Probabilistic models for travel time

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1. Introduction

The future is uncertain, apart from some - mainly uninteresting – certainties. Still most scientists and consultants make and present forecasts as certain. They ignore or eliminate the uncertainties by using deterministic models or by keeping only expectation values as relevant.
When deterministic models are calibrated, the common assumption is that the observations in reality are partly explained by the model and that the unexplained part of the observations is 'noise'. In many cases this 'noise' is 'deafening' and one might admire the model developers for their courage to maintain that there is still regularity in the observations that they use for the calibration of their models.

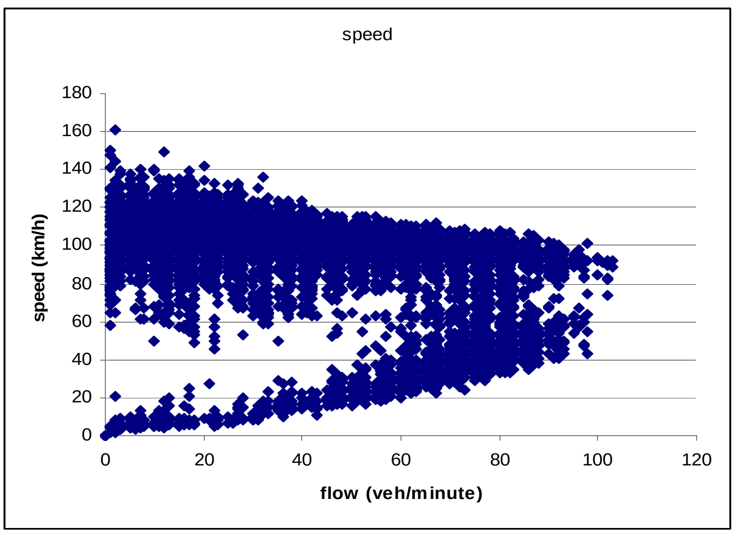


Figure 1 The 'fundamental diagram' with noise or a probabilistic process?

The fundamental diagram for traffic flows is an example of a traffic model with extensive noise. One might wonder whether this 'noise' would not have an intrinsic meaning, that it creates phenomena that are fundamental for traffic flow. As will be shown, this noisy characteristic of traffic flow gives an explanation for the probability distributions for travel time and consequently for the intrinsic uncertainty of travel time on freeways. This travel time uncertainty certainly exists and appears to be even rather regular (Tu et al. 2006).
Travel time is a typical example for which the expectation value alone does not contain sufficient information. Apart from the probabilistic character of traffic flows at freeways, there is another case for uncertainty in travel time, especially for urban trips: the probabilistic character of the delay at intersections. The travel time in urban networks is determined for about 50% by the driving time and about 50% by the delay at intersections. Driving time on urban roads is rather deterministic. The main cause of variation in travel time comes from

queues and the delay at intersections. The delay depends on the arrival in the cycle, the length of the queue and the green and red times (van Zuylen and Viti 2007). Since there is uncertainty about queue length at the moment of arrival and the color of the traffic light at that time, there is an intrinsic uncertainty of delays, as shown in figure 2.

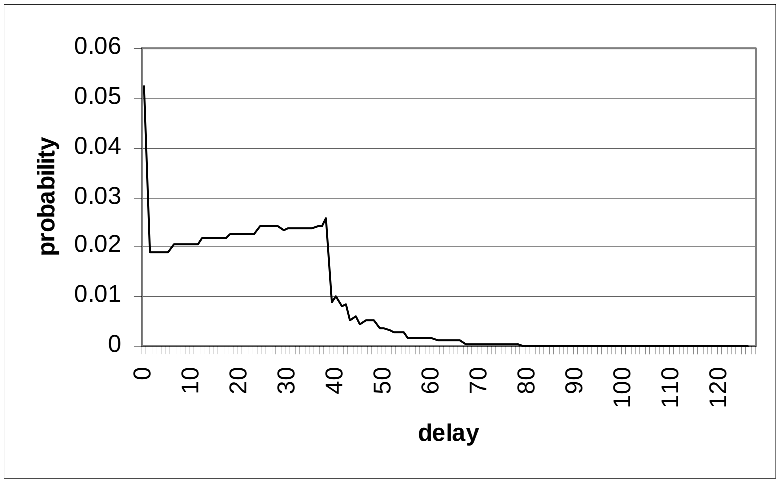


Figure 2 Probability distribution of delays at a (fixed time) controlled intersection (van Zuylen 2006)

