New Resolution Approaches for the Sensor Location Problem

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1 Introduction

Monitoring flows on the network is an important topic in the field of traffic management and control. Observation of flows can be carried out by locating different type of sensors (counting sensors, image sensors, path-ID sensors [8]) either on the nodes (intersections) of the network or on its links (arcs).

The problem of locating sensors on the network to optimize certain objective criteria has been object in the past few years of growing interest. The basic criteria considered can be grouped in the following three main categories:

(i) the estimation of total flow volumes from all origins to all destinations (O/D matrix estimation) (e.g., [3], [6], [7], [8], [12], [13], [15], [16], [17], [19]);

(ii) the estimation of flow volumes on the non-monitored links of the network (e.g., [1], [2], [5], [8]);

(iii) the estimation of flows on routes from origins to destinations (e.g., [8], [9], [10]).

In this paper we focus on the second criterion above mentioned, that is on the specific problem of locating the minimum number of *counting* sensors on the nodes of a network in order to determine the arc flow volume on all the network (the *Sensor Location Problem*).

In particular, given a network $G = (N, A)$, where N is the set on nodes and A is the set of arcs, flow on arcs contains subflows that are generated and/or absorbed from different origin/destination pairs. Among the set of nodes we say v is a centroid node if the flow is generated and/or absorbed by it, otherwise we

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say it is a transfer node.

For each transfer node v, the flow conservation constraints hold:

$$\sum_{(v,w)\in A_v^-} f_{v,w} - \sum_{(w,v)\in A_v^+} f_{w,v} = 0 \tag{1}$$

where A_v^- and A_v^+ are the outgoing and incoming arcs of node v, respectively, and $f_{v,w}$ is the flow volume on arc (v, w).

For each centroid v, we have the following flow conservation constraint:

$$\sum_{(v,w)\in A_v^-} f_{v,w} - \sum_{(w,v)\in A_v^+} f_{w,v} = S_v \tag{2}$$

where S_v is the *balancing flow* at v, that is, a source or a sink flow so that (2) holds. If split ratios[1] at nodes are also known we could define additional equations. Indeed by using split ratios, we can express the total incoming flow $F(v)$ as a function of the flow volume of any outgoing arc. Formally, for each $v \in N$ and each outgoing arc (v, w), by using split ratios at node, we have:

$$f_{v,w} = F(v) \cdot p_{v,w} \tag{3}$$

From (3) and considering any other outgoing arc of v, say (v, z), we obtain:

$$f_{v,z} = \frac{f_{v,w}}{p_{v,w}} \cdot p_{v,z} \tag{4}$$

Consider the network in Figure 1 and assume node 4 and 5 are centroid nodes. By locating a counting sensor on a node we assume to know the flow volumes on all the arcs incident to the node. Let us suppose to locate a sensor on node 1. It is possible to define a system of linear equations where (i) a single unknown variable, representing an outgoing arc flow, is associated with every node, except the monitored node and its adjacent nodes, and, (ii) equations correspond to flow conservation constraints relative to every node of the network. Additional variables S_v, denoting the centroid balancing flows, are also included in the linear system. Note that, the flow conservation equation corresponding to the measured node 1, does not contain any unknown flow, and thus can be omitted from the system. We obtain the following system corresponding to the flow conservation constraints of the network when node 1 is monitored:

[1]The split ratios specify the fraction $0 \leq p_{v,w} < 1$ of the incoming flow $F(v)$ that leaves the node v on each of the outgoing arcs (v, w) [14].

