FORMULATIONS, BOUNDS AND HEURISTIC METHODS FOR A MULTI-ECHELON LOCATION-DISTRIBUTION PROBLEM

Bernard Gendron
Département d'informatique
et de recherche opérationnelle
and
Centre de recherche sur les transports
Université de Montréal
C.P. 6128, succ. Centre-ville
Montreal, Quebec H3C 3J7
bernard@crt.umontreal.ca
Frédéric Semet
LAMIH
Université de Valenciennes et du Hainaut-Cambrésis
Le Mont Houy, 59313 Valenciennes Cedex 9, France
Frederic.Semet@univ-valenciennes.fr

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We consider a multi-echelon location-distribution problem arising from an actual application in fast delivery service, first described by Gendron, Semet and Strozyk [1]: a mail-order company offers several products that must be delivered on time to the customers requesting them. To satisfy these requests, the firm operates a multi-echelon distribution system: starting their trips from a small set of hubs (their locations are assumed known and fixed, following a preliminary strategic analysis), a fleet of medium-size trucks delivers the products to depots, where they are transferred on small-size trucks, and then shipped to satellites, where the products are sorted and delivered to the customers. The company exploits existing facilities for the depots and the satellites, but has to pay to use them. The problem is to ensure that customers' requests are satisfied on time at minimum cost, taking into account the transportation costs and the location costs for using the depots and the satellites.

Typically, satellites are neither owned nor rented by the company. They can be warehouses owned by independent carriers or sites such as car parks where items are transferred from one vehicle to another. More traditional distribution systems, which do not use such satellites, can be very costly to run when the demand varies significantly from one period to another. Moreover, when few depots are present, services such as 24 hour-delivery can be cost ineffective or simply cannot be assured when a wide distribution area is considered. The multi-echelon system we consider allows to satisfy time constraints in a cost-effective way. It is also an *adaptive* system, in the sense that satellites can be opened or closed easily according to demand variations.

We model the problem by defining a network for which the only possible connections are those that ensure on time delivery of the products to the customers. In addition, we assume that for each satellite and each product, the set of customers and the routes used to satisfy their requests have been determined in a preprocessing phase. Hence, the model does not include any routing aspect. Each customer in our model represents a set of customers to which the same product is delivered using a single vehicle. The transportation cost between a satellite and a customer thus corresponds to the cost of the best route determined during this preprocessing phase.

Transportation costs between hubs and depots, and between depots and satellites, vary with the distance travelled, but more importantly, with the number of vehicles used on each arc, each type of vehicle (medium- or small-size truck) having an associated volumetric capacity. A fixed cost is incurred when using any depot, while the satellite location cost increases with the number of batches of products handled at the satellite (a batch corresponds to a fixed number of product units). This cost structure is similar to what can be found in telecommunications network design applications, where multiple facilities, each with an associated capacity, can be installed on the arcs or the nodes. In our problem, vehicles (at the arcs) and product batches (at the satellites) play the role of facilities. Note that this cost structure is more complex than in most location-distribution problems found in the literature, which typically exhibit fixed costs at the nodes and transportation costs that are linear in the number of product units.

Because the locations of the hubs are assumed to be fixed, there are no fixed costs associated to the hubs and we can always assign to each depot its closest hub without losing optimality. This simplification is also performed in the preprocessing phase. Note that we still need to determine how many product units, on how many medium-size vehicles, need to be transported between any depot and its closest hub, but we are now allowed to associate the corresponding decision variables to the depots, instead of the arcs between hubs and depots. The resulting problem can therefore be considered as a *two-echelon* (from depots to satellites, and from satellites to customers) capacitated location-distribution problem.

1 Notation and Formulation

The following sets define the different types of nodes in the network:

D = set of potential sites to locate depots;
S = set of potential sites to locate satellites;
S_i = set of potential sites to locate satellites connected to depot site $i \in D$;
L = set of customers;
L_j = set of customers connected to satellite $j \in S$;
L_i = set of customers connected to depot $i \in D$ from some satellite $j \in S_i$.

The data related to the customer demands and the vehicle capacities are defined as follows (all values are assumed to be positive):

n_l = number of product units to deliver to customer $l \in L$;
v_l = volume of product units to deliver to customer $l \in L$;
Q = capacity (in number of product units) of one batch of products handled at any satellite;
R_p = volumetric capacity of a small-size vehicle transporting product units to any depot, from its closest hub;
R_s = volumetric capacity of a medium-size vehicle transporting product units from any depot to any satellite.

The location and transportation costs are defined as follows (all values are assumed to be nonnegative):

f_i = fixed cost for using and operating depot $i \in D$;
g_j = cost per Q product units for using and operating satellite $j \in S$;
c_i = transportation cost for using one medium-size vehicle to transport product units to depot $i \in D$ from its closest hub;
c_{ij} = transportation cost for using one small-size vehicle from depot $i \in D$ to satellite $j \in S_i$;
c_{jl} = transportation cost between satellite $j \in S$ and customer $l \in L_j$.