Predicting travel time is important for the improvement of the quality of traveling, but one can not deny that there remains a basic uncertainty in the travel time because of an uncertainty in the behavior of the traffic system.
The limitation of causal, deterministic process models is not unique for traffic. In physics it is known for more than a century that causal laws are only applicable for a part of the macroscopic world. The physics on a (sub) atomic level is not deterministic but probabilistic: one can not predict the position of a particle, just a probability distribution. Every measurement has an impact on this probability distribution. Something similar is the case in travel time prediction and predictions of queue lengths, as will be shown in this paper.
The next section explains the probabilistic models for delays at an intersection. It shows that the uncertainty due to stochastic arrivals causes that the future queues can only be predicted as probability distributions and that the classical deterministic models can at best predict the dynamics of the expected queues and delay. Section 3 discusses how the noisy flow-speed diagram is the basic explanation of travel time unreliability. The last section gives a discussion.

2. Probabilistic queues at controlled intersection

Queues at controlled intersections grow during the red phase and diminish during the green phase. If on the average the green phase is long enough to deal with the traffic, the queue at the end of the green phase should have been disappeared. However, due to stochastic fluctuations in the arrival process, a finite probability exists that still a queue remains at the end of green phase. This process has been modeled by e.g. Olszevsky (1990), van Zuylen (1985), and Wu (1990). For simplicity we will limit the analysis to fixed time control, although extension of the theory to traffic actuated control has also been made (Viti and van Zuylen, 2005). The probability $P(m,j)$ of a queue length of m at the end of the green phase of cycle j can be calculated from the probability distribution of the previous cycle $P(m,j\text{-}1)$ and the arrival distribution given by p_l , the probability that l vehicles arrive in this cycle

$$P(m, j) = \sum_{l=0}^{j+s.t_g} p_l P(m-1, j-l+s.t_g) \qquad (1)$$

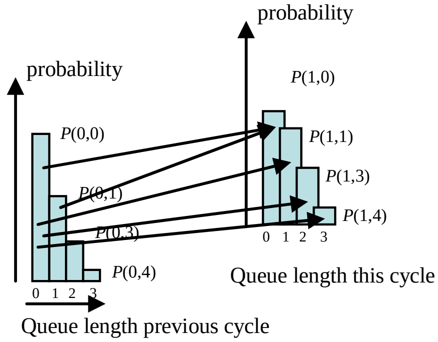


Figure 3 Markov chain process for the probability distribution of the queue length.

If we start with the certainty of a queue with a certain size, the queue size in the future becomes uncertain because the single valued distribution changes in a more dispersed one. The further forward we look, the more uncertain the queue distribution becomes until after some time the distribution reaches equilibrium. It is obvious that as soon as the queue is observed at a certain cycle, the probability distribution changes and the dynamics of the probability distribution will differ until again equilibrium has been established.

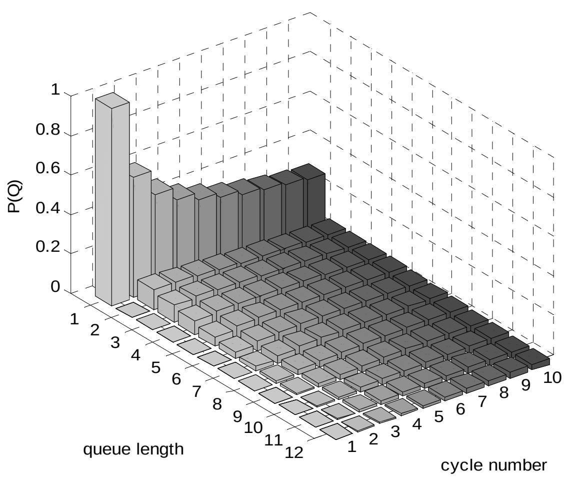


Figure 4 Dynamics of the probability distribution of the queue size after an initial certain zero queue (Viti 2006)

The situation becomes more uncertain if we include the arrival time of a traveler during the cycle. The longest travel time is experienced by a driver arriving in the start of the red phase or at the moment that the queue is too long to be served in the next green phase, as shown by van Zuylen and Viti (2007). Depending on the under- or oversaturation of the green phase the delay probability distribution becomes for undersaturated green phases

$$P^W(W) = \alpha\delta(W) + \beta \text{ for } 0 \le W \le t_r + \{Q_0+1\}/s \qquad (2)$$

were $\alpha = 1 - (t_r + \{Q_0+1\}/s) / \{C(1 - q / s)\}$ and $\beta = \{C(1 - q / s)\}^{-1}$. The delta function is defined by $\delta(x) = 0$ if $x \neq 0$, $\delta(0) = \infty$, and

$$\int_{-\infty}^{+\infty} \delta(x - x_0)\, f(x)\, dx = f(x_0) \qquad (3)$$

For oversaturated green phases with an initial queue Q_0 the delay distribution becomes

$$P^W(W\,|Q_0) = \{C(1\text{-}\ q/s)\}^{-1}\{B(W,\ C + (Q_0 + 1\ - t_g.s)/q\ ,\ t_r + (Q_0+1)/s) +$$

