T to some activity destination, we define $\underline{v}_o^{\max}$ by

$$\underline{v}_o^{\max} = \max\left\{ \ \tilde{v}_p^m(t) : \forall p \in P_{k|o(k)=o}, \forall d(k) \in D, \forall t \in [0,T], \forall m \in \Omega_o \ \right\} \tag{8}$$

where m denotes a traveller with net activity value $\tilde{v}_p^m(t)$ defined by (6).
This DUE condition can be expressed by

$$\tilde{v}_p(t) \begin{cases} = \underline{v}_o^{\max} & \text{if } x_p(t) > 0 \\ \leq \underline{v}_o^{\max} & \text{otherwise} \end{cases} \quad \forall t \in [0,T], \forall p \in P_{k|o(k)=o}, \forall o \in O, \forall k \in K \tag{9}$$

where

$$\sum_{k|o(k)=o} \sum_{p \in P_k} x_p(t) = d_o(t), \forall k \in K, \forall t \in [0,T] \tag{10}$$

3. TIME-DEPENDENT ACO APPROACH

This section is devoted to the computation of the network equilibrium. Many solution methods have been proposed in similar contexts. One method, well suited to multi-agent models, is a metaheuristic approach based on the ant colony algorithm (ACO) concept [17] to solve the time-dependant user equilibrium problem. Extending the ACO algorithm, we propose a time discretization scheme maintaining time-dependent pheromone value corresponding to time-dependent path quality for guiding ant-like agents (travellers) to find better solutions. Each agent selects a departure time interval (for example 5 minutes) according to the departure time choice model (described later). Furthermore, agents departing from different origins use origin-specific pheromone information to reflect correctly the quality of routes used by agents from these different origins.

For the selection of route and destination, agents use the time-dependent pheromone quantity and travel time information over links, indicating at the current time how good it seems to select the next outgoing link in order to optimise the net activity value. These time-dependent pheromone trails are maintained and updated according to pheromone update rules. These rules are composed of a local update rule and a global update rule. The first one is immediately applied to the utilized paths after an agent arrives to some activity destination for which the quantity of added pheromone depends on the quality of solution found by the agent. The second one is applied throughout all links after all agents have found activities. The rule is based on the *cross-entropy pheromone update* concept [18] in order to avoid the scale problem resulting from the variability of numbers of agents and path quality function settings. We introduce a penalty function in the agents' link travel time estimation in order to dissuade agents from using links in a state close to hyper congestion. Concerning the destination choice, we utilize artificial nodes and artificial links connecting all possible destinations. The capacity of artificial links is set to be equal to the number of vacant activities of connected destinations. The flow over artificial links is the amount of occupied activities. We apply the concept of penalty on artificial links in order to model the competition between destinations. The algorithm is stopped when the average quality of solutions has not changed significantly after a certain number of iterations.

We detail the time-dependent ACO approach as follows:

1. Initialize a small quantity of pheromone in each time interval $h \in H_{\overline{T}}$ for all types of pheromone, $o \in O$, and for all links (*i, j*).

$$\tau_{ij}^{o}(h) = c \ \forall (i,j) \in E, \forall o \in O, \forall h \in H_{\overline{T}} \tag{11}$$

where h is a time interval index, $H_{\overline{T}}$ is the set of the intervals of the discretization of time-dependent pheromones, and *o* is the type of pheromone with respect to origin node.

2. The probability for one agent *m* at node *i* to choose next outgoing link (*i, j*) at instant *t*, ($t \in h_s$), according to the following stochastic decision rule (following [17]):

$$P_{ij}^{m}(t) = \begin{cases} \dfrac{[\tilde{\tau}_{ij}^{o}(h_s)]^{\alpha}[\tilde{\eta}_{ij}(t)]^{\beta}}{\sum\limits_{u \in J_i^m(t)}[\tilde{\tau}_{iu}^{o}(h_s)]^{\alpha}[\tilde{\eta}_{iu}(t)]^{\beta}} & \text{if } j \in J_i^m(t) \\ 0 & \text{otherwise} \end{cases} \quad \forall m \in \Omega_o, \forall o \in O, h_s \in H_{\overline{T}} \tag{12}$$

where :

t : route choice decision making time instant when arriving at node *i* within time interval h_s, *s* is an index.

α, β : constant parameters manipulating the relative importance of normalized pheromone value at time interval h_s, $\tilde{\tau}_{ij}^{o}(h_s)$, and normalized visibility value, $\tilde{\eta}_{ij}(t)$, on link (*i, j*) at time instant *t*.

$J_i^m(t)$: set of non-visited outgoing nodes for agent *m* situated at node *i* at time *t*.