To derive the path-based model, the following sets of binary variables are introduced:

$$X_{ijl} = \begin{cases} 1, & \text{if some product units are transported on path } (i,j,l), \\ & i \in D,\ j \in S_i,\ l \in L_j; \\ 0, & \text{otherwise.} \end{cases}$$

$$W_{ij} = \begin{cases} 1, & \text{if some product units are transported between depot } i \in D \\ & \text{and satellite } j \in S_i; \\ 0, & \text{otherwise;} \end{cases}$$

$$Y_i = \begin{cases} 1, & \text{if some product units are transported to depot } i \in D \\ & \text{from its closest hub;} \\ 0, & \text{otherwise.} \end{cases}$$

We also use the following general integer variables to represent the number of batches handled at any satellite and the number of vehicles used on any depot-satellite arc or at any depot:

U_j = number of batches of products handled at satellite $j \in S$;
T_i = number of medium-size vehicles used between depot $i \in D$ and its closest hub;
T_{ij} = number of small-size vehicles used between depot $i \in D$ and satellite $j \in S_i$.

$$\min \sum_{i\in D} f_i Y_i + \sum_{j\in S} g_j U_j + \sum_{i\in D} c_i T_i + \sum_{i\in D}\sum_{j\in S_i} c_{ij} T_{ij} + \sum_{i\in D}\sum_{j\in S_i}\sum_{l\in L_j} c_{jl} X_{ijl} \tag{1}$$

$$\sum_{i\in D}\sum_{j\in S_i} X_{ijl} = 1, \quad \forall l \in L, \tag{2}$$

$$\sum_{i\in D} W_{ij} \le 1, \quad \forall j \in S, \tag{3}$$

$$\sum_{j\in S_i}\sum_{l\in L_j} v_l X_{ijl} \le (\sum_{l\in L_i} v_l) Y_i, \quad \forall i \in D, \tag{4}$$

$$X_{ijl} \le W_{ij}, \quad \forall i \in D, \forall j \in S, \forall l \in L_j, \tag{5}$$

$$W_{ij} \le Y_i, \quad \forall i \in D, \forall j \in S_i, \tag{6}$$

$$\sum_{i\in D}\sum_{l\in L_j} n_l X_{ijl} \le Q U_j, \quad \forall j \in S, \tag{7}$$

$$\sum_{j\in S_i}\sum_{l\in L_j} v_l X_{ijl} \le R_p T_i, \quad \forall i \in D, \tag{8}$$

$$\sum_{l\in L_j} v_l X_{ijl} \le R_s T_{ij}, \quad \forall i \in D, \forall j \in S_i, \tag{9}$$

$$U_j \ge 0 \text{ and integer}, \quad \forall j \in S, \tag{10}$$

$$T_i \ge 0 \text{ and integer}, \quad \forall i \in D, \tag{11}$$

$$T_{ij} \ge 0 \text{ and integer}, \quad \forall i \in D, \forall j \in S_i, \tag{12}$$

$$X_{ijl} \in \{0,1\}, \quad \forall i \in D, \forall j \in S_i, \forall l \in L_j, \tag{13}$$

$$W_{ij} \in \{0,1\}, \quad \forall i \in D, \forall j \in S_i, \tag{14}$$

$$Y_i \in \{0,1\}, \quad \forall i \in D. \tag{15}$$

The objective function, (1), consists in minimising all costs incurred by using and operating depots and satellites, as well as transportation costs between hubs and depots, between depots and satellites, and between satellites and customers. Constraints (2) ensure that each customer is being served by a single satellite. Constraints (3) ensure that any satellite, when it is used, is connected to a single depot. The forcing constraints (4) they ensure that no flow can circulate through a closed depot. Constraints (5) and

(6) are also forcing constraints that link together the different types of binary variables. Constraints (5) ensure that any customer cannot be routed from a satellite that is not connected to some depot. Similarly, constraints (6) ensure that any satellite cannot be connected to a depot that is not used to transport product units. Constraints (7) ensure that the number of product units handled at a satellite cannot exceed the capacity of product batches. Constraints (8) and (9) ensure that the total volume of all product units transported on a network element (depot or depot-satellite arc) cannot exceed the capacity of the vehicles used on that network element. Other constraints specify the nature of the different types of variables.

2 Summary of Results

We show the following structural results:

- An equivalent arc-based model can be derived, but its linear programming (LP) relaxation is dominated by the path-based LP relaxation.
- We compare the so-called *binary relaxations* of the two models, which are obtained by relaxing the integrality constraints for all variables, except for the 0-1 design variables that determine which nodes and which arcs should be used to satisfy customers' requests. Namely, we show that the binary relaxations of the two models, arc-based and path-based, are equivalent.
- The binary relaxation of the path-based model can be reformulated as an equivalent simple plant location problem (SPLP), for which efficient solution methods have been developed (see for instance [2]).
- Finally, we show that the LP relaxation of this SPLP reformulation dominates the LP relaxation of the path-based model.

We present computational results comparing these different formulations and relaxations. We use CPLEX 10 to solve instances derived from an actual application having two hubs, 93 depots, 320 satellites, 722 customers, and 458 014 possible connections between the hubs and the customers.

To solve these large-scale instances, we also developed a heuristic method based on a variable neighborhood approach (see for instance [3]). Following an initial greedy construction procedure, this method iterates over three phases:

1. Node+arc-based neighborhood descent. In this phase, we use four types of neighborhoods: (1) close one depot; (2) close one satellite; (3) serve one customer through a different satellite; (4) serve one satellite through a different depot.
2. Open depot+satellite neighborhood descent. Two types of neighborhoods are used in this phase: (1) open one depot; (2) open one satellite. To evaluate the impact of each move of type (1), we first serve through the open depot every satellite

connected to it, and then we perform phase 1, the node+arc-based neighborhood descent, on the resulting network. The evaluation of each move of type (2) is performed similarly.

3. Diversification. We use an adaptive memory that stores the best solutions found so far, selects some of these solutions and perturbs them by performing random moves in a large-scale neighborhood, defined by all moves that consist in closing k paths, replacing them by k alternative paths.

References

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[3] Mladenovic, N., Hansen, P. (1997). Variable Neighborhood Decomposition Search. *Computers and Operations Research* 24, 1097-1100.