$B(W, 2\, t_r + Q_0\, /s – C\, (1 – q/s), C+ t_r + (Q_0 +1 – t_g.s)/q)\}$ (4)

where the block function $B(W,W_1,W_2)$ is defined by

$B(W, W_1,W_2\,) = 1$ for $W_1 < W < W_2$ and 0 if W has other values

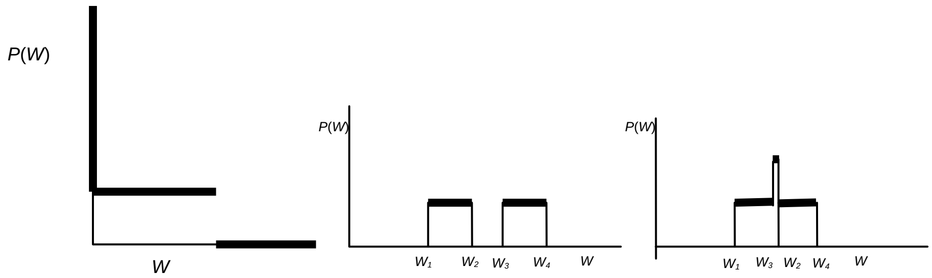
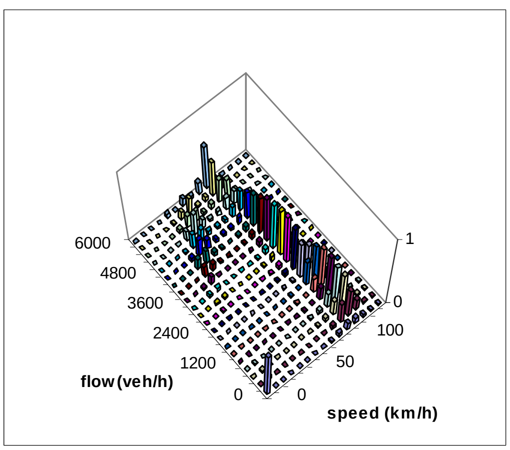


Figure 5 Probability distribution of delays at an intersection with undersaturated (left) and oversaturated green phases (middle and right) (van Zuylen and Viti 2007)

If also the uncertainty of the initial queue is included the probability distribution becomes smoother and more spread out as shown in figure 2.
Replacing these distributions with just the average or expectation value, ignores the fact that travel time is not deterministic and certainly not well represented by the expectation value alone.

3. Travel time distribution on freeways

On freeways the travel time depends on the speed with which vehicles can drive. As shown in figure 1 the speed does not have a unique, well defined value, but it has a probability distribution which depends on e.g. the flow (or the density) on the road.



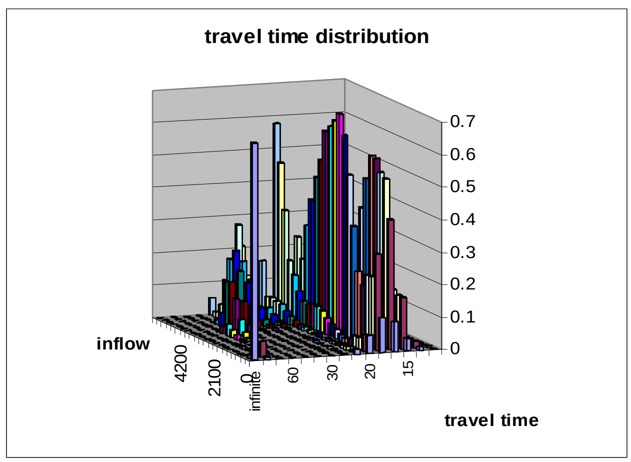
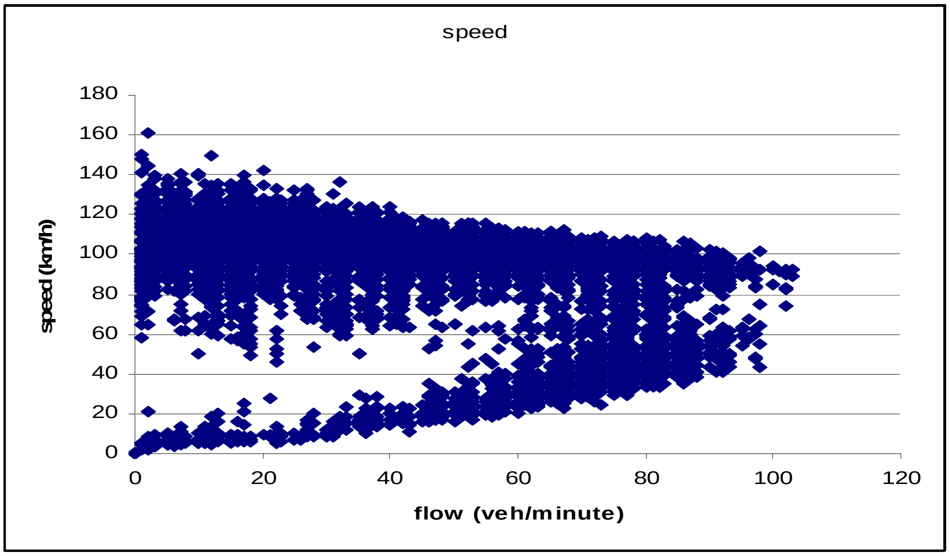