$\tilde{\tau}_{ij}^{o}(h_s)$: normalized type *o* pheromone value on link (*i, j*) at time interval h_s, defined by

$$\tilde{\tau}_{ij}^{o}(h_s) = \frac{\tau_{ij}^{o}(h_s)}{\sum\limits_{u \in J_i^m(t)} \tau_{iu}^{o}(h_s)}$$

$\tilde{\eta}_{ij}(t)$: normalized visibility value on link (*i, j*) at time *t,* defined by $\tilde{\eta}_{ij}(t) = \dfrac{\eta_{ij}(t)}{\sum\limits_{u \in J_i^m(t)} \eta_{iu}(t)}$

in which $\eta_{ij}(t)$ is the inverse of link travel time adding expecting waiting time in queue and a penalty function, $\Gamma_{ij}(t)$, reflecting traffic congestion condition on link (*i, j*), defined by:

$$\eta_{ij}(t) = \frac{1}{t_{ij}^{f} + \dfrac{y_{ij}(t)}{\delta_{ij}(t)} + \Gamma_{ij}(t)} \tag{13}$$

$$\Gamma_{ij}(t) = \begin{cases} \psi[\dfrac{1}{B_{ij} - y_{ij}(t)} - \dfrac{1}{\gamma B_{ij}}] & \text{if } B_{ij}(1-\gamma) \le y_{ij}(t) \le B_{ij} \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

where $y_{ij}(t)$ is the number of agents on link (*i, j*) at time *t*, $\delta_{ij}(t)$ is the temporal outflow capacity, B_{ij} is the vehicle stock capacity at link (*i, j*), $B_{ij} = k^{\max} \times n_{ij} \times L_{ij}$, γ is a parameter controlling the activation of the penalty function, ψ is a parameter defined by $\psi = B_{ij}\zeta$ in order to eliminate the influence of link stock capacity, and ζ is a parameter controlling the influence of the penalty value, $\zeta > 0$. It is necessary to include the penalty function in the expression of the visibility (16), in order to dissuade agents (ants) from using a link close to hyper congestion.

3. After an agent constructs its path to some activity destination, the time-dependent pheromone trail over its path is immediately updated. The time-dependent pheromone in the interval of which the next outgoing link choice decision is taken is updated. The local pheromone update rule is defined by:

$$\Delta\tau_{ij}^{o}(h_s, w) = \sum_{\forall m \in \Omega_o} \Delta\tau_{ij}^{mo}(h_s, w) \quad \forall o \in O, \forall h_s \in H_{\bar{T}} \tag{15}$$

$$\Delta\tau_{ij}^{mo}(h_s, w) = \begin{cases} Q/L_m(w) & \text{if link } (i,j) \in p^m(w) \\ 0 & \text{otherwise} \end{cases} \quad \forall o \in O, \forall h_s \in H_{\bar{T}} \tag{16}$$

where h_s is the time interval of the decision taken for choosing the next link (*i, j*) of agent *m*, $p^m(w)$ is the path used by agent *m* at iteration *w*, *Q* is a constant parameter, and $L_m(w)$ is a solution quality function, defined by general cost minus obtained gross activity value plus the maximum gross activity value in order to guarantee $L_m(w) > 0$.

Concerning the destination choice, we use the virtual nodes and virtual links connecting all destinations. The capacity of virtual links is set to be equal to the number of vacant activities of connected destinations. The flow over virtual links is the amount of occupied activities. We apply the concept of penalty on virtual links to model the competition between destinations. A penalty function similar to equation (13) is applied here.

When all agents have found activities, we apply the global pheromone update rule to all links, defined by

$$\tau_{ij}^{o}(h, w+1) = (1-\rho)\tau_{ij}^{o}(h, w) + \rho\omega \frac{\Delta\tau_{ij}^{o}(h, w)}{\sum_{\forall (u,v) \in E} \Delta\tau_{uv}^{o}(h, w)}, \forall (i,j) \in E, \forall o \in O, \forall h \in H_{\bar{T}} \tag{17}$$

where $\rho \in (0, 1)$ is constant evaporation rate of pheromones and ω is a parameter adjusting the amount of added pheromone.

This global pheromone update rule is based on the *cross-entropy pheromone update* concept [18] in order to avoid the scale problem resulting from the variability of numbers of agents and path quality function settings.