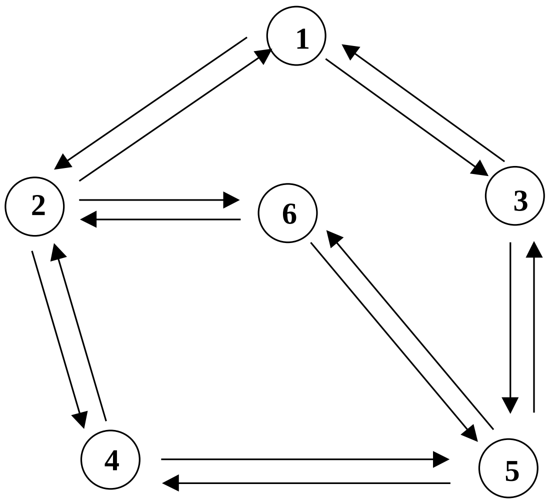

Figure 1: *An example network with six nodes.*

$$
\begin{array}{llcll}
\textit{node } 2 & (f_{2,1} + f_{2,6} + f_{2,4}) & - & (f_{1,2} + x_{6,5} \cdot \frac{p_{6,2}}{p_{6,5}} + x_{4,2}) & = & 0 \\
\textit{node } 3 & (f_{3,1} + f_{3,5}) & - & (f_{1,3} + x_{5,4} \cdot \frac{p_{5,3}}{p_{5,4}}) & = & 0 \\
\textit{node } 4 & (x_{4,2} + x_{4,2} \cdot \frac{p_{4,5}}{p_{4,2}}) & - & (x_{5,4} + f_{2,4}) & = & S_4 \\
\textit{node } 5 & (x_{5,4} + x_{5,4} \cdot \frac{p_{5,3}}{p_{5,4}} + x_{5,4} \cdot \frac{p_{5,6}}{p_{5,4}}) & - & (f_{3,5} + x_{4,2} \cdot \frac{p_{4,5}}{p_{4,2}} + x_{6,5}) & = & S_5 \\
\textit{node } 6 & (x_{6,5} \cdot \frac{p_{6,2}}{p_{6,5}} + x_{6,5}) & - & (f_{2,6} + x_{5,4} \cdot \frac{p_{5,6}}{p_{5,4}}) & = & 0
\end{array}
\qquad (5)
$$

where: the unknown variables are $x_{4,2}$, $x_{5,4}$, $x_{6,5}$, S_4 and S_5, flows $f_{1,2}$, $f_{2,1}$, $f_{1,3}$, $f_{3,1}$ are directly monitored, and flows $f_{2,4}$, $f_{2,6}$ and $f_{3,5}$ are obtained using split ratios. The remaining unknown flows are obtained using equation (4). Note that, in this system, the number of variables is equal to the number of equations and thus, we obtain a unique solution. Let us assume now, node 2 and 6 are centroid too. Because the number of variables is increased (we have the additional variables S_2 and S_6), the system does not have a unique solution anymore. Hence, additional monitored nodes are needed. Obviously, a trivial solution can be obtained by locating a counting sensor on all the centroids. However, it is enough to locate a single counting sensor on node 5 to obtain all flow volumes. Indeed, the corresponding system would be:

$$
\begin{array}{llcll}
\textit{node } 1 & (x_{1,3} \cdot \frac{p_{1,2}}{p_{1,3}} + x_{1,3}) & - & (x_{2,1} + f_{3,1}) & = & 0 \\
\textit{node } 2 & (x_{2,1} + x_{2,1} \cdot \frac{p_{2,4}}{p_{2,1}} + x_{2,1} \cdot \frac{p_{2,6}}{p_{2,1}}) & - & (x_{1,3} \cdot \frac{p_{1,2}}{p_{1,3}} + f_{6,2} + f_{4,2}) & = & S_2 \\
\textit{node } 3 & (f_{3,1} + f_{3,5}) & - & (x_{1,3} + f_{5,3}) & = & 0 \\
\textit{node } 4 & (f_{4,2} + f_{4,5}) & - & (x_{2,1} \cdot \frac{p_{2,4}}{p_{2,1}} + f_{5,4}) & = & S_4 \\
\textit{node } 6 & (f_{6,2} + f_{6,5}) & - & (x_{2,1} \cdot \frac{p_{2,6}}{p_{2,1}} + f_{5,6}) & = & S_6
\end{array}
\qquad (6)
$$

with variables $x_{1,3}$, $x_{2,1}$, S_2, S_4 and S_6.

From the above mentioned example, it should be clear then, that the Sensor Location problem, we focus on, consists in finding a subset of nodes of minimum size such that the associated Flow Conservation System has a unique solution.

Such a problem was formally stated in [1], where two heuristics giving lower and upper bounds to the solution value were also presented. Successively in [2], a combinatorial analysis of the problem was developed and the computational complexity of the problem was studied in different special cases. Moreover, some graph classes, where the problem is polynomially solvable, were presented. Finally, in [5], the problem was further studied and new heuristic algorithms and approximation ones were presented. In this paper we continue the study of the problem by developing an exact branch and bound approach, based on a binary branching rule, that embeds the heuristics presented in [1] to obtain bounds to the solution value. Moreover, we develop a metaheuristic approach to solve the problem. In particular, due to the characteristics of the problem, whose solution is naturally represented by a binary vector, we applied a genetic approach whose efficiency has been showed in the literature to solve 0-1 combinatorial problems (see for example recent applications to network design problems [4], [11] and related sensor location problems [18]). Preliminary results are provided.

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