Figure 6 Probability distribution of speeds and travel time

For the same road also the travel time distribution has been determined (r.h.s. of figure 6) Analyzing this further in terms of travel time reliability we can combine a graphical representation of the flow-speed diagram and the percentiles of the travel time as a function of the inflow of the freeway.



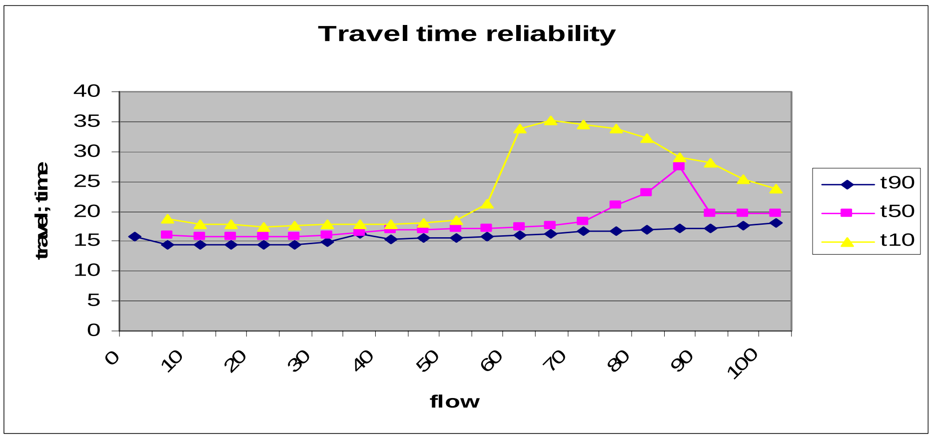


Figure 7 The flow-speed diagram combined with the percentiles of the travel time, showing the impact of the region of synchronized flow on travel time reliability.

The unreliability of travel times is often expressed as the difference between the 'worst' travel time (t_{10}, the travel time that is exceeded by only 10% of the trips) and the 'best' travel time (t_{90}, the travel time that is shorter than 90% of the trips) (Lomax 2003, van Lint and van Zuylen 2005). As can be seen from figure 8, the flow regime where the unreliability of travel time is largest coincides with the regime where in the 'fundamental diagram' the noise is largest. This corresponds to the regime of the 'synchronized flow' (Kerner 2004). The conclusion that can be drawn that the so-called 'noise' in the measurements of states in the fundamental diagram is not just caused by random errors, but is something more fundamental, giving rise to the phenomenon of unreliable travel times.

4. Discussion

Modeling traffic phenomena as deterministic, causal processes where differences between observed traffic behavior and models are considered as errors, gives a poor view on reality. The variability around the assumed model outcomes is as fundamental as the models themselves. Maybe, all traffic processes should be considered as probabilistic processes, for which it is better to search for appropriate models for the probability distribution than for causal models for the expectation value of certain characteristics only.

In the case of delays at controlled intersection, the probabilistic character is caused by the stochastic arrival process of vehicles at the intersection. In the case of the fundamental diagram the causes of the variability is less clear. A preliminary analysis of the dynamics of the states characterized by certain speeds and flows shows that there is a tendency of the traffic states to move from the outer region of the speed-flow space towards the lines that are assumed to represent the fundamental diagram. There is some similarity of the paths followed by the traffic system with the dynamics of chaotic systems that are moving towards attractors (Albert 1995, Prigogine and Stengers 1984).

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