The departure time choice model

For the departure time choice problem, we propose a departure choice model based on an ACO approach similar to the one proposed in the preceding section. We create a graph composed by a set of origin nodes *O* and a set of time interval indices over the demand interval *T*. Links connect origin nodes and all departure time interval indices. Agents start at origin nodes and select for their next node some neighboring node (index of departure time interval), based on pheromone information over links. After all agents find solutions, the pheromone quantity over these departure time interval edges is updated according agents' obtained solution quality (net activity value). The performance of leaving at a certain time interval depends on all agents' behavior and network dynamics. Thus, the obtained net activity value influences the departure time choice in the next day (iteration). In extensions of this scheme (action of information systems), the ACO concept (communication through pheromone) would be replaced by an emulation of the action of information systems in conjunction with the cross entropy concept.

4. NETWORK EXAMPLES

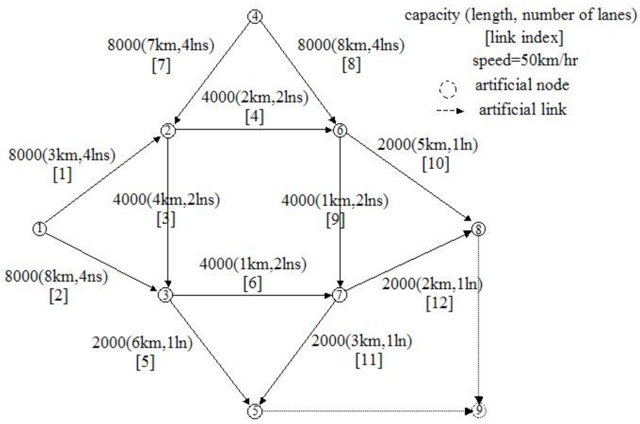

Figure 1: The network

In order to test the proposed algorithm in a congested situation, the capacity of some outgoing links is set to be small and the total demand is set great enough to cause congestion on some links. The demand period is 2 hours (7200 seconds, from 7:00 to 9:00), the unitary departure time choice interval is 5 minutes, and the desired arrival time is 9:00 with a tolerance time window from 8:54 to 9:06. The unit value of time spent in travel is 7 euros/hour, and early/late arrival penalty is respectively 4 euros/hour and 15 euros/hour. The departure time chosen is initialized uniformly in the period of demand.

The test network consists of 8 nodes and 12 links with two origins and two destinations (see figure 1). The demands at the origin nodes (node 1 and node 4) are set to 1500 travellers each. Destination nodes with vacant activities are node 5 and node 8 with 4000 vacant activities each. The gross activity value follows an exponential distribution, with average of 30 euros (node 8) and

20 euros (node 5).

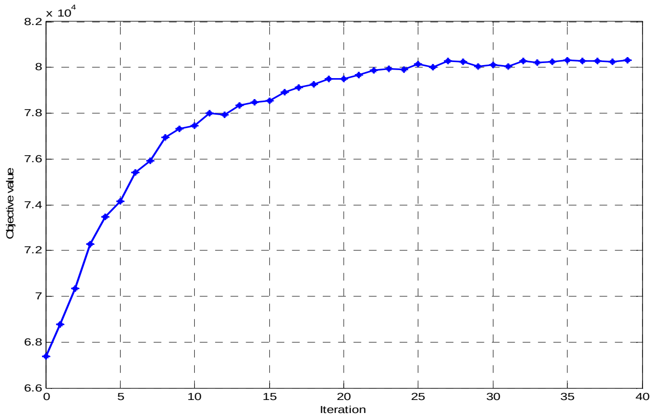

Figure 2 : ACO algorithm convergence

Figure 2 depicts the objective value over 40 iterations. This objective value increases rapidly during early iterations, and then converges to the near optimal value, due to the fact that agents learn their best departure time/route/destination choice through on-route information and the agents' collective experiences.

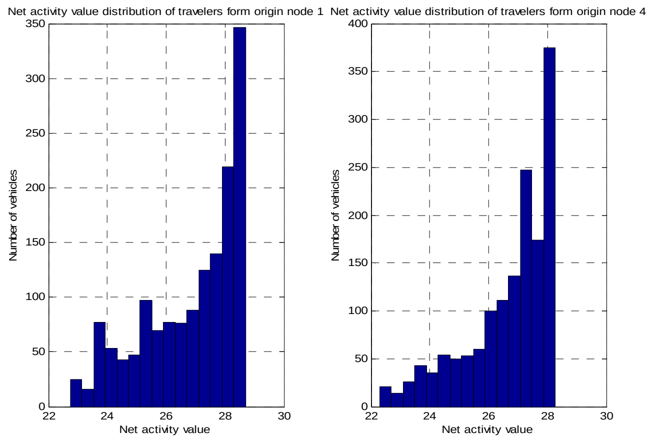

Figure 3: Net activity value frequency plot

Figure 3 depicts the net activity value distribution for travellers from different origins, and shows that, even in a situation of congestion, most agents still find the best net activity value.

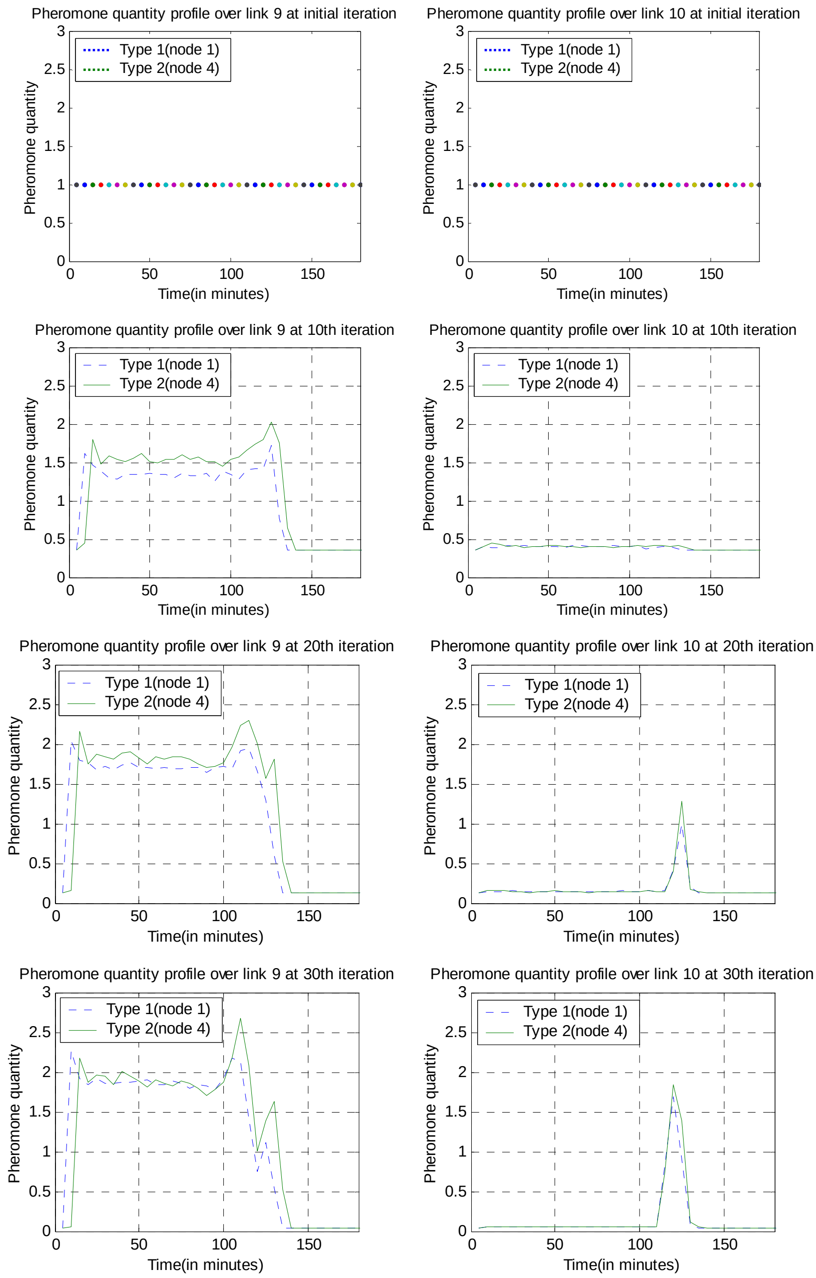

Figure 4: Time variation of pheromone values on some links, for iterations 1, 10, 20, and 30

Figure 4 depicts the evolution of the pheromone quantity on a critical link (link 9) in the shortest paths from origin node 1(or node 4) to destination node 8 (left) and on its alternative link (right). Note that agents start favouring the shortest geometrical paths, where the pheromone quantity increases rapidly. As the critical link becomes congested, the amount of pheromone within the congested period drops quickly. By contrast, the amount of pheromone increases rapidly in alternative link (link 10) within the same period of congestion.

5. CONCLUSION

The model is expected to yield approximate dynamic assignment equilibriums with departure time/route/destination choice in a dynamic network context. We propose a multi-agent approach for the model of the system coupled with an ACO-like algorithm to solve the proposed predictive dynamic traffic assignment problem. Further extensions of this work should include demand modelling on the basis of activity plans, and testing the effect of various information provision or exchange schemes such as road-to-user communication information system and/or user-to-user information system on network operations.

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